THEORY OF FLUCTUATIONS IN A NONEQUILIBRIUM ELECTRON GAS
II. SPATIALLY INHOMOGENEOUS FLUCTUATIONS

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The correlation functions of spatially-inhomogeneous fluctuations in an electron gas located in a strong electric field are calculated for small inelasticity of the collisions. Two limiting cases are considered: 1) the frequency of the interelectronic collisions is equal to zero, or 2) the interelectronic collisions dominate. In the first case the differential conductivity $\sigma_{\alpha\beta}(\mathbf{k}, \omega)$ and the spectral density of the fluctuations of the extraneous currents are found. In the second case the electron temperature approximation is used, and the long wavelength and low frequency fluctuations are represented as fluctuations of the concentration and temperature of the electrons. Transport equations are obtained for these fluctuations, and the correlation functions of the corresponding Langevin sources are calculated.

A knowledge of the correlation functions for the extraneous fluctuations of the spatially-inhomogeneous ($\mathbf{k} \neq 0$) currents in a nonequilibrium gas and of the differential conductivity of such a gas makes it possible to solve a wide range of problems involving fluctuations in a medium containing a nonequilibrium electron gas. A general scheme for the calculation of the fluctuations in a nonequilibrium gas, with collisions taken into consideration (using a Langevin formulation), was developed in a previous article[1] (in what follows, this article will be referred to as I). There the spatially-homogeneous fluctuations were calculated in a semiconductor in which the current carriers were heated by a strong electric field.

In the present article the correlation functions are found for fluctuations with $\mathbf{k} \neq 0$. Two limiting cases are considered: 1) the case when interelectronic collisions are unimportant (Sec. 1) and 2) the case when interelectronic collisions, on the other hand, are important, and notably they determine the form of the symmetric part of the distribution function (Sec. 2).

1. SPECTRAL DENSITY OF THE CURRENT FLUCTUATIONS AND THE DIFFERENTIAL CONDUCTIVITY IN THE ABSENCE OF INTERELECTRONIC COLLISIONS

According to I, the correlation functions for the extraneous fluctuations of the occupation numbers and consequently of the current, the concentration of carriers, and so forth can be expressed in terms of the distribution function (the average value of the occupation number) and the Green's function $g(\mathbf{p}, \mathbf{p}'; \mathbf{k}, \omega)$ for the kinetic equation with a linearized collision integral. Let us consider a non-degenerate electron gas in a strong constant electric field E. For collisions of small inelasticity, when

$$\delta_{\text{eff}} \ll 1 \text{ and, in addition, } kv\tau_1 \ll 1, \quad \omega\tau_1 \ll 1$$

(τ_1 is the relaxation time of the momentum), i.e., in Davydov's approximation the part of the Green's function which is antisymmetric in $\mathbf{p}$ is given by Eq. (I.3.16) (the notation is the same as in I):

$$g_a(\mathbf{p}\mathbf{p}'; \mathbf{k}\omega) = \tau_1(\varepsilon)\left\{\frac{\delta_{\mathbf{p},\mathbf{p}'} - \delta_{-\mathbf{p},\mathbf{p}'}}{2} - \left(i\mathbf{k}\mathbf{v} + e\mathbf{E}\frac{1}{\hbar}\frac{\partial}{\partial \mathbf{p}}\right) g_0(\varepsilon\mathbf{p}'; \mathbf{k}\omega)\right\}, \tag{1.1}$$

and the symmetric part, $g_0(\epsilon\mathbf{p}'; \mathbf{k}\omega)$, as a function of the energy ϵ satisfies the second-order differential equation given by Eqs. (I.3.17) and (I.3.29), which here we write in the form

$$\frac{d}{d\varepsilon}\left[\frac{N(\varepsilon)\varepsilon}{\tau_0(\varepsilon)}\left(T^*\frac{d}{d\varepsilon}+1\right)g_0\right] - \hat{\lambda}(\varepsilon; \mathbf{k}\omega)g_0 = \frac{1}{V}\frac{dF(\varepsilon\mathbf{p}'; \mathbf{k})}{d\varepsilon}. \tag{1.2}$$

Conservation of the number of particles during acceleration by the field and collisions, together with the requirement that g_0 be normalizable leads to two boundary conditions for $\epsilon = 0$ and $\epsilon = \infty$:

$$\lim_{\varepsilon\to 0}\left[\frac{N(\varepsilon)\varepsilon}{\tau_0(\varepsilon)}\left(T^*\frac{d}{d\varepsilon}+1\right)g_0\right] = \lim_{\varepsilon\to\infty}\left[\frac{N(\varepsilon)\varepsilon}{\tau_0(\varepsilon)}\left(T^*\frac{d}{d\varepsilon}+1\right)g_0\right] = 0.$$

According to Eq. (1.2) one of these conditions may be replaced by the equation of continuity

$$\int_0^\infty d\varepsilon\hat{\lambda}g_0 = V^{-1}F(0, \mathbf{p}'; \mathbf{k}). \tag{1.3}$$

In article I, Eq. (1.2) was solved in the low frequency and long wavelength limit, i.e., for

$$\omega\tau_0 \ll 1, \quad ku\tau_0 \ll 1, \quad (kv_T)^2\tau_0\tau_1 \ll 1, \tag{1.4}$$

where τ_0 is the time of scattering of the energy, $\mathbf{u}$ is the drift velocity in the field $\mathbf{E}$, and v_T is the random velocity. Upon fulfilment of the inequalities (1.4) the term λg_0 in Eq. (1.2) is small. However, the equation of continuity (1.3) shows that the function g_0 itself cannot be expanded in powers of small parameters. Nevertheless it turned out to be possible to express it in terms of such solutions of Eq. (1.2), each of which may be found in the form of a series in powers of $\mathbf{k}$ and ω. Namely, Eq. (I.3.30) was obtained, which gives

$$g_0(\varepsilon\mathbf{p}'; \mathbf{k}\omega) = \delta\mathcal{N}^0(\mathbf{p}'; \mathbf{k}\omega)\varphi(\varepsilon; \mathbf{k}\omega) + \psi(\varepsilon\mathbf{p}'; \mathbf{k}\omega), \tag{1.5}$$

where φ denotes the solution of Eq. (1.2) with zero on the right-hand side and ψ denotes a certain particular solution of Eq. (1.2). Each of these functions tends to zero in the necessary way as $\epsilon \to \infty$ and is found by means of successive iterations of Eq. (1.2). In order to find the factor $\delta\mathcal{N}$ which does not depend on ϵ, it is necessary to substitute (1.5) into (1.4). In contrast to I, here we shall use the solution for g_0 containing the functions φ and ψ which satisfy the conditions

$$\int_0^\infty d\varepsilon\, 2N(\varepsilon)\varphi(\varepsilon)=1, \qquad \int_0^\infty d\varepsilon\, N(\varepsilon)\psi(\varepsilon)=0.$$

This is convenient because then $\delta\mathcal{N}(\mathbf{p}';\mathbf{k}\omega)$ is the Fourier amplitude of the change in the concentration of particles. The expression for g_0 obtained as a result only differs from Eq. (I.3.30) by a certain regrouping of the terms.

Knowing $g(\mathbf{pp}';\mathbf{k}\omega)$ one can find the differential conductivity $\sigma_{\alpha\beta}(\mathbf{k}\omega)$ for hot electrons. It is needed, in particular, in order to calculate the correlation functions of the observed current fluctuations with fluctuations of the field taken into account from the correlation functions of the extraneous currents, which we obtain below. For simplicity let us write out only that part of the electrical conductivity tensor which describes the response in an irrotational field. According to Eqs. (I.4.9), (1.1), and (I.3.30–31)

$$\sigma_{\alpha\beta}^{l}(\mathbf{k}\omega)=\sigma_{\alpha\beta'}(\mathbf{k}\omega)\frac{k_{\beta'}k_\beta}{k^2}$$
$$=\sigma_{\alpha\beta}'+\frac{u_\alpha-iD_{\alpha\alpha'}k_{\alpha'}-i(\omega-\mathbf{ku})u_\alpha\tau_0^{(2)}}{\omega-\mathbf{ku}+ik_{\alpha'}D_{\alpha'\beta'}k_{\beta'}+i\mathbf{ku}(\omega-\mathbf{ku})\tau_0^{(2)}}k_\gamma\sigma_{\gamma\beta}'. \quad (1.6)$$

Here $\sigma'_{\alpha\beta}=\sigma\delta_{\alpha\beta}+2(d\sigma/dE^2)E_\alpha E_\beta$ is the static differential conductivity tensor, $\sigma\equiv j/E$, and

$$D_{\alpha\beta}=D_\perp\delta_{\alpha\beta}+u_\alpha u_\beta\tau_0^{(1)}, \qquad D_\perp=\frac{2}{n}\int_0^\infty dx\,N(x)\frac{v^2\tau_1}{3}n_0(x). \quad (1.7)$$

$$\tau_0^{(1)}=\frac{2}{n}\int_0^\infty dx\,N(x)x\tau_0(x)\left(-\frac{dn_0}{dx}\right)$$
$$\times\left[\left(\frac{\mu(x)/\mu(0)-f(x)}{N(x)xn_0(x)/n}\right)^2-\left(\frac{2e\tau_1}{3m\mu(0)}\right)^2\right],$$

$$\tau_0^{(2)}=\frac{2}{n}\int_0^\infty\frac{dx f(x)\tau_0(x)}{T^*(x)}\left[\frac{f(x)-\mu(x)/\mu(0)}{N(x)xn_0(x)/n}+\frac{2e\tau_1(x)}{3m\mu(0)}\right],$$

$$\mu(\varepsilon)=\frac{2e}{n}\int_\varepsilon^\infty dx\,N(x)\frac{2x\tau_1}{3m}\left(-\frac{dn_0}{dx}\right),$$

$$f(\varepsilon)=\frac{2}{n}\int_\varepsilon^\infty dx\,N(x)n_0(x).$$

In the approximation adopted here $(\mathbf{ku}\tau_0\ll 1)$ the pole of the electrical conductivity (1.6) and of the Green's function is determined by the expression

$$\omega=\mathbf{ku}+ik_\alpha D_{\alpha\beta}k_\beta.$$

From here it follows that the tensor $D_{\alpha\beta}$ describes diffusion damping of small perturbations in an electron gas and may be called the tensor of the coefficients of diffusion. The quantity $D_\perp$ is the coefficient of diffusion in a direction perpendicular to the strong field, and

$$D_\parallel=D_\perp+u^2\tau_0^{(1)}$$

is the longitudinal coefficient of diffusion.

Sometimes it is more convenient to use not $\sigma_{\alpha\beta}(\mathbf{k}\omega)$ but a formula for $\delta\mathbf{j}$ which expresses the current in terms of δn, the change in the concentration of carriers. Having made use of Eq. (1.5), we obtain

$$\delta j_\alpha(\mathbf{k}\omega)=\sigma_{\alpha\beta}'\delta E_\beta(\mathbf{k}\omega)+e[u_\alpha-iD_{\alpha\beta}k_\beta-i(\omega-\mathbf{ku})u_\alpha\tau_0^{(2)}]\delta n(\mathbf{k}\omega). \quad (1.8)$$

One can use expression (1.8) for $\delta\mathbf{j}$ provided $\sigma\tau_0\ll 1$. In the opposite case, on the right-hand side of Eq. (1.8) it is necessary to take into account the terms of the order of the product of $\sigma\delta\mathbf{E}$ times the small parameters $i\mathbf{ku}\tau_0$ and $i\omega\tau_0$ to the first and second powers. These corrections arise from terms of first and second order in ψ. Equation (1.8) together with the equation of continuity

$$i\omega e\delta n=i\mathbf{k}\delta\mathbf{j}$$

forms a system of hydrodynamical equations for the gas under consideration, obtained to within the diffusion corrections to the current inclusively.

We obtain the correlation functions of the extraneous currents by substituting expressions (1.1) and (1.5) into formula (I.4.6)

$$\langle\delta j_\alpha^{\mathrm{ex}}(\mathbf{r}_1t_1)\delta j_\beta^{\mathrm{ex}}(\mathbf{r}_2t_2)\rangle_{\mathbf{k}\omega}$$
$$=\frac{2ne^2}{V}\left\{D_{\alpha\beta}+\frac{D_\perp(\omega-\mathbf{ku})(k_\alpha u_\beta+k_\beta u_\alpha)+k_{\alpha'}D_{\alpha'\beta'}k_{\beta'}(u_\alpha u_\beta-k_\alpha k_\beta D_\perp{}^2)}{(\omega-\mathbf{ku})^2+(k_{\alpha'}D_{\alpha'\beta'}k_{\beta'})^2+2k_{\alpha'}k_{\beta'}D_{\alpha'\beta'}(\omega-\mathbf{ku})\mathbf{ku}\tau_0{}^{(2)}}\right\} \quad (1.9)$$

Upon a slight heating of the electron gas, i.e., for $u^2\tau_0\ll v^2\tau_1$, the expression written down goes over into Eq. (I.4.18). If the field is strong, certain terms both in the numerator and in the denominator of (1.9) may be of different order with respect to the small parameter $\mathbf{ku}\tau_0$ depending on the relation between ω and $\mathbf{k}\cdot\mathbf{u}$, and the choice of the tensor indices α and β. However, they must be retained because they depend on the orientations of $\mathbf{k}$ and $\mathbf{u}$ in different ways, and under certain conditions may give a contribution of the order of the first term inside the curly brackets, i.e., of the order of $D_\perp$.

From Eq. (1.9) it follows that the spectral density of the spatially-homogeneous fluctuations of the current at zero frequency in a semiconductor containing hot electrons and in the absence of interelectronic collisions is directly related to the diffusion tensor by[1)]

$$\langle\delta j_\alpha^{\mathrm{ex}}(\mathbf{r}_1t_1)\delta j_\beta^{\mathrm{c}}(\mathbf{r}_2t_2)\rangle_{0,0}=\frac{2ne^2}{V}D_{\alpha\beta}. \quad (1.10)$$

The relation between the spectral density of the transverse fluctuations of the current and the transverse coefficient of diffusion, $D_\perp$, which was obtained in I (see Eq. (I.4.10), is a special case of this formula. From Eq. (1.10) it follows that the spectral density of the current fluctuations in the direction of a strong field is proportional to $D_\parallel$. From the comparison carried out in I (Sec. 4) of the spectral densities of the current fluctuations along the field and perpendicular to the field, it follows that $D_\parallel=D_\perp$ provided the relaxation time τ_1 of the momentum does not depend on the energy, $D_\parallel<D_\perp$ for the scattering of electrons by the deformation potential of the acoustic phonons ($\tau_1\propto\epsilon^{-1/2}$), and $D_\parallel>D_\perp$ for $d\tau_1/d\epsilon>0$.

Let us note one more property of the correlator of the currents which may turn out to be useful. For co-

[1)]This relation is also obtained in an article by Gantsevich, Gurevich, and Katilyus. [2] We are grateful to the authors of this article for sending us a preprint.

incident times it obviously is given by

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \langle \delta j_\alpha^{\text{ex}}(\mathbf{r}_1 t_1) \delta j_\beta^{\text{ex}}(\mathbf{r}_2 t_2) \rangle_{\omega, \mathbf{k}=0} = \frac{2e^2}{V^2} \sum_{\mathbf{p}} (v_\alpha - u_\alpha)(v_\beta - u_\beta) n(\mathbf{p}).$$

This expression is valid for arbitrary inelasticity of the scattering. In principle it enables one, with the aid of measurements of the drift velocity and the integral over frequency of the spectral density of the current fluctuations, to obtain information about the harmonics of second order in the expansion of $n(\mathbf{p})$ in terms of spherical functions.

2. FLUCTUATIONS ASSOCIATED WITH A LARGE FREQUENCY OF THE INTERELECTRONIC COLLISIONS

Now let us go to the case when the interaction between electrons is essential and may be described by a pair collision integral.[2)] Just as in the preceding Section, we shall confine our attention to conditions under which the inelasticity of the scattering is small. Then, as is well-known,[6,7] for a solution of the kinetic equation one can use the electron temperature approximation, according to which the symmetric part of the distribution function has the form

$$n_0(\varepsilon \mathbf{r} t) = \frac{n(\mathbf{r}t)}{N_c(T)} \exp[-\varepsilon / T(\mathbf{r}t)].$$

Here $n(\mathbf{r}t)$ and $T(\mathbf{r}t)$ denote the local concentration and temperature of the electrons, and $N_c(T)$ is the effective density of states. In this case the symmetric part of the fluctuations $\delta n_0(\epsilon \mathbf{r}t)$ in the occupation number may be expressed in terms of fluctuations of the hydrodynamical parameters, that is, in terms of the concentration $\delta n(\mathbf{r}t)$ and temperature $\delta T(\mathbf{r}t)$ of the electrons whose dependence on $\mathbf{r}$ and t is described by the transport equations. Calculation of the correlation functions reduces to a solution of these equations. As a consequence of their obviousness, in the present case such a method is more convenient than the application of the general formulas of article I containing the Green's function found in the electron temperature approximation.

Since the collisions of electrons with each other, with phonons, and with impurities are statistically independent, one can represent Eq. (I.2.7) for the fluctuations of the occupation number in the form

$$\hat{L}(\mathbf{p}\mathbf{r}t)\,\delta n(\mathbf{p}\mathbf{r}t) - S_l'(\mathbf{p}) - S_{ee}'(\mathbf{p}) + \frac{e}{\hbar}\delta \mathbf{E}(\mathbf{r}t)\frac{\partial n}{\partial \mathbf{p}} = \delta J_l(\mathbf{p}\mathbf{r}t) + \delta J_{ee}(\mathbf{p}\mathbf{r}t). \quad (2.1)$$

We have denoted the linearized collision integral and the extraneous fluxes of particles in the state $\mathbf{p}$, $\mathbf{r}$, associated with scattering by phonons and by impurities, by $S_l'(\mathbf{p})$ and $\delta J_l(\mathbf{prt})$, and the corresponding terms associated with interelectronic collisions are denoted by $S_{ee}'(\mathbf{p})$ and $\delta J_{ee}(\mathbf{prt})$. The fluxes δJ_l and δJ_{ee} are not correlated, which corresponds to the additive nature of the collision integrals S_l and S_{ee}.

Let us divide the fluctuations $\delta n(\mathbf{prt})$ into symmetric and antisymmetric parts with respect to $\mathbf{p}$. In the approximation of Davydov they contain, respectively, the harmonics with $l = 0$ and $l = 1$ in the expansion of $\delta n(\mathbf{p})$ in terms of spherical functions.[3)]

For $\omega\tau_1 \ll 1$ and $kv\tau_1 \ll 1$

$$\delta n_a(\mathbf{prt}) = \tau_1(\varepsilon)\left\{\delta J_a(\mathbf{prt}) - \mathbf{v}\frac{\partial \delta n_0}{\partial \mathbf{r}} - \frac{e\mathbf{E}}{\hbar}\frac{\partial \delta n_0}{\partial \mathbf{p}} - \frac{e}{\hbar}\frac{\partial n_0}{\partial \mathbf{p}}\delta \mathbf{E}(\mathbf{r}t)\right\}. \quad (2.2)$$

Here $\delta J_a(\mathbf{p}) = [\delta J_l(\mathbf{p}) - \delta J_l(-\mathbf{p})]/2$, and the terms associated with interelectronic collisions are omitted under the assumption that the frequency of these collisions $\nu_{ee} \lesssim \tau_1^{-1}$, and they do not have any effect on the scattering of momentum[6] (this implies the neglect of viscosity). For $\delta n_0(\epsilon \mathbf{r}t)$ we have

$$\frac{\partial \delta n_0}{\partial t} + \mathbf{v}\frac{\partial \delta n_a}{\partial \mathbf{r}} + \frac{e\mathbf{E}}{\hbar}\frac{\partial \delta n_a}{\partial \mathbf{p}} - S_l'(\mathbf{p}; \delta n_0) - S_{ee}'(\mathbf{p}; \delta n_0) + \frac{e}{\hbar}\frac{\partial n_0}{\partial \mathbf{p}}\delta \mathbf{E}(\mathbf{r}t) = \delta J_0(\varepsilon \mathbf{r}t) + \delta J_{ee0}(\varepsilon \mathbf{r}t), \quad (2.3)$$

where $\delta J_0 = [\delta J_l(\mathbf{p}) + \delta J_l(-\mathbf{p})]/2$.

We shall regard the following as small parameters:

$$(\nu_{ee}\tau_0)^{-1}, \ \omega/\nu_{ee}, \ ku/\nu_{ee}, \ k^2v^2\tau_1/\nu_{ee} \ll 1. \quad (2.4)$$

The expansion of δn_0 in terms of ν_{ee}^{-1} is analogous to the well-known method of a small Hilbert parameter in the kinetic theory of gases.[8,9] In the zero-order approximation the desired function satisfies the equation[4)]

$$S_{ee}'\{\delta n_0\} = 0.$$

This equation has two linearly independent solutions:

$$\partial n_0(\varepsilon \mathbf{r}t)/\partial n \quad \text{and} \quad \partial n_0(\varepsilon \mathbf{r}t)/\partial T,$$

so that the general solution is

$$\delta n_0(\varepsilon \mathbf{r}t) = \frac{\partial n_0(\varepsilon \mathbf{r}t)}{\partial n}\delta n(\mathbf{r}t) + \frac{\partial n_0(\varepsilon \mathbf{r}t)}{\partial T}\delta T(\mathbf{r}t). \quad (2.5)$$

It is easy to understand that the coefficients δn and δT have the meaning of fluctuations in the concentration and temperature of the electrons. The equations for these quantities are obtained, as usual, by integration of (2.3) containing 1 and ϵ, and have the meaning of the equation of continuity and the equation of energy transport for the fluctuations. From the form of δn_0 as given by Eq. (2.5), it follows that the left-hand parts may be obtained from the linearized equations of continuity and energy conservation, which determine the average concentration and temperature of the electrons. In the right-hand sides of the equations for δn and δT appear the corresponding moments from $\delta J_l(\mathbf{prt})$, which play the role of sources of the fluctuations in the resulting Langevin transport equations. The moments of δJ_{ee} are equal to zero because of

2)Strictly speaking, the pair collision integral in a plasma exists only upon taking the dynamical screening of the Coulomb interaction into account.[3-5] However, it is sufficient for us to confine our attention to a model formulation of the problem in which the finite nature of the pair collision integral is guaranteed by a suitable cutoff of the interaction cross section. [5]

3)The applicability of Davydov's approximation to the kinetic equation, in the right-hand side of which spherical harmonics with arbitrary values of l may occur with the same strength, is proved in the same way as the applicability of this approximation to the solution of the equation for the Green's function (I, Sec. 3).

4)Since the spectral density of the fluxes δJ_{ee} (see Eq. (I.2.14)) is proportional to ν_{ee}, one can regard the extraneous flux δJ_{ee} as proportional to $\sqrt{\nu_{ee}}$ and therefore – upon fulfillment of the inequalities (2.4) – as small in comparison with S_{ee}'.

conservation of the number of particles and of the energy during interelectronic collisions.

One can represent the resulting equations in the form

$$\delta\left\{e\frac{\partial n}{\partial t}+\operatorname{div}\mathbf{j}\right\}=0, \tag{2.6a}$$

$$\delta\left\{\frac{\partial n\bar{\varepsilon}(T)}{\partial t}+\operatorname{div}\mathbf{j}_\varepsilon-\mathbf{jE}+P(n,T)\right\}=\tilde{U}(\mathbf{r}t),$$

$$\delta\equiv\delta n\frac{\partial}{\partial n}+\delta T\frac{\partial}{\partial T}+\delta\mathbf{E}\frac{\partial}{\partial\mathbf{E}}. \tag{2.6b}$$

Here $n = n(\mathbf{r}t)$, $T = T(\mathbf{r}t)$, $\bar{\varepsilon}(T)$ is the average energy per electron, and

$$P(n,T)=-\frac{2}{V}\sum_{\mathbf{p}}\varepsilon S_l(\mathbf{p})=\frac{2}{V}\sum_{\mathbf{pp}'}(\varepsilon-\varepsilon')W_{\mathbf{pp}'}n_0(\varepsilon\mathbf{r}t)$$

is the specific power transmitted by the electrons to the phonons. Expressions are obtained for the current density and for the energy flux density of the fluctuations with the aid of formulas (2.2) and (2.5)

$$\delta\mathbf{j}=\delta\{en\mu(T)\mathbf{E}-e\nabla(D(T)n)\}+\tilde{\mathbf{I}}, \tag{2.6c}$$

$$\delta\mathbf{j}_\varepsilon=\delta\left\{\left(\frac{e\mathbf{E}}{T}-\nabla\right)nT^{1/2}\frac{\partial}{\partial T}[T^{1/2}D(T)]\right\}+\tilde{\mathbf{Q}}, \tag{2.6d}$$

$$D=\frac{2}{nV}\sum_{\mathbf{p}}\frac{v^2\tau_1}{3}n_0(\mathbf{p}),\qquad \mu=\frac{2e}{nV}\sum_{\mathbf{p}}\frac{v^2\tau_1}{3}\left(-\frac{dn_0}{d\varepsilon}\right)$$

The Langevin sources of the fluctuations (here we follow the terminology of Lax[10]) are given by

$$\tilde{U}(\mathbf{r}t)=\frac{2}{V}\sum_{\mathbf{p}}\varepsilon\delta J_l(\mathbf{p}), \tag{2.7}$$

$$\tilde{I}_\alpha(\mathbf{r}t)=\frac{2e}{V}\sum_{\mathbf{p}}v_\alpha\tau_1\delta J_l(\mathbf{p}),\qquad \tilde{Q}_\alpha(\mathbf{r}t)=\frac{2}{V}\sum_{\mathbf{p}}\varepsilon v_\alpha\tau_1\delta J_l(\mathbf{p}).$$

They have the following meaning. The Langevin source of energy $\tilde{U}$ describes that part of the fluctuations of the power transmitted from the electrons to the lattice which is associated with the random character of phonon emission and absorption, i.e., the exchange of energy with a thermostat. The current source $\tilde{\mathbf{I}}$ and the source of energy flux $\tilde{\mathbf{Q}}$ are related to the statistical nature of the scattering of the electrons' momentum.

It remains to add the formulas for the correlation functions of the fluctuation sources $\tilde{U}$, $\tilde{\mathbf{I}}$, and $\tilde{\mathbf{Q}}$ to the system of transport equations. After this has been done, the Langevin formulation of the problem of fluctuations in the electron temperature approximation can be regarded as complete. With the aid of Eqs. (2.7) and (I.2.10) we find

$$\langle\tilde{U}(\mathbf{r}_1t_1)\tilde{U}(\mathbf{r}_2t_2)\rangle=\delta(t_1-t_2)\delta(\mathbf{r}_1-\mathbf{r}_2)\frac{2}{V}\sum_{\mathbf{pp}'}(\varepsilon-\varepsilon')^2n_0(\varepsilon)W_{\mathbf{pp}'}$$

$$=2\delta(t_1-t_2)\delta(\mathbf{r}_1-\mathbf{r}_2)n(\mathbf{r}t)P(T)(T_0^{-1}-T^{-1})^{-1}, \tag{2.8a}$$

$$\langle\tilde{I}_\alpha(\mathbf{r}_1t_1)\tilde{I}_\beta(\mathbf{r}_2t_2)\rangle=2\delta(t_1-t_2)\delta(\mathbf{r}_1-\mathbf{r}_2)\delta_{\alpha\beta}e^2n(\mathbf{r}t)D(T), \tag{2.8b}$$

$$\langle\tilde{Q}_\alpha(\mathbf{r}_1t_1)\tilde{Q}_\beta(\mathbf{r}_2t_2)\rangle=2\delta(t_1-t_2)\delta(\mathbf{r}_1-\mathbf{r}_2)\delta_{\alpha\beta}nT^{-3/2}\frac{\partial^2(T^{3/2}D)}{\partial(T^{-1})^2}, \tag{2.8c}$$

$$\langle\tilde{I}_\alpha(\mathbf{r}_1t_1)\tilde{Q}_\beta(\mathbf{r}_2t_2)\rangle=2\delta(t_1-t_2)\delta(\mathbf{r}_1-\mathbf{r}_2)\delta_{\alpha\beta}enT^{1/2}\frac{\partial}{\partial T}(T^{1/2}D), \tag{2.8d}$$

where T_0 denotes the temperature of the phonons. The smallness of the change in an electron's energy during scattering was taken into consideration in the derivation of all expressions.

The correlators $\langle\tilde{U}\tilde{I}\rangle$ and $\langle\tilde{U}\tilde{Q}\rangle$ are expressed in terms of the function $\langle\delta J_0\delta J_a\rangle$, which is proportional to n_a and, for small inelasticity of the scattering, even in a strong field is small in comparison with the functions $\langle\delta J_0\delta J_0\rangle$ and $\langle\delta J_a\delta J_a\rangle$ which are proportional to n_0. Therefore, in problems which are of interest (for example, problems about current fluctuations) the correlators $\langle\tilde{U}\tilde{I}\rangle$ and $\langle\tilde{U}\tilde{Q}\rangle$ give a contribution which differs from the others by an extra power of the inelasticity parameter and which must be discarded.[5)]

The structure of the Langevin transport equations which have been derived agrees with the form of the equations introduced by Landau and Lifshitz[12] in order to describe hydrodynamical fluctuations. Since they considered fluctuations in a one-component liquid, in which the dissipative effects are due only to the viscosity, it is natural that the correlation functions of the extraneous sources obtained in their work can be expressed in terms of kinetic coefficients, which differ from zero only upon taking the interparticle interaction into account. By using the method presented above, one can derive equations for the fluctuations of hydrodynamical quantities from the kinetic equation.

In conclusion we note that if one sets $\delta\mathbf{E} = 0$ in the transport equations (2.6) and determines the current fluctuation $\delta\mathbf{j}$, then it will have the meaning of an extraneous random current $\delta\mathbf{j}^{\mathrm{ex}}$, which comes out in the theory of fluctuations in electrodynamics.[13,14]

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[5)] In article [11] the correlation function $\langle\tilde{U}\tilde{I}\rangle$ was discarded with a reference to the statistical independence of the mechanisms for the scattering of energy and momentum. From what has been said it follows that the results obtained in [11] are always valid when $\tau_1 \ll \tau_0$.

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