SPONTANEOUS EMISSION OF γ QUANTA BY A SYSTEM CONTAINING IDENTICAL NUCLEI

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Spontaneous γ-emission by an excited nucleus, located in a polycrystal or single crystal containing identical (resonance) nuclei, is considered. The problem is solved in the approximation of short wavelengths $\lambda \ll a$ (λ denotes the wavelength of the photon, and a denotes the interatomic distance). Expressions are obtained for the spectrum of γ quanta emitted without recoil (without any change of the phonon state of the lattice) and for the time dependence of the rate of emission in different directions. The results obtained can be experimentally verified by the usual method of measuring the lifetime of the excited nuclear state.

IT is well known that the intensity of γ-radiation which is associated with the decay of an excited nucleus decreases with time according to an exponential law $e^{-\Gamma t}$ (Γ denotes the total width of the nuclear level). However, as is shown in this work, if the excited nucleus is found in a crystal, especially in a perfect single crystal which contains identical nuclei (resonance nuclei), any initial excitation may coherently propagate to other resonance nuclei in the lattice and be converted to a collective excitation whose decay differs appreciably from the decay of an individual nucleus.

The formation and decay of such a collective excited state in a collection of identical nuclei has already been studied in several articles. Podgoretskiĭ and Roĭzen[1] first derived formulas for the decay law of a system of two identical nuclei in a model of a one-dimensional chain. The results of their work pertain to the case of short wavelengths, i.e., $\lambda \ll a$ (λ is the wavelength of the photon, and a is the distance between nuclei). The work of Lyuboshitz[2] and Faĭn and Khanin[3] is devoted to a generalization of the results of article[1].

Interference between the radiation from different nuclei plays a major role in the formation and decay of such a collective excited state. It is obvious that such interference must be clearly exhibited when the nuclei are arranged in a regular system. This was already clear from the results of Podgoretskiĭ and Roĭzen for the model of a one-dimensional chain of identical nuclei. Therefore an investigation of the properties of γ emission from a real source in the form of a crystal (single- or polycrystal) containing identical nuclei is of interest. In certain respects this problem is similar to the problem of the propagation of particles or quanta in a single crystal, which was considered in a series of articles by Kagan and Afanas'ev (see, for example,[4]).

At the initial moment and at the lattice site $\mathbf{r}_0 = 0$, let an excited nucleus be produced (with the aid, for example, of a previous γ cascade or β-transition) with spin component i_0, the lattice being in the phonon state n_0. We shall not consider the case when many nuclei are excited simultaneously. Such an approximation is valid when the activity of the source is not too large in comparison with $\Gamma/\hbar$. The propagation of an excitation which was initially localized at the site $\mathbf{r}_0 = 0$ to other crystal sites is characterized by the amplitude C_{si_sn} of the state which corresponds to an excitation of only the nucleus at the site s with spin component i_s; here the lattice is in the phonon state n.

We shall denote the amplitude of the state when all nuclei are unexcited and a photon exists with wave vector $\mathbf{k}$ and polarization σ by $C_{\mathbf{k}\sigma n}$. In order to simplify the discussion, we assume for the time being that the spin of the ground state of the nucleus is equal to zero ($J = 0$).

One can write the equations of motion for these amplitudes in the energy representation in the form[4,5] ($\hbar = c = 1$)

$$(E - E_k - \Delta E_n) C_{\mathbf{k}\sigma n}(E) = \sum_{s,\, i_s,\, n'} H^{\mathbf{k}\sigma n}_{si_sn'} C_{si_sn'}(E),$$
$$(E - E_0' - \Delta E_n + {}^1/_2\, i\Gamma_c) C_{si_sn}(E) = \delta_{s0}\delta_{i_s i_0}\delta_{nn_0} + \sum_{\mathbf{k}\sigma n'} H^{si_sn}_{\mathbf{k}\sigma n'} C_{\mathbf{k}\sigma n'}(E), \tag{1}$$

where Γ_c denotes the conversion width of the level, and ΔE_n denotes the energy of the phonons measured from the energy of the initial phonon state n_0 (we note that $\Gamma \ll \Delta E_n \ll E_0'$);

$$H^{\mathbf{k}\sigma n}_{si_sn'} = (H^{si_sn'}_{\mathbf{k}\sigma n})^* = M^{\mathbf{k}\sigma}_{i_s}\; e^{-i\mathbf{k}\mathbf{r}_s} (e^{-i\mathbf{k}\mathbf{u}_s})_{nn'}, \tag{2}$$

where $\mathbf{r}_s$ denotes the coordinate of the s-th lattice site, $\mathbf{u}_s$ denotes the displacement of the s-th nucleus, and $M^{\mathbf{k}\sigma}_{i_s}$ denotes the matrix element of the γ transition. The matrix elements (2) are associated with the parameters of the nuclear level by the relation

$$\sum_{\mathbf{k}\sigma n''} \frac{H^{si_sn}_{\mathbf{k}\sigma n''} H^{\mathbf{k}\sigma n''}_{si_s'n'}}{E - E_k - \Delta E_{n''} + i\varepsilon} = \left(-\frac{i\Gamma_R}{2} + \Delta E_0\right) \delta_{nn'}\delta_{i_s i_s'}, \tag{3}$$

where ϵ is an infinitesimal real number, Γ_R is the radiative width of the nuclear level, $\Gamma_R + \Gamma_c = \Gamma$, and ΔE_0 denotes the change in the energy of the excited nuclear state associated with the presence of its self-field of spontaneous γ emission:

$$\Delta E_0 = E_0 - E_0', \tag{4}$$

where E_0 denotes the energy of the excited nuclear state.

In writing down (1) it was assumed that the nuclear levels are completely degenerate. We also note that the second equation contains the initial condition indicated above.

The system (1) is solved by the method of iteration. First of all one can show that the amplitude $C_{0i_0n_0}(E)$ has the form

$$C_{0i_0n_0}(E) = [E - E_0 + i\Gamma/2 + R]^{-1}, \tag{5}$$

where one can represent R in the form of a series whose first term is of order

$$\Gamma_R\left(\frac{\lambda}{a}\right)^3 \frac{\Gamma_R}{E - E_0 + i\Gamma/2}$$

In the approximation of short wavelengths ($\lambda \ll a$) one can neglect R, and thus the total probability for the decay of the system under consideration does not differ from the case of an isolated nucleus. We note that since in the general case R depends on the energy, the time dependence of the total probability $|C_{0i_0n_0}(t)|^2$ for decay of the system differs from the usual exponential law.

Now let us go on to a calculation of the amplitude of the state $C_{\mathbf{k}\sigma n}(E)$. After certain transformations, from the system of equations (1) we have

$$C_{\mathbf{k}\sigma n}(E) = \frac{C_{0i_0n_0}(E)}{E - E_k - \Delta E_n + i\varepsilon}\Big\{M_{i_0}^{\mathbf{k}\sigma}(e^{-i\mathbf{k}\mathbf{u}_0})_{nn_0} + \frac{1}{E - E_0 + i\Gamma/2}\sum_{i_1} M_{i_1}^{\mathbf{k}\sigma}\sum_{\mathbf{r}_1\neq 0}(e^{-i\mathbf{k}\mathbf{u}_1})_{nn_0}L_{i_1,i_0}(\mathbf{r}_1 - \mathbf{r}_0)$$
$$+ \frac{1}{(E - E_0 + i\Gamma/2)^2}\sum_{i_2,i_1} M_{i_2}^{\mathbf{k}\sigma}\sum_{\mathbf{r}_1\neq\mathbf{r}_2\neq 0}(e^{-i\mathbf{k}\mathbf{u}_2})_{nn_0}L_{i_2,i_1}(\mathbf{r}_2 - \mathbf{r}_1)L_{i_1,i_0}(\mathbf{r}_1 - \mathbf{r}_0) + \ldots\Big\}, \tag{6}$$

where

$$L_{i_b,i_a}(\mathbf{r}_b - \mathbf{r}_a) = \sum_{\mathbf{k}'\sigma'n'} \frac{M_{\mathbf{k}'\sigma'}^{i_b} M_{i_a}^{\mathbf{k}'\sigma'}}{E - E_{k'} - \Delta E_{n'} + i\varepsilon}\cdot\exp\{-i(\mathbf{k} - \mathbf{k}')(\mathbf{r}_b - \mathbf{r}_a)\}$$
$$\times(\exp\{-i\mathbf{k}'\mathbf{u}_a\})_{n_0n'}(\exp\{i\mathbf{k}'\mathbf{u}_b\})_{n'n_0} \tag{7}$$

(in what follows we shall discard ΔE_n in Eqs. (6) and (7) since $\Delta E_n \ll E_k$). Formula (6) clearly indicates the formation and decay of a collective, excited state of the system. In fact, different channels contribute to the emission of the photon $\mathbf{k}\sigma n$. The first term of the series (6) corresponds to the emission of a photon directly from the initially excited nucleus ($\mathbf{r}_0 = 0$, $i = i_0$, $n = n_0$). The second term describes the emission of a photon by the other nuclei ($\mathbf{r}_1 \neq 0$) which are excited with the aid of the photon which is originally emitted by the excited nucleus, and so forth.

Certain special cases are considered below:

1. The source does not contain resonance nuclei. For a comparison of the results obtained below, let us consider this case first. Only the first term is left in the series (6), and

$$C_{\mathbf{k}\sigma n}(E) = \frac{M_{i_0}^{\mathbf{k}\sigma}(e^{-i\mathbf{k}\mathbf{u}_0})_{nn_0}}{(E - E_k + i\varepsilon)(E - E_0 + i\Gamma/2)}. \tag{8}$$

Formula (8) contains all information about the emission by an "isolated nucleus," in particular the well-known Lorentzian shape of the energy distribution (spectrum) of the γ-quanta emitted without recoil ($n = n_0$):

$$W(E_k) \sim |C_{\mathbf{k}\sigma n_0}(t)|^2_{t\to\infty} = \frac{|M_{i_0}^{\mathbf{k}\sigma}|^2 f_{\mathbf{k}}}{(E_k - E_0)^2 + \Gamma^2/4}, \tag{9}$$

where $f_{\mathbf{k}} = [(e^{-i\mathbf{k}\cdot\mathbf{u}_0})_{n_0n_0}]^2$ is the probability for the emission of γ quanta without recoil in the direction $\mathbf{k}$, and $C_{\mathbf{k}\sigma n}(t)$ denotes the amplitude obtained by taking the Fourier transform of the amplitude $C_{\mathbf{k}\sigma n}(E)$:

$$C_{\mathbf{k}\sigma n}(t) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} e^{-i(E-E_k)t}C_{\mathbf{k}\sigma n}(E)\,dE. \tag{10}$$

The time dependence of the rate of emission (or the flux) in different directions is an important characteristic of γ emission:

$$\omega_{\mathbf{k}\sigma n}(t) = \frac{A}{2\pi}\frac{d}{dt}\int |C_{\mathbf{k}\sigma n}(t)|^2 dE_k \tag{11}$$

(A denotes a certain normalization factor). Substituting (8) and (10) into (11) one can easily see that this rate of emission decreases with time according to the law $e^{-\Gamma t}$.

2. The resonance nuclei are distributed randomly in the source (this also refers to the case of a polycrystalline source). The sum of the phase factors in (6) and (7) is given by

$$\sum_{\mathbf{r}\neq 0} e^{-i\mathbf{r}\Delta\mathbf{k}} \to 0 \text{ for } \Delta\mathbf{k} \neq 0.$$

Therefore, for the amplitude of the state $C_{\mathbf{k}\sigma n_0}(E)$ the sum over $\mathbf{k}'$ and n' in (7) reduces to the single term with $\mathbf{k}' = \mathbf{k}$ and $n' = n_0$:

$$L_{i_b,i_a}(\mathbf{r}_b - \mathbf{r}_a) = \sum_{\sigma'} \frac{M_{\mathbf{k}'\sigma'}^{i_b} M_{i_a}^{\mathbf{k}'\sigma'}}{E - E_{k'} + i\varepsilon}\exp\{-i(\mathbf{k} - \mathbf{k}')(\mathbf{r}_b - \mathbf{r}_a)\}$$
$$\times(\exp\{-i\mathbf{k}'\mathbf{u}_a\})_{n_0n_0}(\exp\{i\mathbf{k}'\mathbf{u}_b\})_{n_0n_0} \quad (\mathbf{k}' \parallel \mathbf{k}). \tag{7'}$$

The absence of the summation over n' in Eq. (7′) in comparison with (7) is associated with the fact that a transition accompanied by a change of the phonon state ($n' \neq n_0$) gives an additional phase factor $\exp\{-i\mathbf{q}\cdot(\mathbf{r}_b - \mathbf{r}_a)\}$, where $\mathbf{q}$ is the wave vector of the phonon. In a polycrystal the nuclei are not randomly distributed in each crystal; however the sum of the terms with $\mathbf{k}' \neq \mathbf{k}$ in (7) nevertheless gives a small contribution of order $(\lambda/a)^2$ upon averaging over the orientation of the crystals. Such an averaging is realized in analogy to the calculation of the total effective cross section for coherent scattering of a neutron in polycrystals[6] (here it should be understood that the summation over the lattice sites in (6) is taken over $\mathbf{r} \neq 0$).

With the aid of these simplifications the series (6) for the state amplitude $C_{\mathbf{k}\sigma n_0}(E)$ reduces to the expression

$$C_{\mathbf{k}\sigma n_0}(E) = \frac{M_{i_0}^{\mathbf{k}\sigma}(e^{-i\mathbf{k}\mathbf{u}_0})_{n_0n_0}e^{\alpha z}}{(E - E_k + i\varepsilon)(E - E_0 + i\Gamma/2)}, \tag{12}$$

where

$$\alpha = -i(2I + 1)\rho f\frac{\lambda^2}{8\pi}\frac{\Gamma_R}{E - E_0 + i\Gamma/2}, \tag{13}$$

ρ denotes the density of the resonance nuclei, z is the distance from the initial excited nucleus to the surface of the source along the given direction $\mathbf{k}$ of the photon's flight. From (12) one can easily find the spectrum for recoilless emission of γ quanta:

$$W(E_k) \sim |C_{\mathbf{k}\sigma n_0}(t)|^2_{t\to\infty} = \frac{|M_{i_0}^{\mathbf{k}\sigma}|^2 f}{(E_k - E_0)^2 + \Gamma^2/4} \exp\left\{-\frac{\sigma_0 \rho f z \Gamma^2/4}{(E_k - E_0)^2 + \Gamma^2/4}\right\}, \tag{14}$$

where

$$\sigma_0 = \frac{\Gamma_R}{\Gamma}(2I+1)\frac{\lambda^2}{2\pi}. \tag{15}$$

The value of $W(E_k)$ depends on the position z of the initially excited nucleus. The emission spectrum averaged over z is observed experimentally. If the initially excited nuclei are uniformly distributed over the entire source, then an averaging of expression (14) over the initially excited nuclei gives

$$W(E_k) \sim f\left[1 - \exp\left\{-\frac{\sigma_0 \rho f d \Gamma^2/4}{(E_k - E_0)^2 + \Gamma^2/4}\right\}\right], \tag{16}$$

which is in agreement with the previously known result for the recoilless emission spectrum of γ quanta in the presence of self-absorption in the source[7] (d is the linear dimension of the source in the direction $\mathbf{k}$). We note that for a wide interval of variation of the parameter $c_S = \sigma_0 \rho f d$ the function $W(E_k)$ is close to a Lorentzian curve with a half-width greater than Γ.[7]

Substituting (12) and (10) into (11) gives the following expression for the rate of recoilless emission of γ quanta in the direction $\mathbf{k}$:

$$\omega_{\mathbf{k}\sigma n_0}(t) = Af\frac{|M_{i_0}^{\mathbf{k}\sigma}|^2}{4\pi^2}\left|\int \frac{e^{\alpha z}}{E - E_0 + i\Gamma/2} e^{-iEt} dE\right|^2. \tag{17}$$

The integral (17) is calculated by Lynch, Holland, and Hamermesh.[8] Using the results of article[8] we arrive at the expression

$$\omega_{\mathbf{k}\sigma n_0}(t) = Af|M_{i_0}^{\mathbf{k}\sigma}|^2 e^{-\Gamma t}|J_0(\sqrt{\sigma_0 \rho f \Gamma z t})|^2,$$

where J_0 denotes the Bessel function of zero order. Averaging this expression over the initially excited nuclei (z and i_0) and summing over the polarization (σ) give

$$\omega_{\mathbf{k}n_0}(t) = Af\langle|M_{i_0}^{\mathbf{k}\sigma}|^2\rangle e^{-\Gamma t}\{|J_0(\sqrt{\sigma_0 \rho f \Gamma d t})|^2 + |J_1(\sqrt{\sigma_0 \rho f \Gamma d t})|^2\},$$

where J_1 denotes the Bessel function of first order, and the angular brackets on the right hand side denote averaging over i_0 and summation over σ.

We note that the rate of emission of γ quanta accompanied by a change of the phonon state of the lattice $\omega_{\mathbf{k}n}(t)$ ($n \neq n_0$) decreases with time according to the usual law $e^{-\Gamma t}$; therefore if all quanta of a given γ transition are detected in the experiment, then the rate of emission in the direction $\mathbf{k}$ is given by

$$\omega_{\mathbf{k}}(t) = A\langle|M_{i_0}^{\mathbf{k}\sigma}|^2\rangle e^{-\Gamma t}\{1 - f + f[|J_0(\sqrt{\sigma_0 \rho f \Gamma d t})|^2 + |J_1(\sqrt{\sigma_0 \rho f \Gamma d t})|^2]\}. \tag{18}$$

3. A source in the form of a single crystal. It is well known that for a single crystal one can write

$$\sum_{\mathbf{r}} e^{-i\mathbf{r}\Delta\mathbf{k}} = \frac{(2\pi)^3}{v_0}\sum_{\mathbf{b}} \delta(\Delta\mathbf{k} - 2\pi\mathbf{b}), \tag{19}$$

where v_0 denotes the volume of the elementary cell, and $\mathbf{b}$ is a reciprocal lattice vector. Thus the sum $\Sigma e^{-i\mathbf{r}\cdot\Delta\mathbf{k}}$ over $\mathbf{r}$ may reach a large value when the difference of the wave vectors $\Delta\mathbf{k}$ is close to $2\pi\mathbf{b}$. This always happens for "resonant forward scattering", $\mathbf{k}' = \mathbf{k}(\mathbf{b} = 0)$. Therefore, for a single crystal it is necessary to distinguish two cases.

A. The wave vector $\mathbf{k}$ does not satisfy the Bragg condition: $\mathbf{k} \neq \mathbf{k}' + 2\pi\mathbf{b}$ for all $\mathbf{k}'$ where $\mathbf{b}$ denotes a nonvanishing reciprocal lattice vector. In this case each sum over $\mathbf{k}'$ in (7) consists of two different parts: the term with $\mathbf{k}' = \mathbf{k}$ which satisfies the Bragg condition (with $\mathbf{b} = 0$), and the sum of the terms with $\mathbf{k}' \neq \mathbf{k}$ which do not satisfy the Bragg condition.

If the thickness of the source is small, i.e., when $c_S = \sigma_0 \rho f d \ll 1$, the terms with $\mathbf{k}' = \mathbf{k}$ in the series (6) give small contributions and we may neglect them. Then, by using the kinematic theory of the emission of γ quanta[9] it is not difficult to show that in this limiting case formula (6) for the amplitude $C_{\mathbf{k}\sigma n_0}(E)$ turns into a geometrical series whose sum is given by

$$C_{\mathbf{k}\sigma n_0}(E) = \frac{M_{i_0}^{\mathbf{k}\sigma}(e^{-i\mathbf{k}\mathbf{u}_0})_{n_0 n_0}}{(E - E_k + i\varepsilon)(E - E_0 + i\Gamma/2 + L)}, \tag{20}$$

where

$$L = \sum_{\sigma',\mathbf{k}'\neq\mathbf{k}} \frac{|M_{i_0}^{\mathbf{k}'\sigma'}|^2}{E - E_{k'} + i\varepsilon} \sum_{\mathbf{r}_s \neq 0} \exp\{-i(\mathbf{k} - \mathbf{k}')\mathbf{r}_s\}(\exp\{i\mathbf{k}'(\mathbf{u}_s - \mathbf{u}_0)\})_{n_0 n_0}. \tag{21}$$

Comparing (20) with (8) we immediately see that the radiation in a given direction $\mathbf{k}$ is similar to the radiation of an "isolated nucleus" upon the replacement of Γ by $\Gamma' = \Gamma + 2\,\mathrm{Im}\,L$ and E_0 by $\tilde{E}_0 = E_0 - \mathrm{Re}\,L$. In particular, the rate of recoilless emission of γ quanta decreases with time according to the law $e^{-\Gamma' t}$. At lower temperatures $T \ll \Theta_D$ (Θ_D denotes the Debye temperature of the crystal) one can regard the phonon matrix element in (21) as equal to unity, and then one can easily show that

$$\Gamma' = \Gamma - \eta\Gamma_R, \tag{22}$$

where η denotes the concentration of resonant nuclei in the source ($\eta \leq 1$). In the opposite limiting case $T \gg \Theta_D$ one will find $\Gamma' \to \Gamma$. An analysis of expression L for intermediate temperatures is given in article[4].

In the general case of a source of finite thickness each term of n-th order in the series (6) is converted into a sum of 2^p components of n-th order, differing from each other by a sequence of "forward scattering" ($\mathbf{k}' = \mathbf{k}$) and "scattering to the side" ($\mathbf{k}' \neq \mathbf{k}$) processes. In spite of the apparent inconvenience, by grouping the corresponding components of different orders in separate series and summing them in sequence, we can sum all of the terms in the series (6). As a result we have

$$C_{\mathbf{k}\sigma n_0}(E) = \frac{M_{i_0}^{\mathbf{k}\sigma}(e^{-i\mathbf{k}\mathbf{u}_0})_{n_0 n_0}}{E - E_k + i\varepsilon}\,\frac{e^{\alpha' z}}{E - E_0 + i\Gamma/2 + L}, \tag{23}$$

where

$$\alpha' = -i(2I+1)\rho f_{\mathbf{k}}\frac{\lambda^2}{4\pi}\,\frac{\Gamma_R}{E - E_0 + i\Gamma/2 + L}. \tag{24}$$

From (23) it is not difficult to derive an expression for the spectrum of a γ quantum emitted without recoil in a given direction $\mathbf{k}$ (averaged over z):

$$W(E_k) \sim f_{\mathbf{k}}\left\{1 - \exp\left[-\frac{\sigma_0 \rho f_{\mathbf{k}} d \Gamma'^2/4}{(E_k - E_0)^2 + \Gamma'^2/4}\right]\right\}. \tag{25}$$

Comparison of (23) with (12) immediately gives an expression for the time dependence of the rate of emission of γ quanta in the direction $\mathbf{k}$. If all the quanta of a given γ transition are detected in the experiment, then

$$\omega_{\mathbf{k}}(t) = A\langle|M_{i_0}{}^{\mathbf{k}\sigma}|^2\rangle e^{-\Gamma t}\{1 - f_{\mathbf{k}} + f_{\mathbf{k}}[|J_0(\sqrt{\sigma_0\rho f_{\mathbf{k}}\Gamma' dt})|^2 + |J_1(\sqrt{\sigma_0\rho f_{\mathbf{k}}\Gamma' dl})|^2]\}. \quad (26)$$

B. The wave vector **k** satisfies the Bragg condition. Let us assume for simplicity that $\mathbf{k} = \mathbf{k}_1 + 2\pi\mathbf{b}$ with one specific nonvanishing reciprocal lattice vector **b**. In this case the series (6) diverges, and the amplitude $C_{\mathbf{k}\sigma n_0}(E)$ cannot be determined from formulas (20) and (23). However, in analogy to Eqs. (8) and (20) we shall seek this amplitude in the form

$$C_{\mathbf{k}\sigma n_0}(E) = \frac{M_{i_0}{}^{\mathbf{k}\sigma}(e^{-i\mathbf{k}\mathbf{u}_0})_{n_0 n_0}}{(E - E_k + i\varepsilon)(E - E_0 + i\Gamma/2 + L')}, \quad (27)$$

where L' will be determined from Eqs. (1). It is natural that here the amplitude $C_{\mathbf{k}_1\sigma n_0}(E)$ has a similar form:

$$C_{\mathbf{k}_1\sigma n_0}(E) = \frac{M_{i_0}{}^{\mathbf{k}_1\sigma}(e^{-i\mathbf{k}_1\mathbf{u}_0})_{n_0 n_0}}{(E - E_{k_1} + i\varepsilon)(E - E_0 + i\Gamma/2 + L')}. \quad (28)$$

Deriving from the system of equations (1) an equation which holds between the amplitudes $C_{\mathbf{k}\sigma n}(E)$ and substituting formulas (19), (27), and (28) into it, we obtain

$$(E - E_k)\, C_{\mathbf{k}\sigma n_0}(E) = (e^{-i\mathbf{k}\mathbf{u}_0})_{n_0 n_0}\Bigg[\frac{M_{i_0}^{\mathbf{k}\sigma}}{E - E_0 + i\Gamma/2} + \frac{\eta N}{(E - E_0' + i\Gamma_c/2)(E - E_0 + i\Gamma/2 + L')} \times \sum_{i_s,\sigma'} (f_{\mathbf{k}} M_{i_s}^{\mathbf{k}\sigma} M_{\mathbf{k}\sigma'}^{i_s} M_{i_0}^{\mathbf{k}\sigma'} + f_{\mathbf{k}_1} M_{i_s}^{\mathbf{k}\sigma} M_{\mathbf{k}_1\sigma'}^{i_s} M_{i_0}^{\mathbf{k}_1\sigma'})\Bigg] \quad (29)$$

where ηN is the number of resonant nuclei in the source, referred to an area of 1 cm^2 perpendicular to **k**. In the short wavelength approximation $\mathbf{k}_1$ differs little from **k**, and then Eq. (29) can be appreciably simplified:

$$(E - E_k)C_{\mathbf{k}\sigma n_0}(E) = (e^{-i\mathbf{k}\mathbf{u}_0})_{n_0 n_0} M_{i_0}{}^{\mathbf{k}\sigma}\left[\frac{1}{E - E_0 + i\Gamma/2} - \frac{i\beta\Gamma_R}{(E - E_0' + i\Gamma_c/2)(E - E_0 + i\Gamma/2 + L')}\right], \quad (30)$$

$$\beta = \eta N f_{\mathbf{k}}(2I + 1)\lambda^2/4\pi. \quad (31)$$

Substitution of (27) into (30) gives

$$L' = i\beta\Gamma_R \frac{E - E_0 + i\Gamma/2}{E - E_0' + i\Gamma_c/2},$$

and therefore

$$C_{\mathbf{k}\sigma n_0}(E) = \frac{M_{i_0}{}^{\mathbf{k}\sigma}(e^{-i\mathbf{k}\mathbf{u}_0})_{n_0 n_0}}{E - E_k + i\varepsilon} \frac{E - E_0' + i\Gamma_c/2}{(E - E_0 + i\Gamma/2)(E - E_0' + i\Gamma_c/2 + i\beta\Gamma_R)} \quad (32)$$

Let us find the rate of emission of γ quanta in a given direction **k**. Substituting (32) into (10) and (11) instead of (18) and (26) we obtain

$$\omega_{\mathbf{k}}(t) = A\langle|M_{i_0}{}^{\mathbf{k}\sigma}|\rangle\Bigg\{(1 - f_{\mathbf{k}})e^{-\Gamma t} + f_{\mathbf{k}}\frac{\Delta E_0^2 + \Gamma^2/4}{\Delta E_0^2 + (2\beta - 1)\Gamma_R^2/4} \times \Bigg[e^{-\Gamma t} + \frac{\beta^2\Gamma_R^2}{\Delta E_0^2 + \Gamma^2/4}\exp\{-(\Gamma_c + 2\beta\Gamma_R)t\} - \frac{\beta\Gamma_R}{\Delta E_0^2 + \Gamma^2/4}\exp\left\{-\left(\frac{\Gamma + \Gamma_c}{2} + \beta\Gamma_R\right)t\right\} \times (2\Delta E_0 \sin\Delta E_0 t + \Gamma_R\cos\Delta E_0 t)\Bigg]\Bigg\}, \quad (33)$$

where ΔE_0 is determined by formula (4). In cases of practical interest $\beta \gg 1$; then from (33) it is seen that $\omega_{\mathbf{k}}(t)$ decreases with time basically according to the law $\exp\{-(\Gamma_c + 2\beta\Gamma_R)t\}$.

One can generalize all of the obtained results to the case of nonvanishing spin of the ground state of the nucleus ($J \neq 0$). Such a generalization does not present any difficulties in principle although it complicates the discussion markedly. In the case $J \neq 0$ it is necessary to multiply α, α', σ_0, and β (formulas (13), (23), (15), and (31)) by $(2J + 1)^{-1}$ and $\eta\Gamma_R$ (formula (22)) by $(2I + 1)/(2l + 1)(2J + 1)$ (l denotes the multiplicity of the γ transition).

Returning to expressions (18), (26), and (33) we see that the results obtained permit experimental verification with the aid of the usual method of measuring the lifetime of the excited nuclear state. The time dependence of the rate of emission of γ quanta by a system of identical nuclei differs markedly from the ordinary law $e^{-\Gamma t}$ for an "isolated nucleus," and this difference is larger the smaller the fraction of γ quanta emitted with a change in the phonon state of the lattice ($T \ll \Theta_D$). In this connection we note that the application of resonance counters[10] in experiments gives the possibility to completely exhibit the characteristic properties of radiation from a system of identical nuclei.

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