DEPOLARIZATION OF NEUTRONS PASSING THROUGH A FERROMAGNET

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Depolarization of neutrons passing through a ferromagnetic sample in the saturated state is considered. The depolarization is caused by random magnetic fields produced by thermal motion of the atomic spins. In the most interesting case, when the neutron polarization is parallel to the external field, the depolarization is expressed in terms of the magnetization fluctuation correlator. It is shown that the depolarization is determined by fluctuations averaged over the quantum uncertainty of the neutron position in the beam; it thus depends strongly on the angular divergence of the beam and on its degree of monochromaticity. For a wide and nonmonochromatic beam, the depolarization is proportional to the total cross section for inelastic magnetic scattering. Formulas for depolarization are derived on basis of spin-wave theory, and it is noted that an experimental study of depolarization should permit one to determine the main parameters of the theory. Depolarization in the critical region is discussed qualitatively.

1. INTRODUCTION

AT the present time, the scattering of slow neutrons is widely used for the investigation of the dynamics of atomic and magnetic lattices in crystals. Usually the spectra of the phonons and of the spin waves are determined by investigating the angular and energy distributions of the scattered neutrons (see, for example[1]). In many cases, however, such a direct method is not applicable. Thus, it is impossible to investigate by this method the long-wave part of the magnon spectrum, since the change of the neutron energy is negligible[2]. For the same reason, it is impossible to carry out an energy analysis of the critical scattering of neutrons near the Curie point, where the average transferred energy is also very small (see[3]). In all such cases, one studies only the angular distribution of the scattered neutrons, which is then compared with the distribution calculated on the basis of some model.

In this paper we discuss a new method of investigating the dynamics of a spin system in magnetic crystals, by studying the depolarization of neutrons passing through the sample.

Assume that a beam of polarized neutrons is incident on a ferromagnetic crystal situated in an external magnetic field **H**. The neutron polarization vector $\mathbf{P}_0$ is parallel to **H**, and we are interested in the polarization of the transmitted beam.

If the magnetic field is weak, then the ferromagnet consists of a large number of domains in which the magnetization **M** is directed to different sides. In each such domain, the spin of the neutron is rotated around the induction vector **B** through a certain angle, and in the case of a sufficiently long sample, the beam is completely depolarized. This question was investigated in 1941 by Halpern and Holstein[4] in detail, and will not be discussed further here.

We now consider the case of a strong external field ($H > 4\pi M$), such that the sample is magnetized to saturation, i.e., there are no domains. At first glance it seems that the polarization of the transmitted beam should be the same as that of the incident beam, since the neutron is acted upon by a constant field $\mathbf{B}_0 \parallel \mathbf{P}_0$. In fact, however, this is not so, since the ferromagnet still contains random magnetic fields due to the thermal motion of the spins. Obviously, the action of these random fields on the polarized neutrons should lead to a certain depolarization of the transmitted beam. As will be shown below, a study of this fluctuation depolarization as a function of the external field, the temperature, and the neutron energy makes it possible to obtain in a number of cases the same information as an investigation of the scattering of neutrons through small angles. The advantage of the proposed method is that it is much simpler to measure the polarization of the transmitted beam than to measure the small-angle neutron scattering.

We note also that this method obviously is applicable not only in the case of ferromagnets, but also for arbitrary magnetic crystals.

2. GENERAL FORMULAS FOR DEPOLARIZATION

The equation of motion for the neutron polarization vector $\mathbf{P} = \langle \sigma \rangle$ is[5]

$$\frac{d\mathbf{P}}{dt} = g_n[\mathbf{PB}], \qquad (1)^*$$

where $g_n = 2\mu_n$ ($\hbar = 1$), μ_n is the magnetic moment of the neutron, and $\mathbf{B}(t)$ is the magnetic induction of the sample at the location of the neutron. The vector **B** obviously consists of two parts, the constant field $\mathbf{B}_0 = \langle \mathbf{B} \rangle$ and the alternating field $\mathbf{b} = \mathbf{B} - \mathbf{B}_0$, due to the fluctuations (obviously, $\langle \mathbf{b} \rangle = 0$). The field $\mathbf{B}_0$ causes rotation of the vector **P** around $\mathbf{B}_0$ with frequency $\omega_0 = 2\mu_n B_0$, and the length of **P** and its projection on $\mathbf{B}_0$ are conserved. As will be shown below, the action of the fluctuation addition $\mathbf{b}(t)$ leads to depolarization, i.e., to a decrease of the vector $\mathbf{P}(t)$.

Let us rewrite Eq. (1) in integral form

$$\mathbf{P}(t) = \mathbf{P}_0 + g_n \int_0^t dt' [\mathbf{P}(t')\mathbf{B}(t')]. \qquad (2)$$

*$[\mathbf{PB}] \equiv \mathbf{P} \times \mathbf{B}$.

We choose as the time reference the instant of entry of the neutron into the sample. We solve this equation by successive approximations:

$$\mathbf{P} = \mathbf{P}_0 + g_n \int_0^t dt' \langle [\mathbf{P}_0 \mathbf{B}(t')] \rangle + g_n^2 \int_0^t dt' \int_0^{t'} dt'' \langle [[\mathbf{P}_0 \mathbf{B}(t'')] \mathbf{B}(t')] \rangle + \ldots \quad (3)$$

In this equation, in order to obtain a physically observable value of the polarization, we averaged over the states of the sample. We confine ourselves below only to the experimentally most interesting situation, when $\mathbf{P}_0 \parallel \mathbf{B}_0$, and consider two cases, when the neutron velocity v is either parallel or perpendicular to $\mathbf{B}_0$. The depolarization is then expressed only in terms of the correlator of the transverse fluctuations $\mathbf{b}_\perp(t)$ (those perpendicular to $\mathbf{B}_0$):

$$\frac{P - P_0}{P_0} = -\frac{g_n^2}{2} \int_0^t dt' \int_0^{t'} dt'' \langle \mathbf{b}_\perp(t') \mathbf{b}_\perp(t'') + \mathbf{b}_\perp(t'') \mathbf{b}_\perp(t') \rangle. \quad (4)$$

The macroscopic treatment of the magnetization fluctuations implies that the vector $\mathbf{b}(\mathbf{r}, t)$ is classical. In order for formula (4) to be applicable in the quantum region, where the vector $\mathbf{b}$ is regarded as an operator (see the Appendix), the integrand in (4) has been written out in symmetrized form.

Thus, the depolarization of a neutron beam passing through a magnet is determined by the spatial and temporal correlations of the transverse components of the magnetic induction along the neutron trajectory. If the time of travel of the neutron through the sample is large compared with the characteristic times of these correlations of the magnetization, then the degree of the depolarization of the transmitted beam is proportional to the length of the sample L, and can be represented in the form

$$\frac{\Delta P}{P_0} = \frac{g_n^2 L}{2v} \int_{-\infty}^{\infty} dt \langle \mathbf{b}_\perp(t) \mathbf{b}_\perp(0) \rangle, \quad (5)$$

where v is the neutron velocity. Representing the integrand of this formula in the form of a Fourier integral, we obtain

$$\frac{\Delta P}{P_0} = \frac{g_n^2 L}{2v} \frac{1}{(2\pi)^3} \int d\mathbf{q} [K_{xx}(\mathbf{q}, \mathbf{qv}) + K_{yy}(\mathbf{q}, \mathbf{qv})],$$
$$K_{\alpha\beta}(\mathbf{q}, \omega) = \int d\mathbf{r}\, dt\, e^{-i(\mathbf{qr} - \omega t)} \langle b_\alpha(\mathbf{r}, t) b_\beta(0, 0) \rangle; \quad (6)$$

The z axis is chosen here along the field $\mathbf{B}_0$. In the derivation of this formula, we took into account the fact that $\mathbf{r} = \mathbf{v}t$ along the neutron trajectory.

So far we have regarded the neutron as a classical particle having a definite trajectory. Actually, in experiments with thermal neutrons, one uses beams characterized by a momentum $\mathbf{p}$, by an energy uncertainty (nonmonochromaticity) ΔE, and an angle width ϑ_0. The neutrons in such a beam must be described by means of a wave packet with a width $\Delta l \sim p^{-1} E/\Delta E = \lambda\!\!\!^{-} E/\Delta E$ along the beam direction and with transverse dimensions $\Delta\rho \sim 1/p\vartheta_0 = \lambda\!\!\!^{-}\vartheta_0$. In other words, the uncertainty of the beam-neutron momentum is $\Delta p_l \sim p\Delta E/E$ in the longitudinal direction and $\Delta p_\perp \sim p\vartheta_0$ in the transverse direction. Obviously, formula (6), which is obtained on the basis of the classical considerations, can be used only when the packet is small compared with the dimensions of the magnetization fluctuations, i.e., the quantities q_x, q_y, and q_z which are important in the integral (6) are small compared with Δp_l and $\Delta p_\perp$. We shall call such a beam nonmonochromatic and broad. If this condition for the classic treatment of the beam is not satisfied, the fluctuation field $\mathbf{b}(\mathbf{r}, t)$ must be averaged over the wave packet of the neutron, and consequently it is necessary to limit in a definite manner the region of integration with respect to $\mathbf{q}$ in formula (6). This is done by introducing a cutoff factor $\Delta^2(\mathbf{q})$, which characterizes the shape of the packet in momentum space (see Appendix I).

In particular, for a narrow nonmonochromatic beam with angular divergence ϑ_0 it is necessary to make the following substitution in (6):

$$\int d\mathbf{q} F(\mathbf{q}, \mathbf{qv}) \rightarrow \int \Delta^2(\varkappa) d\varkappa \int dq_l F(q_l, q_l v) \approx \pi (p\vartheta_0)^2 \int dq_l F(q_l, q_l v), \quad (7)$$

where κ is the part of the vector $\mathbf{q}$ perpendicular to the beam. The physical cause of the dependence of the depolarization on the form of the beam is connected with the fact that the spin of the neutron in the magnet is acted upon by the fluctuation magnetic field smoothed over its wave packet. For a narrow beam the uncertainty of the position of the neutron in a plane perpendicular to the beam is large, and therefore it is acted upon by fluctuations averaged over the dimensions of this uncertainty. On the other hand, if the beam is broad, then the uncertainty of the position is small, and the depolarization is due to the non-averaged fluctuations, so that naturally the depolarization is much larger in this case than for a narrow beam.

We have thus expressed the depolarization of neutrons in a ferromagnet in terms of the correlation function of the transverse components of the magnetic induction $\mathbf{b}_\perp(t)$. On the other hand, as is well known[1], the cross section for the magnetic scattering of neutrons is expressed in terms of the correlator of the atomic spins. Using the usual formula[1] and recognizing that the Fourier component of the induction $\mathbf{b}_q$ is connected with the corresponding component of the magnetization vector $\mathbf{m}_q$ by the formula[6]

$$\mathbf{b}_\mathbf{q} = 4\pi \left[\mathbf{m}_\mathbf{q} - \frac{\mathbf{q}(\mathbf{q}\mathbf{m}_\mathbf{q})}{q^2} \right], \quad (8)$$

we can represent the total cross section of the magnetic inelastic scattering of unpolarized neutrons through small angles ($qa \ll 1$, $\vartheta_{max} \lesssim \lambda\!\!\!^{-}/a$, where a is the lattice constant) in the form[6]

$$\sigma = \frac{g_n^2 \mathscr{V}_0}{4v} \frac{1}{(2\pi)^3} \int d\mathbf{q}\, K_{\alpha\alpha}(\mathbf{q}, E_\mathbf{p} - E_{\mathbf{p}-\mathbf{q}}). \quad (9)$$

Here $\mathscr{V}_0$ is the volume of the unit cell, and E_p and E_{p-q} are the energies of the incident and scattered neutrons.

The integrals in (6) and (9) differ in two respects: first, because of the additional term in (9) with the longitudinal correlator K_{zz}, and second, because the energy arguments $\mathbf{q} \cdot \mathbf{v}$ and $E_p - E_{p-q} = \mathbf{q} \cdot \mathbf{v} - q^2/2m$ are not equal. However, if the neutron energy is sufficiently high, so that the main contribution in the integral of (10) is made by $q \ll p$, the second difference is of no importance. Further, if we consider the paramagnetic phase (a ferromagnet or an antiferromagnet above the Curie point, or else a paramagnet), then obviously $K_{xx} = K_{yy} - K_{zz}$, and by comparing (6) with (9) we obtain

$$\frac{\Delta P}{P_0} = \frac{4}{3} \frac{L\sigma_{par}}{\mathscr{V}_0}, \quad (10)$$

where σ_{par} is the cross section for scattering in the paramagnet. In the ferromagnetic phase there is no such a simple connection between the depolarization and cross section. We can only state that

$$\Delta P / P_0 \sim L\sigma / \mathscr{V}_0. \tag{11}$$

3. CALCULATION OF DEPOLARIZATION IN ACCORDANCE WITH SPIN WAVE THEORY

Taking (8) into account, we can express the depolarization in terms of the correlation functions of the magnetic-moment density fluctuations

$$\frac{\Delta P}{P_0} = \frac{g_n^2 L}{4\pi v} \int dq\, \Delta^2(q)\, \{(1+e_z^4)[G_{+-}(q, qv) + G_{-+}(q, qv)]$$
$$-(1+e_z^2)[e_-^2 G_{++}(q, qv) + e_+^2 G_{--}(q, qv)] + 4e_z^2(1-e_z^2) G_{zz}(q, qv)\},$$
$$G_{\alpha\beta}(q, \omega) = \int dr\, dt\, e^{-i(qr-\omega t)} \langle m_\alpha(r, t) m_\beta(0, 0)\rangle, \tag{12}$$

where $\mathbf{e} = q/q$ and $a_\pm = a_x \pm i a_y$. We have introduced into this formula the factor $\Delta^2(q)$, which takes into account the form of the wave packet of the incident neutrons (see Appendix I). At the present time, the correlators $G_{\alpha\beta}$ can be reliably calculated in a wide interval of temperatures only with the aid of spin-wave theory, in a form recently proposed by Vaks, Larkin, and Pikin[7]. Since the polarization is due to the long-wave spin waves, we use an improved variant of this theory, which takes into account not only the exchange interaction of the spins but also the dipole-dipole interaction (see[8] and[9]). As a result, we obtain after simple calculations

$$\frac{\Delta P}{P_0} = \frac{2g_n^2 \mu M L}{v} \int dq\, \Delta^2(q) \frac{A_q}{\varepsilon_q}\left(n_q + \frac{1}{2}\right) \delta(\varepsilon_q - qv)$$
$$\times \left[1 + \cos^4\vartheta_q - \frac{4\pi\mu M}{A_q} \sin^4\vartheta_q (1 + \cos^2\vartheta_q)\right], \tag{13}$$
$$A_q = Aq^2 + 2\mu H + q 4\mu M \sin^2\vartheta_q,$$
$$\varepsilon_q = [(Aq^2 + 2\mu H)(Aq^2 + 2\mu H + 8\pi\mu M \sin^2\vartheta_q)]^{1/2}.$$

Here $n_q = (\exp(\epsilon_q/T) - 1)^{-1}$, ϵ_q is the energy of the spin wave, M(T) is the saturation magnetization at the temperature T, A(T) is a temperature-dependent renormalized exchange-interaction constant, connected with the effective mass of the spin wave calculated in[7-9] by the equation $A^{-1}(T) = 2m_{eff}(T)$, ϑ_q is the angle between q and B_0. We have left out from (13) the term with G_{zz}, which enters in (12). It is shown in Appendix II that this term is small and can be neglected.

The fact that (13) contains a δ-function of $\epsilon_q - \mathbf{q}\cdot\mathbf{v}$ admits of a simple physical interpretation. Namely, the depolarization is due only to those spin waves for which the component of the phase velocity in the direction of motion of the neutron coincides with the neutron velocity. It follows from the condition $\epsilon_q = \mathbf{q}\cdot\mathbf{v}$ that for sufficiently slow neutrons expression (13) vanishes, since $\epsilon_0 \neq 0$. Thus, there should exist a certain threshold value of the neutron energy E, below which there is no depolarization in order of the calculation.

In the cases considered by us, when (i) the velocity v of the neutrons is parallel to B_0 and (ii) the velocity v is perpendicular to B_0, simple calculations similar to those contained in a paper by one of the authors[2] 1) lead to the following values of the threshold energy:

$$E_\parallel = 2\mu H\alpha, \quad (\mathbf{v} \parallel \mathbf{H})$$
$$E_\perp = \mu H\alpha\left(1 + \frac{2\pi M}{H} + \sqrt{1 + \frac{4\pi M}{H}}\right), \quad (\mathbf{v} \perp \mathbf{H}), \tag{14}$$

where a = 2mA(T) and m is the neutron mass. We note that these values of the threshold energies for the depolarization coincide in the limit as $\alpha \gg 1$ with the expressions for the thresholds for scattering with emission or absorption of one spin wave[2]. Actually the spin-wave theory contains two parameters: M(T, H)—the saturation magnetization and A(T)—the renormalization constants of exchange interaction. The quantity M is determined experimentally, for example, from ordinary magnetic measurements, and it can be regarded as known. Therefore real interest attaches to a determination of A(T); in particular, a study of its temperature dependence and the comparison with the results of modern theory[7-9].

We shall now calculate the depolarization in a number of limiting cases. To this end, it is convenient to rewrite (13) in the following form:

$$\frac{\Delta P}{P_0} = CI(a, b), \quad C = \frac{4\pi g_n^2 L p^3}{v}\left(\frac{\mu M T}{\alpha E^2}\right) \approx 0.2 \frac{T^0}{\alpha E^0} L(\text{cm}) M_T(\text{kG}),$$
$$I(a, b) = \frac{1}{2\pi} \int dy \frac{(y^2 + a + {}^1/{}_2 b \sin^2\vartheta)\Delta_\parallel^2(y_\parallel)\Delta_\perp^2(y_\perp)}{(y^2 + a)(y^2 + a + b\sin^2\vartheta)}$$
$$\times\left[1 + \cos^4\vartheta - \frac{b \sin^4\vartheta(1 + \cos^2\vartheta)}{2(y^2 + a + {}^1/{}_2 b \sin^2\vartheta)}\right]$$
$$\times \delta\{2y\cos\psi - \sqrt{(y^2 + a)(y^2 + a + b\sin^2\vartheta)}\}, \tag{15}$$

where $a = 2\mu H\alpha/E$, $b = 8\pi\mu M\alpha/E$, $y = \alpha q/p$, and ψ is the angle between q(y) and v; the factor $\Delta^2(y)$ introduced above has been broken up here into two factors $\Delta_\parallel^2(y_\parallel)$ and $\Delta_\perp^2(y_\perp)$, describing the formula of the packet in a direction parallel and perpendicular to the neutron velocity, respectively. In addition, in the derivation of (15) we have replaced $n_q + 1/2$ by T/ϵ_q, which is permissible at sufficiently high temperatures ($T \gg E/\alpha$). It follows already from (15) that the depolarization is small, since $\alpha \sim 10$–100 and $I \sim 1$, and even under favorable conditions $\Delta P/P_0$ is of the order of several per cent.

Let us consider the first case, when $\mathbf{v} \parallel \mathbf{H}$. Then $\cos\psi = \cos\vartheta$; the integral in the expression for I can be readily evaluated, and we obtain

$$I(a, b) = 2\int_{y_-}^{y_+} dy \frac{y^3}{4y^2 + b(y^2 + a)} \frac{y^2 + a + {}^1/{}_2 b\sin^2\tilde\vartheta}{(y^2 + a)(y^2 + a + b\sin^2\tilde\vartheta)}$$
$$\times\left\{1 + \cos^4\vartheta - \frac{b}{2}\frac{\sin^4\tilde\vartheta(1 + \cos^2\tilde\vartheta)}{y^2 + a + {}^1/{}_2 b\sin^2\tilde\vartheta}\right\}\Delta_\parallel^2(y\sin\tilde\vartheta)\Delta_\perp^2(y\cos\tilde\vartheta),$$
$$y_\pm = 1 \pm \sqrt{1 - a}, \quad \cos^2\tilde\vartheta = \frac{(y^2 + a)(y^2 + a + b)}{4y^2 + b(y^2 + a)}.$$

In these formulas $a \leq 1$, corresponding to expression (14) for the threshold.

Near the threshold, when $(1 - a) \ll 1$, or, which is the same, $E - E_\parallel \ll E_\parallel$, we have $\cos\tilde\vartheta \approx 1$ and it is

1)We take the opportunity to note certain errors in [2]. Formulas (2) and (4) should include terms containing the product $u^*_q\vartheta_q$ and $u_q\vartheta^*_q$. Allowance for these terms does not change the conclusions, which are based only on conservation laws, but at energies close to the threshold these terms affect the polarization and the cross section.

easy to find the conditions under which the factors $\Delta_{\parallel}^2$ and $\Delta_{\perp}^2$ can be disregarded in the integrand. Namely, the factor $\Delta_{\parallel}^2$ is negligible when

$$\Delta E / E \gg 1/\alpha, \tag{17}$$

and the factor $\Delta_{\perp}^2$ when

$$\vartheta_0 \gg \frac{1}{\alpha}\sqrt{\frac{E-E_{\parallel}}{E_{\parallel}}}. \tag{18}$$

The inequalities (17) and (18) are the conditions for a nonmonochromatic broad beam near the threshold, for which we have

$$I \approx \frac{H}{H+2\pi M}\sqrt{\frac{E-E_{\parallel}}{E_{\parallel}}}. \tag{19}$$

In this case the depolarization is very small. If even one of the conditions, (17) or (18), is violated, then the depolarization is even smaller, and the corresponding expressions will therefore not be given here. For a narrow nonmonochromatic beam we shall derive below a formula that is valid both near and far from the threshold.

Far from the threshold, when $E \gg E_{\parallel}$, we have $a \ll 1$, and accordingly $b \ll 1$, since H and $4\pi M$ are quantities of the same order. Since now $y \sim 1$ and $\sin^2\tilde{\vartheta} \sim \cos^2\tilde{\vartheta} \sim 1$, the criterion for the nonmonochromaticity of the beam (17) remains the same, and the condition for the broadness of the beam takes the form

$$\vartheta_0 \gg 1/\alpha. \tag{20}$$

In this case the depolarization can also be readily calculated by taking into account the fact that when $a \ll 1$ and $b \ll 1$ the values of y that are of importance in the integral I lie in the interval $2 \gg y \gg \sqrt{a}$. As a result we obtain

$$I \approx \frac{1}{4}\left(\ln\frac{4}{a}+\frac{1}{2}\right) = \frac{1}{4}\left(\ln\frac{4E}{E_{\parallel}}+\frac{1}{2}\right). \tag{21}$$

As expected, the depolarization far from the threshold is much larger than near the threshold, and increases with increasing distance from the threshold.

If the beam width is decreased, then the factor $\Delta_{\perp}^2(y_{\perp})$, which cuts off the integration region in (16), comes into play at $\vartheta_0 \sim 1/\alpha$. This leads at first mainly to a slight decrease of the number in formula (21) under the logarithm sign, namely 4 is replaced by $(\alpha\vartheta_0)^2$. However, when $\alpha\vartheta_0$ becomes $\sim\sqrt{a}$, the logarithm vanishes completely, and when $\vartheta_0 \ll \sqrt{a}/\alpha$ it is necessary to use in place of formula (21) the expression

$$I = \frac{(\alpha\vartheta_0)^2}{2a\sqrt{1-a}} = \frac{(\alpha\vartheta_0)^2}{2}\frac{E}{E_{\parallel}}\sqrt{\frac{E}{E-E_{\parallel}}}. \tag{22}$$

The latter is obtained from (13) for a narrow beam by using (7). We note that the same expression (22) is valid for a narrow beam also near the threshold, and in the entire region it is small compared with the value of I for a broad beam.

Let us consider now the second case, $\mathbf{v} \perp \mathbf{H}$. In this case $\cos\psi = \sin\vartheta\cos\varphi$, and after integrating with respect to φ in I we can represent it in the form

$$I = \frac{1}{\pi}\int y^2\,dy\int d\cos\vartheta\,\frac{y^2+a+{}^1/_2\,b\sin^2\vartheta}{(y^2+a)(y^2+a+b\sin^2\vartheta)}$$
$$\times\frac{\Delta_{\parallel}^2(y,\cos\vartheta)\,\Delta_{\perp}^2(y,\sin\vartheta)}{[4y^2\sin^2\vartheta-(y^2+a)(y^2+a+b\sin^2\vartheta)]^{1/2}}$$
$$\times\left[1+\cos^4\vartheta-\frac{b\sin^4\vartheta(1+\cos^2\vartheta)}{2(y_{\parallel}^2+a+{}^1/_2\,b\sin^2\vartheta)}\right]. \tag{23}$$

In this integral, the region of integration is determined by the requirement that the radicand be positive:

$$y_- \leqslant y \leqslant y_+,$$
$$y_\pm^2 = 2-a-\frac{b}{2}\pm\sqrt{(2-b/2)^2-4a},$$
$$1 > \sin^2\vartheta > \frac{(y^2+a)^2}{4y^2-b(y^2+a)}. \tag{24}$$

From the fact that y_+^2 is real and y_-^2 is positive we get expression (14) for the threshold energy $E_{\perp}$. It is easy to show that the criteria for the nonmonochromatic and broad beam remain the same in the entire region as in the case when $\mathbf{v} \perp \mathbf{H}$.

For a broad nonmonochromatic beam near the threshold $(E - E_{\perp}) \ll E_{\perp}$, where $y^2 \approx 2 - a - b/2$ and $\sin^2\tilde{\vartheta} \sim 1$, we get from (23)

$$I \approx \frac{\sqrt{2-b/2-a}}{4-b^2/4}\sqrt{(2-b/2)^2-4a} = \frac{(1+2\pi M/H)^{1/2}}{2(1+4\pi M/H)^{1/4}}\sqrt{\frac{E-E_{\perp}}{E_{\perp}}}. \tag{25}$$

Analogously, just as when $\mathbf{v} \parallel \mathbf{H}$, we can calculate I far from the threshold, where $E \gg E_{\perp}$, $a \ll 1$, and $b \ll 1$; as a result we get for a broad non-monochromatic beam the formula

$$I \approx \frac{11}{32}\ln\frac{4}{a}-\frac{9}{64} = \frac{11}{32}\ln\frac{2E}{\mu H a}-\frac{9}{64}, \tag{26}$$

which differs little from (21).

In exactly the same manner as in the case of $\mathbf{v} \parallel \mathbf{H}$, when the beam becomes very narrow ($\vartheta_0 \ll \sqrt{a+b}/\alpha$), it is necessary to use in place of (26) the expression

$$I = \frac{\alpha^2\vartheta_0^2(2+b)}{4(a+b)\sqrt{(2-b/2)^2-4a}} \tag{27}$$

the latter being valid both far and near the threshold.

4. DEPOLARIZATION CONNECTED WITH SCATTERING

We have considered the depolarization of the transmitted neutrons, due to the influence exerted on the neutrons by the fluctuating magnetic fields. In particular, it has turned out that this depolarization is largest if the incident neutron beam is sufficiently broad, i.e., its aperture angle ϑ_0 is large compared with $1/\alpha$. On the other hand, it is well known[2] that inelastic magnetic scattering occurs in a narrow corner of angles with an aperture not exceeding $2/\alpha$. Thus, if $\vartheta_0 \gg 1/\alpha$, there will be present in the transmitted beam also neutrons that experience scattering. The change of the neutron energy upon scattering is $\Delta E \lesssim E/\alpha$, and therefore, in the case of a broad nonmonochromatic beam, it is impossible to separate the scattered neutrons from the transmitted ones. However, as shown in[2], scattering is accompanied by a rotation of the neutron spin, and should obviously cause a change in the observed polarization of the transmitted beam.

We are interested in that part of the polarization which is parallel to the field B_0. It is easy to show, using standard methods, that its form is

$$\frac{\langle P\rangle_{\text{scat}}}{P_0} = \left(\int d\mathbf{q}\,K_{\alpha\alpha}(\mathbf{q},\omega_{\mathbf{q}})\right)^{-1}\left\{\int d\mathbf{q}\,[2K_{zz}(\mathbf{q},\omega_{\mathbf{q}})-K_{\alpha\alpha}(\mathbf{q},\omega_{\mathbf{q}})]\right.$$
$$\left.+\frac{1}{2}\int d\mathbf{q}\,[K_{+-}(\mathbf{q},\omega_{\mathbf{q}})-K_{-+}(\mathbf{q},\omega_{\mathbf{q}})]\right\},$$
$$\omega_{\mathbf{q}} = E_{\mathbf{p}}-E_{\mathbf{p}-\mathbf{q}} \approx \mathbf{q}\mathbf{v}. \tag{28}$$

Knowing $\langle P\rangle_{scat}$, we can readily determine the depolarization due to scattering

$$\left(\frac{\Delta P}{P_0}\right)_{scat} = \frac{[P_0 - \langle P\rangle_{scat}]}{P_0}\frac{\sigma L}{\mathcal{V}_0}. \tag{29}$$

The term in (28) containing the difference $(K_{+-} - K_{-+})$ can be neglected, since it vanishes in the classical limit and consequently is small at high temperatures, and only in this case is the depolarization noticeable at all. Then, taking (9) into account, we obtain

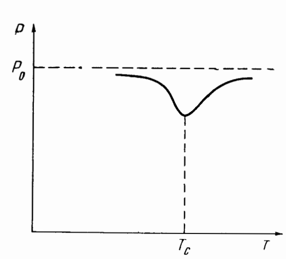
$$\left(\frac{\Delta P}{P_0}\right)_{scat} = \frac{g_n^2 L}{2v}\frac{1}{(2\pi)^3}\int d\mathbf{q}\,[K_{xx}(\mathbf{q}, \omega_{\mathbf{q}}) + K_{yy}(\mathbf{q}, \omega_{\mathbf{q}})]. \tag{30}$$

Thus, the depolarization due to scattering practically coincides with the depolarization (6) for a broad beam. Therefore the actually observed depolarization of a broad beam is twice as large as that calculated in the preceding section. The same result is valid also for a narrow beam. Indeed, when measuring the depolarization of a narrow beam one registers simultaneously only the neutrons scattered through an angle not exceeding ϑ_0. The limitation imposed thereby on the region of integration in (30) is similar to the limitation imposed by the factor $\Delta^2(q)$ in (12).

5. CONCLUSION

As we have already noted, the depolarization is small. According to (15), it is proportional to $TM(T)/\alpha(T)$. But in the roughest approximation $\alpha(T) \propto M(T)$, and therefore the depolarization is mainly proportional to the temperature. Our formulas can be used only at those temperatures when the theory of spin waves is applicable, i.e., not very close to the Curie point. In this region, its measurement makes it possible to determine the quantity A(T), which is the main parameter of the theory, and in this respect depolarization experiments yield the same information as scattering experiments. We note also that the depolarization is due mainly to spin waves with $q \lesssim p/\alpha$. This circumstance must be taken into account when our formulas are used near T_c, with the region of applicability of the spin-wave theory depends essentially on q[7]. We now discuss qualitatively the question of the depolarization near the Curie point T_c, when the spin-wave theory is known not to be applicable.

In the critical region, the long wave fluctuations increase strongly, reaching a maximum at $T = T_c$, and the short-waves remain practically unchanged. Therefore, as $T \to T_c$, the depolarization should have a maximum due to the long-wave excitations, and the polarization behaves in the manner shown in the figure. To observe the left side $(T < T_c)$ of this curve, the sample should be in a saturating magnetic field. This condition was not satisfied in[3,10], where depolarization near T_c was investigated.



Apparently, the most interesting would be an experimental study, in this temperature region, of the dependence of the polarization on the beam width. This would make it possible to determine directly the dimensions of the critical fluctuations, since a sharp decrease of the depolarization with decreasing width of the beam should be observed at angles ϑ_0 connected with the dimension of the fluctuations r_c by the relation

$$r_c \sim 1/p\vartheta_0 = \bar{\lambda}/\vartheta_0. \tag{31}$$

The meaning of the quantity r_c which enters in this formula depends on the relation between the lifetime of the fluctuation τ_c and the time t_c within which the neutron passes through it. If $t_c \ll \tau_c$, then r_c coincides with the static correlation radius, and in the opposite limiting case the connection between r_c and the parameters of the ferromagnet cannot be established so far reliably. In both cases, the question calls for further experimental and theoretical investigation.

We note also that if the system contains nuclei with a spin, then nuclear depolarization occurs. It is easy to show that its form is

$$(\Delta P/P_0)_{nuc} \approx \vartheta_0^2\, \sigma_{incoh}^{(spin)} L/\mathcal{V}_0, \tag{32}$$

where $\sigma_{incoh}^{(spin)}$ is that part of the incoherent nuclear scattering which is due to spins. This depolarization is very small and does not depend on the temperature, and is therefore of no interest.

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APPENDIX I

We present a consistent quantum-mechanical derivation of the formula for the depolarization.

We describe the neutron by means of a wave packet

$$\psi_{\mathbf{p}}(\mathbf{r}, t) = e^{i(\mathbf{pr} - E_{\mathbf{p}}t)} X_{\mathbf{p}}(\mathbf{r} - \mathbf{v}t), \qquad \int d\mathbf{r}\, |X_{\mathbf{p}}(\mathbf{r})|^2 = 1, \tag{AI.1}$$

where the function $X_p(r)$ differs from zero in the region

$$\Delta x \lesssim \bar{\lambda} E_{\mathbf{p}}/\Delta E_{\mathbf{p}}, \quad \Delta\rho \lesssim \bar{\lambda}\vartheta_0. \tag{AI.2}$$

Here x is the coordinate in the direction of motion of a neutron and ρ is the radius vector in the perpendicular direction. The effective Hamiltonian of the interaction between the neutrons and the medium is obtained by averaging the energy of the magnetic interaction V(r) over the state (AI.1):

$$V_{eff}(t) = \int d\mathbf{r}\, |X_{\mathbf{p}}(\mathbf{r} - \mathbf{v}t)|^2 V(\mathbf{r}) = \frac{1}{(2\pi)^3}\int d\mathbf{r}\, d\mathbf{q}\, |X_{\mathbf{p}}(\mathbf{r})|^2 e^{-i\mathbf{q}(\mathbf{r} - \mathbf{v}t)} V(\mathbf{q}),$$

$$V(\mathbf{q}) = -8\pi\mu\mu_n \sum_{(l)} \{\mathbf{S}_l - (\mathbf{e}\mathbf{S}_l)\mathbf{e}\}\, \sigma e^{i\mathbf{q}\mathbf{R}_l} F(\mathbf{q}) = \sigma \mathbf{D}(\mathbf{q}), \tag{AI.3}$$

where $\mathbf{S}_l$ and $\mathbf{R}_l$ are the spin and coordinates of the l-th atom, F(q) is the magnetic form factor, $\mathbf{e} = \mathbf{q}/q$, V(q) is the energy matrix element of the magnetic interaction of the neutron, and is well known from the theory of magnetic scattering[1]. Using the customary methods (see[11]), it is easy to find the time dependence of the spin of a neutron moving in a medium

$$\sigma(t)=e^{i\mathscr{H}_0 t}U^{-1}(t)\sigma(0)U(t)e^{-i\mathscr{H}_0 t},$$

$$U(t)=1-i\int_0^t dt' V_{\text{eff}}(t')+(-i)^2\int_0^t dt'\int_0^{t'} dt'' V_{\text{eff}}(t')V_{\text{eff}}(t'')+\ldots, \quad \text{(AI.4)}$$

where $\mathscr{H}_0$ describes the interaction between the neutron and the external magnetic field. From this expression, in second order of perturbation theory, we obtain in the case when $P_0 \perp B_0$ and the velocity v is parallel or perpendicular to B_0,

$$\frac{\Delta P}{P_0}=\frac{2}{(2\pi)^6}\int_0^t dt'\int_0^{t'} dt''\int d\mathbf{r}'\,d\mathbf{r}''\,d\mathbf{q}'\,d\mathbf{q}''\,|X_{\mathbf{p}}(\mathbf{r}')X_{\mathbf{p}}(\mathbf{r}'')|^2$$
$$\times\{\langle \mathbf{D}_\perp(\mathbf{q}',t')\mathbf{D}_\perp(\mathbf{q}'',t'')+\mathbf{D}_\perp(\mathbf{q}'',t'')\mathbf{D}_\perp(\mathbf{q}',t')\rangle + i\varepsilon_{z\alpha\beta}\langle[D_\alpha(\mathbf{q}',t'),D_\beta(\mathbf{q}'',t'')]\rangle\}. \quad \text{(AI.5)}$$

The mean value in this equation differs from zero only when $q' + q'' = 2\pi\tau$, where τ is the reciprocal-lattice vector. The terms with $\tau \neq 0$ are small, since they contain the factor $\exp(i\tau\cdot vt'')$, as a result of which the integral with respect to t'' converges and its order of magnitude is $1/\tau v \sim \mathscr{V}_0^{1/2}/v$, whereas the term with $\tau = 0$ is proportional to the large quantity $l = L/v$.

The last term in (AI.5) corresponds fully to the discarded term in (28), and we neglect it. The first two terms in (AI.5) at $q' + q'' = 0$ can be readily reduced to the expression used in the main text of the article, if it is recognized that

$$\mathbf{m}(\mathbf{R}_l)=\frac{2\mu}{\mathscr{V}_0}\mathbf{S}(\mathbf{R}_l);$$

it turns out here that

$$\Delta^2(\mathbf{q})=\int d\mathbf{r}\,e^{i\mathbf{q}\mathbf{r}}|X_{\mathbf{p}}(\mathbf{r})|^2.$$

APPENDIX II

We now calculate the contribution made to the depolarization by the last term of formula (12). In this term, the principal role is played by spin waves with energy on the order of T, and it is possible to neglect from the very beginning the dipole-dipole interaction. As a result we obtain

$$\left(\frac{\Delta P}{P_0}\right)_{zz}=\frac{g_n^2L}{\pi v}\frac{\mu^2}{(2\pi)^5}\int d\mathbf{q}\,d\mathbf{q}'\,\Delta^2(\mathbf{q})e_z^2(1-e_z^2)$$
$$\times n_{\mathbf{q}}(1+n_{\mathbf{q}-\mathbf{q}'})\delta(\varepsilon_{\mathbf{q}}-\varepsilon_{\mathbf{q}-\mathbf{q}'}-\mathbf{q}\mathbf{v}). \quad \text{(AII.1)}$$

In this expression, the integral with respect to q' can be readily evaluated, and we have

$$\left(\frac{\Delta P}{P_0}\right)_{zz}=\frac{2g_n^2L}{(2\pi)^4}\frac{\mu_l^2T^2}{A^3}J(\Lambda),$$

$$J(\Lambda)=\frac{1}{2\pi}\int_0 y\,dy\int d\Omega\,e_z^2(1-e_z^2)\frac{\Delta^2(2py\sqrt{T/\alpha E})}{\exp(xy/\sqrt{\Lambda})-1}$$
$$\times\ln\frac{1-\exp[-(y+x/\sqrt{\Lambda})^2]}{1-\exp[-(y-x/\sqrt{\Lambda})^2]},\quad x=\cos\psi. \quad \text{(AII.2)}$$

Here ψ is the angle between y and v, $\Lambda = \alpha T/E \gg 1$. In the integral J, the principal role is played by $y \sim 1$, therefore the conditions under which the beam is broad and nonmonochromatic take the form

$$\vartheta_0 \gg \sqrt{T/\alpha E},\quad \Delta E/E \gg \sqrt{T/\alpha E}, \quad \text{(AII.3)}$$

i.e., they are much more stringent than (17), (18), and (20).

When these conditions are satisfied, J can be readily calculated, if it is recognized that $xy/\sqrt{\Lambda} \ll 1$, and if we integrate first with respect to y and then with respect to the angles. As a result we obtain

$$J=\begin{cases}{}^8/_{15}(\ln\Lambda+3^1/_{15}) & (\mathbf{v}\parallel\mathbf{H}),\\ {}^8/_{15}(\ln\Lambda+5^1/_{15}) & (\mathbf{v}\perp\mathbf{H}).\end{cases} \quad \text{(AII.4)}$$

We see that J is of the order of unity; the factor preceding J in (AII.2) is small compared with the corresponding factor in (12), so that the quantity $(\Delta P/P_0)_{zz}$ can be neglected. This conclusion is in full agreement with the conclusion of smallness of two-magnon scattering[12]. We note also that inasmuch as the difference $\epsilon_q - \epsilon_{q-q'}$ can vanish, it follows that $(\Delta P/P_0)_{zz}$ has no threshold.

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28