IMAGE CONVERSION FROM THE INFRARED TO THE VISIBLE BY NONLINEAR OPTICS METHODS

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Image conversion from the infrared to the visible range in a nonlinear crystal is analyzed. The resolving power is estimated. A conversion from 1.06 to 0.53 μ is obtained experimentally in a KDP crystal; the resolving power is 18 lines per millimeter.

NONLINEAR optics methods yield radiation at a resultant frequency from the interaction of two waves in a nonlinear crystal. The incident radiation has a frequency ω_2 corresponding to the infrared region of the spectrum and the frequency sum, $\omega_3 = \omega_1 + \omega_2$, where ω_1 is the pumping frequency, lies in the visible region.

If a plane pumping wave is used, each plane wave with frequency ω_2 and wave vector confined to some solid angle corresponds to a crystal-generated plane wave with frequency ω_3 and wave vector whose direction is identical with that of the wave vector of the signal wave. This permits us to obtain an image at the frequency sum if the information is contained in the angular spectrum, i.e., the field with frequency ω_3 corresponds to radiation from an object at infinity.

It is this possibility that was used in[1-3] for image conversion. The image of the object was obtained at infinity using an ordinary optical system.

As we show in this paper, it is possible to obtain an image of an object at a sum (or difference) of frequencies when the object is at a finite distance from the nonlinear crystal. This preserves the information on the depth distribution of objects, i.e., we can obtain an image of solid objects. Furthermore we show that when the object (or its image projected by an ordinary optical system) is situated near the crystal the high resolving power requirements imposed on pump divergence and monochromaticity of radiation become much less stringent.

The resolving power achieved in[1-3] is far from the possible limit and is determined by the pump divergence.

In our present work on image conversion from 1.06 to 0.53 μ with a near-field object, the resolving power obtained was close to its limiting value. In our system the object information is contained in the spatial rather than in the angular field structure. The possibilities of such a system were analyzed theoretically and the experimental results confirmed this analysis.

1. RESOLVING POWER IN IMAGE CONVERSION

We consider the resolving power problem using the following converter model (Fig. 1). The nonlinear crystal has a thickness z_0 and large dimensions along the x and y axes (the y axis is normal to the plane of

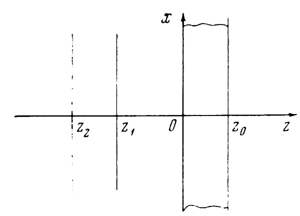

FIG. 1. Conversion diagram.

the diagram) so that it fills the space between the planes $z = 0$ and $z = z_0$.

A strong plane pumping wave with a wave vector along the z axis and frequency ω_1 is generated in the crystal; match conditions are satisfied for a signal wave with frequency ω_2 and wave vector confined to a cone coaxial with the z axis. The cone angle $\Delta\varphi$ outside the crystal amounts to a few degrees in the best case for crystal lengths z_0 of the order of 1 cm. Thus the resultant frequency wave has a wave vector lying within a cone with a cone angle of $(\omega_2/\omega_3)\Delta\varphi$.

Since conversion in a nonlinear crystal always entails a definite polarization, the field can be described by scalar functions.

We assume that in the plane $z = 0$ there is a Gaussian diaphragm with amplitude transparency $R(x, y)$, where

$$R(x,y) = \exp\left\{-\frac{x^2+y^2}{a^2}\right\}. \quad (1)$$

The effective radius of the diaphragm is denoted by a.

Let us assume a known field with frequency ω_2 in the plane $z = z_1$. The field is thus given in the entire space and can be computed in any plane $z = \text{const}$ by resolution in terms of plane waves propagating from left to right. The plane $z = z_1$ can lie within the crystal or even can pass to the right of the crystal. In this case we set a field in the plane $z = z_1$ in the absence of both crystal and diaphragm. The field generated by the crystal at the frequency ω_3 is considered in some plane $z = z_2$. It is a converted field when $z < z_0$.

To consider the resolving power problem we merely analyze the case when the field in the plane $z = z_1$ is described by a δ-function:

$$E_1(x, y, z_1) = \varepsilon\delta(x - x_0, y - y_0). \quad (2)$$

Resolution of the field in the plane $z = z_1$ into a twodimensional Fourier integral has the form:

$$E_1(k_x, k_y, z_1) = \frac{\varepsilon}{2\pi} e^{i(k_x x_0 + k_y y_0)}, \tag{3}$$

where k_x and k_y are wave vector components of the plane waves resulting from the analysis of the field.

The spectrum E_1 of the field in the plane $z = 0$ can be thus written as

$$E_1(k_x, k_y, 0) = \frac{\varepsilon}{2\pi} \exp\left\{ i(k_x x_0 + k_y y_0) - \frac{icz_1}{2\omega_2}(k_x^2 + k_y^2) \right\}, \tag{4}$$

where c is the velocity of light.

The phase factor does not depend on k_x and k_y and is dropped here and henceforth as insignificant. Furthermore we assume that the z component of the plane wave vector equals

$$k_z = \frac{\omega_2}{c} - \frac{c}{2\omega_2}(k_x^2 + k_y^2), \tag{5}$$

which is valid for small k_x and k_y. Terms of a higher order of smallness dropped in (5) can be shown to cause aberrations. However, these aberrations are negligible when z_1 is of the order of several z_0. For large $|z_1|$ the deviation of the parabolic wave front from spherical can exceed $\lambda_2/4$, where $\lambda_2 = 2\pi c/\omega_2$ is the radiation wavelength of frequency ω_2, and the aberrations can become significant. They attain maximum for $|z_1|$ of the order of $2a/\Delta\varphi$, so that when $|z_1| > 2a/\Delta\varphi$ the maximum values of k_x and k_y are determined by the diaphragm dimension rather than by the angle $\Delta\varphi$. When the diaphragm diameter and crystal length are of the order of 1 cm the aberrations are insignificant for all values of $|z_1|$.

The angular field spectrum beyond the diaphragm in the plane $z = 0$ has the form

$$E_1'(k_x, k_y, 0) = \frac{\omega_2 a^2 \varepsilon}{2\pi(\omega_2 a^2 + 2icz_1)} \exp\left\{ i(k_x x_0 + k_y y_0) - \frac{icz_1}{2\omega_2}(k_x^2 + k_y^2) - \frac{\omega_2}{\omega_2 a^2 + 2icz_1}\left[\left(\frac{cz_1 k_x}{\omega_2} - x_0\right)^2 + \left(\frac{cz_1 k_y}{\omega_2} - y_0\right)^2\right]\right\}. \tag{6}$$

Using the results of[4] the angular field spectrum at the frequency sum in the plane $z = z_2$ can be written as

$$E_2(k_x, k_y, z_2) = gE_1'(k_x, k_y, 0)\frac{\sin {}^1\!/_2 \varkappa z_0}{{}^1\!/_2 \varkappa z_0} \exp\left[-\frac{i}{2} z_0 \varkappa + \frac{ic}{2\omega_3}(k_x^2 + k_y^2)\left(z_2 - z_0 + \frac{z_0}{n_3}\right)\right]. \tag{7}$$

Here g is a constant determined by the pump power and crystal nonlinearity, and n_1, n_2, and n_3 are refraction indices for waves with frequencies ω_1, ω_2, and ω_3, and wave vector a along the z axis

$$\varkappa = -\frac{n_1\omega_1 c}{2n_2 n_3 \omega_2 \omega_3}(k_x^2 + k_y^2). \tag{8}$$

The mismatch κ is expressed by (5) when the crystal anisotropy is small (which is always the case) and the angle $\Delta\varphi$ is maximum. Using (6) and (7) we can analyze the resolving power and the size of the visual field.

We limit the analysis to the resolving power at the center of the visual field for various $|z_1|$.

When $x_0 = y_0 = 0$ and $a^2 \gg 2|z_1|c/\omega_2$, which corresponds to the Fresnel zone of the aperture diaphragm, the expression for the angular spectrum in the plane $z = z_2$ has the form

$$E_2(k_x, k_y, z_2) = \frac{\varepsilon g \sin {}^1\!/_2 \varkappa z_0}{\pi \varkappa z_0} \exp\left[\left(-\frac{z_1^2 c^2}{a^2 \omega_2^2} + \frac{iz_l c}{2\omega_3}\right)(k_x^2 + k_y^2)\right]. \tag{9}$$

Here

$$z_l = z_2 - z_1\frac{\omega_3}{\omega_2} - z_0\left(1 - \frac{\omega_3}{2n_2\omega_2} - \frac{1}{2n_3}\right). \tag{10}$$

Varying z_2 we can find the field in various cross sections. Optimum focusing corresponds to $z_l = 0$, i.e.,

$$z_2 = z_1\frac{\omega_3}{\omega_2} + z_0\left(1 - \frac{\omega_3}{2n_2\omega_2} - \frac{1}{2n_3}\right). \tag{11}$$

In this plane the angular spectrum does not contain the phase factor that depends on k_x^2 and k_y^2 and is real. In two planes z = const at equal distances from the focal plane the field is reflected by a complex conjugate field as in the focus of an ideal lens.

It follows from (11) that a change of the z_1 coordinate by the increment Δz_1 is equivalent to a change of z_2 by $(\omega_3/\omega_2)\Delta z$. Thus the longitudinal scale in the image space is larger for frequencies ω_3 and ω_2 in comparison to the object space. The transverse scale is unchanged since k_x and k_y are the same in wave E_2 as in wave E_1'.

If $\Delta\varphi$ is determined from the condition $\sin {}^1\!/_2 \kappa z_0 = 0$, we have

$$\Delta\varphi = \sqrt{\frac{\pi n_2 n_3 \omega_3 c}{n_1 \omega_1 \omega_2 z_0}}. \tag{12}$$

In the region $|z_1| \ll 2a/\Delta\varphi$ (9) can be written for $z_l = 0$ as

$$E_2(k_x, k_y, z_2) = \varepsilon g \frac{\sin {}^1\!/_2 \varkappa z_0}{\pi \varkappa z_0}. \tag{13}$$

The reverse Fourier transform of this expression yields a scattering function, i.e., a point image

$$\Gamma(x, y) = \frac{\varepsilon g}{\pi} Q\left[\frac{\pi}{2} - \operatorname{Si} Q(x^2 + y^2)\right], \quad Q = \frac{n_2 n_3 \omega_2 \omega_3}{n_1 \omega_1 c z_0}. \tag{14}$$

The function $f(\xi) = {}^1\!/_2\pi - \operatorname{Si}\xi^2$ has the same character as the function describing diffraction from a round hole and turns to zero for the first time when $\xi \approx 1.4$. Using a criterion analogous to the Raleigh criterion we find for the resolving power

$$\Delta x_0 = \Delta y_0 = 1{,}4\ Q^{-1/2}. \tag{15}$$

The Raleigh criterion cannot be used throughout the Fresnel region, since the Fourier transform of (9) can be obtained only by numerical methods.

Since any criterion is very approximate and the complete characteristic is represented by the scattering function or its spectrum (9), the resolving power can be determined in terms of the effective spectral width. The effective width of the angular spectrum Δk is readily found from (9) if it is determined by the drop of $E_2(k_x, k_y, z_2)$ from the maximum value at $k_x = k_y = 0$ to the 0.7 level. The values of Δx and Δy are related to Δk by

$$\Delta x = \Delta y = 2\pi / \Delta k. \tag{16}$$

According to this criterion in the region $|z_1| \ll 2a/\Delta\varphi$

$$\Delta x = \Delta y = 1{,}32\ Q^{-1/2}, \tag{17}$$

which is in agreement with (15).

Figure 2 shows $\Delta x Q^{1/2}$ as a function of $a^{-1}|z_1|\Delta\varphi$ computed from (16).

According to this diagram the resolving power remains constant for $|z_1| < 2a/\Delta\varphi$ and corresponds to (15) or (17), while for $|z_1| > 2a/\Delta\varphi$

$$\Delta x \approx \frac{\lambda_2 |z_1|}{1.2a}, \tag{18}$$

where $\lambda_2 = 2\pi c/\omega_2$ is the wavelength of light with frequency ω_2.

We can find from (7) that the visual field for $|z_1| < 2a/\Delta\varphi$ is determined by the diaphragm and has the area πa^2, while for $|z_1| > 2a/\Delta\varphi$ it is determined by the angle $\Delta\varphi$ and has the area $\frac{1}{4}\pi(\Delta\varphi)^2 z_1^2$. Taking (17) and (18) into account we find that the number of resolved elements equal to the ratio of the visual field to Δx^2 is the same in both cases and equals

$$N_{max} = a^2 Q. \tag{19}$$

In the Fraunhofer region where $2|z_1|c/\omega_2$ is of the order of or larger than a^2 we can no longer regard the location of the image plane as $z = z_2$. The object and its image can be considered at infinity so that we must take the angular field of vision and the angular resolving power into account.

The angular field of vision in the Fraunhofer zone is $\Delta\varphi$ and the angular resolving power is determined by the diaphragm spectrum $R(x, y)$ whose number of resolved elements is the same as in the Fresnel zone.

We can show that the converter model used in our analysis, with a diaphragm in the plane $z = 0$, is equivalent to a model with a diaphragm in the plane $z = z_0$ or to a model without a diaphragm, but with a pumping field distribution corresponding to the transparency of the diaphragm. The equivalence occurs for $a \gg \Delta x_0$ which is always the case. Therefore the analysis also applies to a converter with Gaussian pumping.

Thus a transformation on an object image effected by an ordinary optical system, and in particular its removal to infinity (Fourier transformation), fails to be of any advantage with respect to the number of resolved image elements. At the same time, the far-object scheme used in[1-3] is very weak. The pumping divergence and the distortion of phase fronts due to the optical inhomogeneity of the crystal sharply deteriorate the resolving power. In fact the number of resolved elements in the field of vision obtained in[1-3] is much lower than the possible limit N_{max}.

We can show that if the pumping direction inside the crystal is rotated through the angle $\delta\beta$ the image in the plane $z = z_2$ is displaced by δx, where

$$\delta x = [z_0 + (z_2 - z_0)n_3 - z_1 n_2]\delta\beta. \tag{20}$$

It follows from (20) that z_1 and z_2 related by (20) can be selected so as to render $\delta\kappa = 0$. In this case small variations in pumping direction and wave front distortion fail to displace the image and degrade the resolving power.

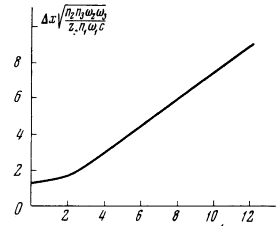

FIG. 2. Resolving power for various z_1.

Furthermore the displacement and degradation of the resolving power are very small for any $|z_1|$ and $|z_2|$ of the order of several z_0.

The same consideration applies to the effect of non-monochromaticity of the radiation on the resolving power.

2. METHOD OF CRYSTAL ADJUSTMENT

We consider the conditions under which frequency conversion by a nonlinear crystal is the least sensitive to variation in signal wave direction.

The wave vectors of frequencies ω_1, ω_2 and $\omega_3 = \omega_1 + \omega_2$ in the crystal are designated by $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$. Here the waves with frequencies ω_1 and ω_2 are assumed ordinary and the frequency sum wave is extraordinary. Let the z axis be normal to the plane of crystal face and lie at a small angle to the direction of one-dimensional match in the crystal for waves with frequencies ω_1, ω_2, and ω_3. The x and y axes are selected so as to contain the optical axis of the crystal in the plane xz.

The power conversion coefficient is proportional to τ:

$$\tau = \frac{\sin^2(\frac{1}{2}\kappa z_0)}{(\frac{1}{2}\kappa z_0)^2}, \tag{21}$$

where z_0 designates the length of the crystal and $\kappa = k_{1z} + k_{2z} - k_{3z}$. Here $k_{1x} + k_{2x} - k_{3x} = 0$, and $k_{1y} + k_{2y} - k_{3y} = 0$. Since the angles between the vectors k_j ($j = 1, 2, 3$) and the z axis are small, when the crystal anisotropy is weak we can write the following with an accuracy up to terms of a higher order of smallness with respect to k_x and k_y[4]:

$$\varkappa = \varkappa_0 + k_3\gamma\beta_x - \frac{k_1 k_2}{2k_3}\left[\left(\alpha_x - \frac{k_3}{k_1}\gamma\right)^2 + \alpha_y^2\right]. \tag{22}$$

The symbol κ_0 designates a constant determined by the angle between the z axis and the optical axis of the crystal for the extraordinary wave of the frequency sum ω_3: k_1, k_2, and k_3 are the lengths of wave vectors k_j when their angles with the z axis are small; $\alpha_x = k_{2x}/k_2 - k_{1x}/k_1$ and $\alpha_y = k_{2y}/k_2 - k_{1y}/k_1$ are angles between the directions of wave vectors $\mathbf{k}_2$ and $\mathbf{k}_1$; $\beta_x = k_{1x}/k_1$ is the angle between the projection of vector $\mathbf{k}_1$ on plane xz and the z axis, and γ is the angle of crystal anisotropy.

Assuming that $\alpha_x = \alpha_y = 0$ and $\kappa = 0$ we express κ_0 by the angle β_x^0 between the direction of exact one-dimensional match (in the xz plane) and the z axis:

$$\varkappa_0 = k_3\gamma\left(\frac{k_2\gamma}{2k_1} - \beta^0_x\right). \tag{23}$$

It follows from (21) and (22) that the conversion coefficient is constant at the circles

$$\left(\alpha_x - \frac{k_3}{k_1}\gamma\right)^2 + \alpha_y^2 = \rho^2. \tag{24}$$

We determine the region of allowable deviations from the exact match direction by the condition $|\frac{1}{2}\kappa z_0| \leq \pi$. The limiting values of ρ_1 and ρ_2 satisfy the relation

$$|\rho_2^2 - \rho_1^2| = 8\pi k_3 / k_1 k_2 z_0. \tag{25}$$

Thus the total solid angle of allowable deviations from

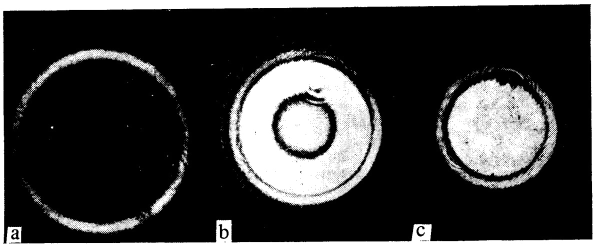

FIG. 3. Image in the focal plane of the lens for various crystal orientations: a – crystal turned 10′ from optimal position; b – crystal turned 3′ from optimal position; c – image at optimal orientation.

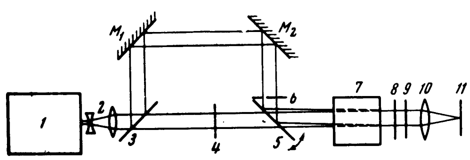

FIG. 4. Diagram of experimental setup. 1 – Nd glass laser; 2 – telescope; 3, 5 – K-8 glass plates; M_1, M_2 – mirrors; 4 – image plane; 6 – resolution grid; 7 – KDP crystal; 8 – SZS-21 filter; 9 – NS-2 filter; 10 – lens; 11 – film.

match $\Omega' = \pi|\rho_2^2 - \rho_1^2|$ does not depend on κ_0 and β_x and the region of allowable deviations is a ring with the center at $\alpha_x = k_3\gamma/k_1$ and boundary radii ρ_1 and ρ_2. The width of the ring increases with decreasing radii ρ_1 and ρ_2. If

$$\beta_x - \beta_x^0 = -k_2\gamma / 2k_1, \tag{26}$$

the match condition $\kappa = 0$ is satisfied for

$$\alpha_x = k_3\gamma / k_1, \quad \alpha_y = 0, \tag{27}$$

and the ring degenerates into a circle with a radius

$$\rho_0 = 2\gamma\sqrt{\pi k_3 / k_1 k_2 z_0}. \tag{28}$$

It is under these conditions that the resolving power is maximum for image conversion of an object near the crystal. In the case of a far object under these conditions the field of vision has the form of a ring rather than a circle. In order to account for all the angles in the space outside the crystal we must multiply the refraction coefficients of the corresponding waves by α and β. Then we obtain (12) from (28).

It can be shown that if (26) is valid the surface defined by vector $\mathbf{k}_1 + \mathbf{k}_2$ is tangent to the surface defined by vector $\mathbf{k}_3$. This proposition was formulated by Warner[2] as a condition for the maximum deviation angle.

Condition (26) can be realized in the following manner. The crystal is rotated about the y axis through a small angle until generation at the frequency sum with a one-dimensional match is attained. The crystal is then rotated about the y axis so as the change angle β_x in the crystal by the amount $-k_2\gamma/2k_1$.

In the degenerate case ($\omega_1 = \omega_2$) that occurred in this work this angle is equal to $-\gamma/2$. In this case the angle between pumping and signal in the crystal with exact match is $k_3\gamma/k_1 \approx 2\gamma$, and the angle between pumping and the direction of wave vector $\mathbf{k}_3$ approximately equals γ.

Exact adjustment was accomplished by placing a lens or ground glass across the signal beam. This produced the image shown in Fig. 3 in the focal plane of the lens behind the crystal. Rotation of the crystal through a small angle can change the circle into a ring.

The effect can be used to increase the field of vision in the case of a far object (and also when the object is removed to infinity by the optical system, i.e., when a Fourier transformation was performed). The field of vision is scanned by rotating the crystal.

3. EXPERIMENTAL RESULTS

Figure 4 shows a diagram of the experimental setup used to convert the image.

A Q-switched single-mode neodymium laser generating at 1.06 μ was used as the signal and pump source. A telescopic system reduced divergence and increased the diameter of the beam up to 9 mm. The pump power incident on the crystal was 0.5 MW. The nonlinear material was represented by a KDP crystal (15 × 15 × 30 mm) cut at the angles $\theta = 41°$ and $\varphi = 45°$. A variable-spacing resolution chart used for testing lenses represented the object. The converted image was photographed on MZ-2 film placed behind the lens that was focused on the image plane $z = z_2$. The grid was 20 cm from the crystal. According to (10) the lens was focused on a plane approximately 40 cm from the crystal. The photograph was taken during a single flash of the laser.

The crystal was adjusted according to the method described in Sec. 2. The pumping beam was found to deviate from the direction of one-dimensional match to such an extent that a single pumping pulse generated a harmonic that was much weaker than that obtained from the sum of pump and signal. The pump emission was finally freed of the parasitic harmonic by diaphragms in front of the lens and in the focal plane of the lens.

Figure 5 shows a typical photograph of the grid image in visible (0.53 μ) light. Each square measures 0.75 × 0.75 mm. The use of a variable-pitch grid enabled us to determine the resolution achieved in the experiment; it was found to be 18 lines/mm. The conversion efficiency was 0.1%. The resolving power computed from (15) yields the same value as that obtained from the experiment. Thus the obtained resolving power was maximum for the given crystal.

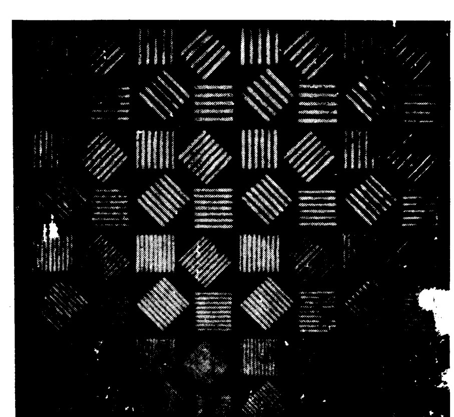

FIG. 5. Photograph of test charge at the summary frequency.

According to (15) any further increase in the resolving power can be obtained by reducing the crystal length or the pumping frequency. In the latter case a multistage conversion may be necessary to obtain a sufficiently high summary frequency.

The total number of resolved elements was determined by the field of vision and amounted in our experiment to 150×150 elements.

We also obtained an image in scattered light. For this purpose ground glass was inserted in front of the grid. In this case the image was degraded because it consisted of points. This always occurs when an image is formed in coherent light[5]. The discrete structure of the image can be removed by averaging during several laser pulses at various positions of the ground glass. This was indeed observed although averaging over 10 pulses proved to be insufficient to obtain the maximum resolution that was obtained without the ground glass.

The ground glass method simulates the conversion of images of scattering solid objects.

In conclusion the authors are indebted to R. V. Khokhlov for suggestion of the subject and discussion of results.

[1] J. E. Midwinter, Appl. Phys. Lett. **12**, 68 (1968).
[2] J. Warner, Appl. Phys. Lett. 13, 360 (1968).
[3] L. Campel, IEEE, J. Quant. Electronics **QE4**, 354 (1968).
[4] Yu. A. Il'inskiĭ and Yu. A. Yanait, Izv. vyssh. uch zav. Radiofizika (in press).
[5] P. S. Considine, J. Opt. Soc. Am. 56, 1001 (1966).

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