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QUANTUM MODEL OF A LASER WITH NONLINEAR ABSORPTION

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Fluctuations of the amplitude and phase associated with spontaneous emission are investigated in a laser with nonlinear absorption. The noise intensity in such a laser turns out to be substantially larger than in an ordinary laser. Near the hysteresis threshold the fluctuations may lead to an instability of generation. A statistical description of the laser radiation in the hysteresis region is equivalent to the picture of a liquid–gas transition near the critical point.

1. INTRODUCTION

In the present article the general theory of quantum fluctuations of laser radiation[1-5] will be applied to the laser with nonlinear absorption which was experimentally studied in the articles by Lisitsyn and Chebotaev.[6,7] In these articles a He-Ne laser was investigated which had inside the resonator, together with the active cell, an absorbing cell containing neon. A theoretical analysis of the operation of a nonlinear laser is carried out in the article by Rautian and the authors,[8] where in particular the effect of noncoincidence of the points of initiation and cessation of generation associated with a change of the populations in the active and passive parts of the laser, which was observed in[6], is explained.

Here we consider "quantum noise," i.e., fluctuations associated with spontaneous emission of the amplitude and phase in a laser with nonlinear absorption. It turns out that the intensity of noise in such a laser is generally higher than in an ordinary laser. In addition, near the hysteresis threshold the quantum fluctuations may lead to an instability of the radiation.

If the investigation is limited to the case, characteristic of a He-Ne laser, when the photon lifetime $1/\nu$ in the resonator is large in comparison with the lifetime τ of the excited atom,

$$\nu\tau \ll 1, \tag{1}$$

then the problem reduces to an investigation of the Fokker-Planck equation for the photon distribution function $\rho(z, t)$ (a bar denotes complex conjugation)

$$\frac{\partial\rho}{\partial t} = 2\nu\left\{\frac{\partial}{\partial\xi}\left(A\rho + B\frac{\partial\rho}{\partial\xi}\right) + \frac{1}{\xi}\frac{\partial}{\partial\varphi}\left(C\rho + \frac{D}{\xi}\frac{\partial\rho}{\partial\varphi}\right)\right\}, \tag{2}$$

$$z = \sqrt{\xi}e^{i\varphi}, \quad \bar{z} = \sqrt{\xi}e^{-i\varphi}.$$

The distribution function $\rho(z, t)$ of a quantum oscillator in the representation of coherent states is the weight function in the expression for the density matrix $\hat{R}$ of the oscillator if it is represented in diagonal form[9]

$$\hat{R} = \int d^2z\, \rho(z, t)\, |z\rangle\langle z|. \tag{3}$$

In Eq. (2) the coefficients $A(\xi)$ and $C(\xi)$ are classical quantities which determine the active and reactive parts of the radiated power, and the coefficients $B(\xi)$ and $D(\xi)$ are associated with quantum fluctuations and describe the diffusion of photons in the z plane in the radial and azimuthal directions. The explicit form of these coefficients was found in the authors' article[5] for the simplest model of a laser in which the atoms were assumed to be fixed and their relaxation was described by a single relaxation time τ. Taking the thermal motion of the atoms and the difference in relaxation times (longitudinal and transverse) into account is not a major factor since it only leads to a certain renormalization of the coefficients a, b, and c (see Eqs. (9)).

Expanding the oscillator distribution function in a Fourier series

$$\rho(z, t) = \frac{1}{\pi}\sum_{m=-\infty}^{+\infty}\rho_m(\xi, t)\, e^{-im\varphi}, \quad \rho_m = \bar{\rho}_{-m}, \tag{4}$$

it is not difficult to see that in this expansion the zero harmonic $\rho_0(\xi, t)$ determines the distribution of photons. The average number of photons n and the dispersion Δn are related to $\rho_0(\xi, t)$ in the following way:

$$n = \int_0^\infty d\xi\, \xi\rho_0(\xi, t), \quad \int_0^\infty d\xi\, \rho_0(\xi, t) = 1, \tag{5}$$

$$(\Delta n)^2 = \int_0^\infty d\xi\, \xi^2\rho_0(\xi, t) - n^2 + n. \tag{6}$$

Near the generation threshold (it is precisely this case we consider later) $\Delta n \gg \sqrt{n}$, and $\rho_0(\xi, t)$ becomes the usual distribution function of the photons. In the course of time all harmonics of the distribution function except $\rho_0(\xi, t)$ are damped due to the phase fluctuations.

In the second section of this article the steady state distribution function $\rho(\xi)$ of the photons is found which, in the regime of hysteresis, has two sharp maxima corresponding to classically stable states with n = 0 and n ≠ 0. Because of quantum fluctuations these states are metastable. In the third section the time for a transition from one metastable state to the other is estimated.

2. DISTRIBUTION FUNCTION OF THE PHOTONS

For maximum simplification of the formulas we will assume that in the active cell in the absence of radiation all atoms are in the upper level and their number is N_1, but in the passive cell N_2 atoms are in the lower level, and generation takes place in the center of the line. The relaxation time in the active cell is τ_1, and in the passive cell it is τ_2. As is well known,[8] for the appearance of hysteresis it is necessary that τ_2 be much larger than τ_1.

In a rigorously stationary state, when no probability current is present, from Eq. (2) we obtain

$$A(\xi)\rho(\xi) + B(\xi)d\rho(\xi)/d\xi = 0. \tag{7}$$

Near the generation threshold we approximately have the following expressions[5] for the coefficients A and B:

$$A(\xi) = -\xi(a\xi^2 - b\xi + c), \quad B(\xi) = \xi\eta_1; \tag{8}$$

$$a = \eta_1\beta_1^2 - \eta_2\beta_2^2, \quad b = -\eta_2\beta_2 + \eta_1\beta_1, \quad c = \eta_1 - \eta_2 - 1,$$
$$\eta_i = \beta_i N_i / 4\nu\tau_i, \quad \beta_i = 2\tau_i\omega d^2/\hbar V, \quad i = 1, 2. \tag{9}$$

Here d is the dipole moment of the operating transition at the frequency ω, V denotes the volume of the system, and β_i has the meaning of the saturation parameter in the i-th cell. Since the phenomena under consideration are more noticeable at small radiation power, here we have confined our attention to precisely such a case. Formulas (8) and (9) are obtained under the assumption of a weak saturation effect in both cells,

$$\beta_i\xi \ll 1. \tag{10}$$

As we see later, this is the condition for nearness to the generation threshold

$$|c| = |\eta_1 - \eta_2 - 1| \ll \beta_1/\beta_2. \tag{11}$$

We note that $\beta_1 < \beta_2$ since $\tau_1 < \tau_2$.

The quantity $-A(n)/n$ is the classical coefficient of gain. In order to describe hysteresis phenomena in $A(n)/n$ it is necessary to retain the quadratic term in the radiated power.[8] From Eq. (7) we obtain

$$\rho(\xi) = Z^{-1}e^{f(\xi)}, \quad f(\xi) = ({}^1/_3 a\xi^3 - {}^1/_2 b\xi^2 + c\xi)/\eta_1, \tag{12}$$

$$Z = \int_0^\infty d\xi\, e^{f(\xi)}.$$

In the region of classical generation $f(\xi)$ is large because $f(\xi) \sim \xi$. Therefore in this region it is important to know the extremal points of the function $\rho(\xi)$, which are determined from the condition

$$\frac{df}{d\xi} = a\xi^2 - b\xi + c = 0, \quad \xi_{1,2} = \frac{b \pm \Delta}{2a}, \tag{13}$$

$$\Delta = \sqrt{b^2 - 4ac}.$$

In order for hysteresis to exist, it is necessary that $\xi_1 \geq 0$ and $\xi_2 > 0$; hence follows the condition which determines the region of existence of hysteresis:[8]

$$a < 0, \quad -2\sqrt{ac} \leq b < 0, \quad c \leq 0. \tag{14}$$

Only the point $\xi_2 = (b - \Delta)/2a$ corresponds to a stable state, but the point $\xi_1 = (b + \Delta)/2a$ is unstable (from this state the system goes into one of the states with $\xi \approx \xi_2$ or $\xi \approx 0$).

In the classical domain the average number of photons is determined from the condition $A(n) = 0$. The dependence of n on η_1 is shown in Fig. 1, i.e., the dependence on the number of atoms in the active cell, for certain characteristic values of the parameter η_2 which denotes the number of atoms in the passive cell. Curve 1 corresponds to $\eta_2 = 0$, i.e., to ordinary generation. Curve 3 corresponds to the value $\eta_2 = \eta_2^0$ at which the coefficients b and c simultaneously vanish at the threshold of generation, and is the boundary between the curves of type 2 without hysteresis and the curves of type 4 with hysteresis. From the condition b = c = 0 we find the hysteresis threshold

$$\eta_1^0 = \frac{\beta_2}{\beta_2 - \beta_1}, \quad \eta_2^0 = \frac{\beta_1}{\beta_2 - \beta_1}. \tag{15}$$

Later it will be convenient to introduce the notation $\eta_1 = 1 + \eta_2 + \eta$ and $\eta_2 = \eta_2^0(1 + \theta)$. In this connection, in virtue of condition (11) the generation parameter η must be rather small: $|\eta| \ll \beta_1/\beta_2$. Condition (10) for the weakness of saturation means that we consider the case of small hysteresis when the distance between the points of triggering (η_3) and cutoff (η_c) of generation are small, and the condition $-1 \leq \theta \leq \theta_0 > 0$, $\theta_0 \leq 1$ must be imposed on the parameter θ. In terms of this notation the coefficients a, b, and c take the following form in the region of the hysteresis threshold:

$$a = -\beta_1\beta_2, \quad b = -\beta_1(\theta - \eta) \quad c = \eta. \tag{9$'$}$$

After these preliminary remarks, it is not difficult to appreciably simplify the distribution function (12) in the following three characteristic ranges of the parameter θ.

A. The region below the hysteresis threshold ($-1 \leq \theta < 0$). Curves 1 and 2 in Fig. 1 refer to this region. In this case the nonlinear term $a\xi^2$ in the gain coefficient is small, and one can neglect it. Thus, we have to deal with ordinary laser emission near the threshold for generation. Below the threshold for classical generation ($\eta < 0$, $\sqrt{\beta_1} \ll |\eta| \ll 1$) the distribution function has the usual Planck form

$$\rho(\xi) = n^{-1}e^{-\xi/n}, \tag{16}$$

$$n = |\eta|^{-1}, \quad \Delta n = n. \tag{17}$$

At the threshold for classical generation ($\eta = 0$) the distribution function has a Gaussian form:

$$\rho(\xi) = \frac{2}{\pi n}\exp\left\{-\frac{\xi^2}{\pi n^2}\right\}, \tag{18}$$

$$n = \sqrt{\frac{2}{\pi\beta_1}}\Big/|\theta|, \quad \Delta n = n\sqrt{\frac{\pi}{2} - 1}. \tag{19}$$

Finally, above the generation threshold ($\eta \gg \sqrt{\beta_1}$) we have

$$\rho(\xi) = \frac{1}{\sqrt{2\pi}\,\Delta n}\exp\left\{-\frac{(\xi - n)^2}{2(\Delta n)^2}\right\}, \tag{20}$$

$$n = \xi_2 = \frac{\eta}{\beta_1|\theta|}, \quad \Delta n = \sqrt{\frac{\eta_1}{\beta_1|\theta|}} = \sqrt{\frac{\eta_1}{\eta}}\,n. \tag{21}$$

For $\theta = -1$ formulas (18)–(21) agree with the corresponding well-known formulas for an ordinary laser. We note that in these three cases the width of the transition region with respect to η amounts to a quantity of the order of $\sqrt{\beta_1}$.

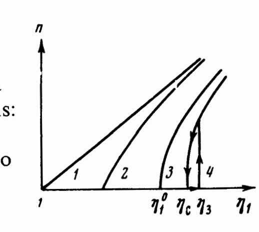

FIG. 1. Dependence of the number of photons on the number of active atoms for a fixed value of the number of absorbing atoms: curve 1 corresponds to $\eta_2 = 0$, curve 2 corresponds to $0 < \eta_2 < \eta_2^0$, curve 3 corresponds to $\eta_2 = \eta_2^0$, and curve 4 corresponds to $\eta_2 > \eta_2^0$.

B. At the hysteresis threshold ($\theta = 0$). In this case we have the Planck distribution (16) below the threshold for classical generation ($\eta < 0$, $\sqrt{\beta_1} \ll |\eta| \ll 1$).

At the threshold for classical generation ($\eta = 0$)

$$\rho(\xi)=\frac{1}{Z}\exp\left\{-\left(\frac{\xi}{\xi_0}\right)^3\right\},\quad \xi_0{}^3=\frac{3}{\beta_1(\beta_2-\beta_1)},\tag{22}$$

$$n=\frac{\Gamma(^2/_3)\,\xi_0}{\Gamma(^1/_3)},\quad \Delta n=\frac{\sqrt{\Gamma(^1/_3)-\Gamma^2(^2/_3)}}{\Gamma(^2/_3)}n\approx 0{,}68n.\tag{23}$$

Here $\Gamma(x)$ denotes Euler's gamma function. As is clear from here, the fluctuations in the number of photons are on the order of $\Delta n \sim \beta_1^{-1/3}$. In a He-Ne laser a typical value of the parameter β_1 is $\sim 10^{-6}$ so that the level of fluctuations at the generation threshold in case B is approximately an order of magnitude higher than in case A.

Above the generation threshold ($(\beta_1\beta_2)^{1/3} \ll \eta \ll 1$) we have the Gaussian distribution (20), where

$$n=\sqrt{\frac{\eta}{\beta_1\beta_2}},\quad \Delta n=\sqrt{\frac{\beta_2 n}{2(\beta_2-\beta_1)\eta}}.\tag{24}$$

If $\beta_2 \gg \beta_1$ then the fluctuations $(\Delta n)^2/n$ are two times smaller than in the usual case. For very similar values of the saturation parameters the quantity $(\Delta n)^2/n$ may be large, which is associated with an increase in the total number of atoms ($N_1 + N_2 \gg N_1 - N_2$). We note that the width of the transition region with respect to the parameter η now is of the order of $(\beta_1\beta_2)^{1/3}$, which is approximately an order of magnitude smaller than in case A.

C. The hysteresis region ($0 < \theta \ll 1$). Hysteresis corresponds to the curve of type 4 shown in Fig. 1. Generation appears at $\eta = \eta_3 = 0$ and ceases at $\eta = \eta_c = -\beta_1\theta^2/4\beta_2$, i.e., at the point where the discriminant Δ vanishes. Thus, in the hysteresis region the parameter η varies within the limits $\eta_c \le \eta \le 0$.

The dependence of the coefficient of gain, $-A(\xi)/\xi$, on ξ is shown in Fig. 2 for different values of η. The approximate shape of the distribution function $\rho(\xi)$ for $\eta_c < \eta < 0$ is shown in Fig. 3. This function has two maxima, at $\xi = 0$ and at $\xi = \xi_2$, and a minimum at the point $\xi = \xi_1$. As η approaches η_c the value of $\rho(0)$ increases but $\rho(\xi_2)$ decreases, and for $\eta \approx \eta_c$ the inequality $\rho(0) \gg \rho(\xi_2)$ is satisfied. In this connection the points ξ_1 and ξ_2 come together and coincide when $\eta = \eta_c$. The function $\rho(\xi)$ has an inflection at the point $\xi = \xi_1 = \xi_2$. When $\eta \to 0$ the value of $\rho(0)$ decreases, but $\rho(\xi_2)$ increases and at sufficiently small values of η the opposite inequality holds, $\rho(\xi_2) \gg \rho(0)$. Finally, for $\eta > 0$ the function $\rho(\xi)$ has only one maximum at the point $\xi = \xi_2$.

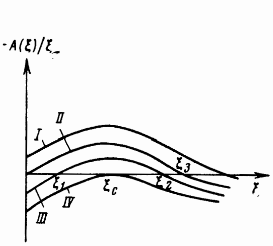

FIG. 2

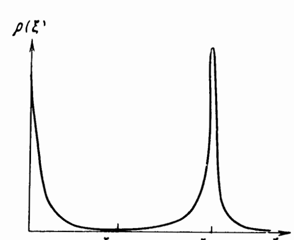

FIG. 3

FIG. 2. The dependence of the coefficient of gain on ξ in the hysteresis region. Curve I corresponds to $\eta > 0$; curve II corresponds to $\eta = 0$; curve III corresponds to $\eta_c < \eta < 0$; curve IV corresponds to $\eta = \eta_c$.

FIG. 3. Approximate shape of the distribution function $\rho(\xi)$ of the photons in the hysteresis region corresponding to the coefficient of gain given by curve III in Fig. 2.

In the hysteresis region, not too close to its boundaries, $\rho(\xi_1)$ is exponentially small relative to $\rho(0)$ and $\rho(\xi_2)$, and thus the distribution function $\rho(\xi)$ has two very sharp maxima which correspond to classical stationary states. The time for establishment of equilibrium inside the boundaries of each maximum is of the order of $1/\nu$ whereas the time for a transition from one state to the other is exponentially large (see formulas (31) and (32) below). Therefore one can regard each state as metastable with a long lifetime. For time intervals which are small in comparison with the lifetime of the state, it makes sense to talk about the statistical properties of this state (fluctuations in the number of photons, width of the emission line, etc.) Here only the region of classical generation ($\xi \approx \xi_2$) is of interest; the region below the generation threshold differs only slightly from case (17).

Since $|\eta| \le |\eta_c| \sim \theta^2 \ll \theta$ in the hysteresis region, then the coefficient $b = -\beta_1\theta$. The following formulas hold for n and Δn:

$$n=\left(\sqrt{|\eta_c|+\eta}+\sqrt{|\eta_c|}\right)/\sqrt{\beta_1\beta_2},\tag{25}$$

$$(\Delta n)^2={}^1/_2[\beta_2/\beta_1(\beta_2-\beta_1)^2(|\eta_c|+\eta)]^{1/2}\quad(\eta_c\leqslant\eta\gtrless 0).\tag{26}$$

In contrast to an ordinary laser, near the generation threshold (at which, according to Eq. (21), we have $(\Delta n)^2 = 1/\beta_1$ for $\theta = -1$ and $\eta_1 \approx 1$) the dispersion Δn of the distribution function now depends on the generation parameter η, i.e., on the radiated power. Upon approach to the cutoff point ($\eta = \eta_c$) the fluctuations increase markedly; however, formula (26) is obtained under the assumption that the fluctuations are sufficiently small ($\Delta n \ll \xi_2 - \xi_1 \sim [(|\eta_c| + \eta)/\beta_1\beta_2]^{1/2}$). Hence the restriction $|\eta_c| + \eta \gg (\beta_1\beta_2)^{1/3}$ follows. A small neighborhood of the cutoff point, where the concept of a metastable state is violated, is thereby excluded. Substituting $|\eta_c| + \eta \sim (\beta_1\beta_2)^{1/3}$ into formula (26), let us estimate the level of fluctuations near the cutoff point itself; it is not difficult to see that in this case $(\Delta n)^2$ exceeds the level of fluctuations in an ordinary laser by roughly $(\beta_1\beta_2)^{-1/6}$ times. Thus, for small θ the fluctuations in the hysteresis region are substantially larger than in an ordinary laser.

The increase of fluctuations in a nonlinear laser is due to two causes. In the first place it is associated with the increase of the coefficient of diffusion, which is proportional to the number N_1 of excited atoms. This effect is described by the factor $\beta_2/(\beta_2 - \beta_1)$ in formulas (24) and (26). The other cause consists in the fact that the slope of the coefficient of gain $-A(\xi)/\xi$ is decreased at the point $\xi = \xi_2$, especially in the hysteresis region.

As to the distribution function at $\xi \approx 0$, in this region $\rho(\xi)$ has the Planck form (16) and

$$n=\beta_2/(\beta_2-\beta_1)|\eta|,\quad \eta<0.\tag{27}$$

For very small $|\eta| \lesssim \sqrt{\beta_1}$ formula (27) is violated since Δn is comparable with ξ_1.

Now let us consider the system during a time interval which is large in comparison with the lifetimes of the metastable states $\xi \approx 0$ and $\xi_2 \approx 0$. Then obviously the probability of the state with generation is proportional to $\exp\{f(\xi_2)\}$, and the probability of the state without generation $\sim \exp\{f(0)\}$. A transition between these states occurs at that value $\eta = \tilde{\eta}$ at which $f(\xi_2) = f(0)$. From here we find

$$\tilde{\eta} = {}^3/_4 \eta_c. \tag{28}$$

This means that for a very slow variation of the parameter η the hysteresis disappears—generation appears and stops at the point $\eta = \tilde{\eta}$. This transition is represented in Fig. 4 by the solid vertical line.

Let us again consider the damping of the average field due to quantum fluctuations of the phase and the width of the generation line corresponding to this damping. The average field is determined by the harmonic $\rho_1(\xi, t)$ in the expansion (4). Solving Eq. (2) in the quasistatic approximation, we find the following width $\Delta\nu$ of the generation line:

$$\Delta\nu = \nu\beta_2 / 2\xi_2(\beta_2 - \beta_1).$$

In contrast to the dispersion of the photons' distribution function, the width of the generation line does not have any singularities associated with the approach to the cutoff point $\eta = \eta_c$, in the same way as it does not have singularities upon approach to the region of classical instability.[5]

3. LIFETIME OF A METASTABLE STATE

Now let us determine the time for a transition due to fluctuations from one metastable state to another. Formally this is equivalent to a determination of the time for passage of a Brownian particle through a potential barrier.[10] For this purpose let us consider an approximate solution of the diffusion equation (2) with nonvanishing probability current

$$j = -2\nu(A\rho + B d\rho / d\xi) \tag{29}$$

In the initial stage of the transition one can regard j as a constant quantity equal to T^{-1}, where T denotes the transition time. In order to be definite, let us consider a transition from the state $\xi \approx 0$ to the state $\xi \approx \xi_2$ (spontaneous initiation of generation). Integrating (29) with respect to ξ from n_0 up to ξ_2 (n_0 denotes the average number of photons in the state $\xi \approx 0$) and setting $\rho(\xi_2) = 0$, we obtain

$$T(0 \to 2) = \frac{n_0}{2\nu} \int_{n_0}^{\xi_2} d\xi \frac{e^{-f(\xi)}}{B(\xi)}. \tag{30}$$

The region $\xi \approx \xi_1$ introduces the major contribution to the integral (30). Evaluating the integral by the method of steepest descent we obtain ($x = \eta/|\eta_c|$)

$$T(0\to 2) = \frac{\sqrt{\pi}\beta_2^{5/4}\beta_1^{1/4}}{\nu(\beta_2 - \beta_1)|\eta_c|^{7/4}x^2} \times \exp\left\{\frac{|\eta_c|^{3/2}}{3(\beta_1\beta_2)^{1/2}}(1 + 2\sqrt{1+x})(1 - \sqrt{1+x})^2\right\}. \tag{31}$$

The exponential in this expression is assumed to be large. It is also not difficult to exactly find the time for a transition from the state $\xi \approx \xi_2$ to the state $\xi \approx 0$ (spontaneous cutoff of generation):

$$T(2\to 0) = \frac{\sqrt{\pi}\beta_2^{3/4}\beta_1^{-1/4}}{\nu(\beta_2 - \beta_1)|\eta_c|^{1/4}x}(1 + \sqrt{+x})\exp\left\{\frac{4}{3}\frac{|\eta_c|^{3/2}(1+x)^{3/2}}{(\beta_1\beta_2)^{1/2}}\right\}. \tag{32}$$

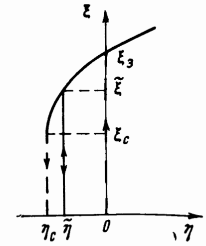

FIG. 4. Picture of the hysteresis region. Dependence of the generated power ξ on the generation parameter η. The dashed line with arrows indicates the region of classical hysteresis. The solid $\eta = \tilde{\eta}$ corresponds to the pulsed regime of the laser.

At the point $\eta = \tilde{\eta}$ the arguments of the exponentials in (31) and (32) coincide and here, of course $T(0 \to 2) \approx T(2 \to 0)$. Thus, at $\eta = \tilde{\eta}$ generation takes place in the pulsed regime, and the generation time approximately coincides with the time in the absence of generation. Such pulsations bear, of course, a random statistical character.

Now let us estimate the order of magnitude of the quantity T. According to formula (31) the triggering time is of the order of one second for the following values of the parameters: $\nu = 10^6$ Hz, $|\eta_c| = 10^{-2}$, $x = 1/3$, $\beta_1 = (1/2)\beta_2 = 10^{-6}$. From here it follows that in order to observe the spontaneous appearance (or disappearance) of generation it is necessary to get into the region of parameters very close to the hysteresis threshold ($|\eta_c| \lesssim 10^{-2}$). Such a task is apparently actually feasible. Thus, in article[11] the generation parameter of an ordinary laser was fixed near the generation threshold with an accuracy of the order of 10^{-2}.

4. CONCLUSION

The investigation carried out indicates that the fluctuations of the intensity of the radiation in a nonlinear laser in the hysteresis region are substantially larger than in an ordinary laser. This effect, which is associated with a decrease in the slope of the coefficient of gain, is especially noticeable at the point of cutoff of classical generation.

The statistical description of laser radiation in the hysteresis region is equivalent to the picture of a liquid—gas phase transition near the critical point.[12] Here the generation parameter plays the role of the pressure, and the radiation energy plays the role of the volume of the system (see Fig. 4). The metastable states in the hysteresis regime are analogous to superheated and supercooled phases. Thus, in the present article the fluctuations are calculated within the framework of the single-mode model of a phase transition.

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