PASSAGE OF ELECTRONS AND ATOMS THROUGH A MAGNETIZED MEDIUM

V. G. BARYSHEVSKIĬ and I. D. FERANCHUK

Belorussian State University

Submitted May 25, 1969

Zh. Eksp. Teor. Fiz. 57, 2107–2111 (December, 1969)

It is shown that the rotation frequency of the spin of an electron beam traversing a medium with polarized electrons is not determined by the macroscopic magnetic field of the medium. The effect of this circumstance on splitting of the ground and excited levels of atoms moving in a polarized paramagnetic gas is discussed.

LET a beam of electrons be incident on a substance with polarized atomic electrons. At first glance it appears that just as in the case of neutrons, for example, the spin of the incident electrons will precess with a frequency determined by the macroscopic magnetic field **B** of the sample. We note, however, that owing to the identity of the electrons, there exists for them, unlike for neutrons, not only the direct process of elastic coherent scattering, which leads to spin precession in the field **B**, but also an additional exchange scattering that depends on the spin state of the colliding particles. The contribution made to the exchange scattering comes from two processes connected respectively with the exchange Coulomb and exchange magnetic scatterings. According to[1], the presence of interaction that depends on the spin state of the colliding particles signifies that in a polarized target the beam is acted upon by a certain effective field, which leads to precession of the spin of the incident particles with frequency

$$\omega = kv(n_{\uparrow\uparrow} - n_{\downarrow\uparrow}), \tag{1}$$

where k is the wave vector of the beam particles, v is their velocity, $n_{\uparrow\uparrow}$ is the refractive index of the particle with spin parallel to the target polarization vector, and $n_{\downarrow\uparrow}$ is the same for a particle with a spin antiparallel to the target polarization vector.

Using the well known connection between the refractive index and the amplitude of elastic coherent scattering forward (see, for example,[2]), we find that the contribution of the exchange processes to the refractive index of an electron with spin directed parallel to the polarization vector of the target electrons is

$$\delta n_{\uparrow\uparrow} = -\frac{2\pi N}{k^2}(f'_{\uparrow\uparrow} + f''_{\uparrow\uparrow}), \tag{2}$$

where $f'_{\uparrow\uparrow}$ and $f''_{\uparrow\uparrow}$ are the amplitudes of elastic coherent scattering forward due to exchange Coulomb and magnetic interaction, respectively, and the minus sign is connected with the fact that in the triplet state the exchange amplitude is subtracted from the amplitude of the direct process; it is assumed for simplicity that the electrons of the medium are completely polarized.

The contribution of the exchange to the real part of the refractive index of the electrons with spin antiparallel to the polarization vector of the scatterers, $\delta n_{\downarrow\uparrow}$, is equal to zero. This is connected with the fact that in this case, as a result of the exchange, the spins of both the incident electron and the electron of the medium reverse direction. Consequently, such a process is incoherent and leads only to absorption. Thus, the additional contribution to the difference $n_{\uparrow\uparrow} - n_{\downarrow\uparrow}$, in comparison with the difference of the refractive indices $n^{(0)}_{\uparrow\uparrow} - n^{(0)}_{\downarrow\uparrow}$ of the electrons in the magnetic field of the target **B**, can be written in the form

$$\delta n_{\uparrow\uparrow} - \delta n_{\downarrow\uparrow} = -\frac{2\pi N}{k^2}(f'_{\uparrow\uparrow} + f''_{\uparrow\uparrow}). \tag{3}$$

Using (1) and (3), we find that the change produced in the electron spin precession frequency by the exchange scattering is

$$\Delta\omega = -\frac{2\pi N\hbar}{m}(f'_{\uparrow\uparrow} + f''_{\uparrow\uparrow}), \tag{4}$$

where m is the electron mass. Thus, the frequency of the spin precession of the electron beam in a polarized target is

$$\omega = \omega_B + \Delta\omega,$$

where ω_B is the electron spin precession frequency in the ordinary macroscopic field **B**.

For sufficiently fast electrons (with energy on the order of several keV and higher) at $ka \gg 1$ (a—dimension of the atom), the values of $f'_{\uparrow\uparrow}$ and $f''_{\uparrow\uparrow}$ practically coincide with the amplitudes of elastic exchange scattering forward by a free electron initially at rest. As a consequence, we can write the following expressions for $f'_{\uparrow\uparrow}$ and $f''_{\uparrow\uparrow}$:

$$f'_{\uparrow\uparrow} = -\frac{e^2}{mv^2}, \quad f''_{\uparrow\uparrow} = \frac{2m\mu^2}{\hbar^2}\sin^2\vartheta,$$

where μ is the magnetic moment of the electron, and ϑ is the angle between the incident-electron momentum and the polarization vector of the electrons of the substance. Hence

$$\omega = \omega_B + \frac{2\pi N\hbar}{m}\left(\frac{e^2}{mv^2} - \frac{2m\mu^2}{\hbar^2}\sin^2\vartheta\right). \tag{5}$$

According to (5), the exchange process leads to a dependence of ω on the electron velocity in the direction of their propagation relative to the target polarization vector. The dependence on the velocity is connected in this case with the exchange Coulomb scattering, and the dependence on the angle ϑ is connected with the magnetic exchange scattering. We recall that in the nonrelativistic case the frequency ω_B of the spin precession in a magnetic field **B** does not depend on either the velocity of the incident particle or on the direction of its motion. On the other hand, owing to the dependence of **B** on the shape of the sample, the frequency ω_B also depends on the shape of the target.

For electrons with energy $E \approx 10$ keV and $N \approx 10^{22}$, the contribution to ω due to the Coulomb exchange scattering is

$$\Delta\omega' = \frac{2\pi N\hbar}{m}\frac{e^2}{mv^2} \approx 10^{12}\ \text{sec}^{-1}.$$

The corresponding effective field (compare with[1]) $G' = \hbar\Delta\omega'/2\mu \approx 10^5$ G and increases with decreasing beam energy.

The contribution to the spin precession frequency from the magnetic exchange interaction does not depend on the energy, and at $\vartheta = \pi/2$ it equals $\Delta\omega'' = 4\pi N\mu^2/\hbar \approx 10^{10}\ \text{sec}^{-1}$. The corresponding effective field $G'' = \hbar\Delta\omega''/2\mu \approx 10^3$G and has the same order of magnitude as the macroscopic magnetic field produced by the polarized electrons of the medium (in the case of scattering in a plate, with the electrons polarized parallel to its surface, we have $G'' = (\frac{1}{2})(B - H)\sin^2\vartheta$).

Thus, owing to the exchange scattering, the spin of the incident electrons in a polarized substance is acted upon not by the magnetic field $\mathbf{B}$, but by a certain effective field $\mathbf{J} = \mathbf{B} + \mathbf{G}' + \mathbf{G}''$. For electrons with energy ≈ 10 keV and less we have $G' \gg B, G''$. As a consequence, $\Delta\omega' \gg \omega_B, \Delta\omega''$ and $\omega \approx \Delta\omega'$. Thus, in this case the beam spin precession is not determined by the magnetic field $\mathbf{B}$ at all. With increasing energy of the incident particle, the field G' decreases and at an incident particle energy ~ 1 MeV it becomes comparable with the fields B and G''. In this case $\omega_B \simeq \Delta\omega' \approx \Delta\omega''$ and the frequency ω is determined by all three types of interaction.

The length over which complete reversal of the beam spin takes place is $l = 2\pi v/\omega$. For electrons with energy $E \lesssim 10$ keV, the precession is determined only by the exchange Coulomb frequency $\Delta\omega'$ and $l \approx 2\pi v/\Delta\omega' \lesssim 10^{-2}$ cm. For electrons with energy 1 MeV the frequencies are $\omega_B \approx \Delta\omega' \approx \Delta\omega'' \approx 10^{10}\ \text{sec}^{-1}$ and the length $l = 20$ cm.

So far, we have not taken into account incoherent processes. It is important to note that incoherent elastic and inelastic scattering (for example, ionization) is not accompanied by a change in the spin state of the beam, although it does lead to deceleration of the electrons, without destroying the precession of their spin in the effective exchange field. This means that not only the spin of the rapidly attenuating coherent wave will precess, but also the spin of the electrons that slow down as a result of the inelastic scattering. The situation is perfectly analogous here to precession of decelerated electrons in an ordinary magnetic field. Processes of scattering with spin flip may be dangerous. However, it turns out that the spin-reversal length l is much shorter than the absorption length of one of the components of the beam, $L = 1/N\sigma_{\uparrow\downarrow}$ ($\sigma_{\uparrow\downarrow}$ is the cross section for scattering with spin flip; $\sigma_{\uparrow\downarrow} = \sigma^1_{\downarrow\uparrow} + \sigma^2_{\downarrow\uparrow} + \sigma^3_{\downarrow\uparrow}$, where $\sigma^{1,2,3}_{\downarrow\uparrow}$ are the cross sections for the scattering respectively with spin flip due to the Coulomb-exchange, magnetic, and magnetic-exchange scattering. Indeed, $\sigma^{2,3}_{\downarrow\uparrow} \approx r_0^2 \approx 10^{-25}$ cm^2 (r_0—classical radius of the electron), i.e., $N\sigma^{2,3}_{\downarrow\uparrow} \approx 10^{-3}$ and we have $l N\sigma^{2,3}_{\downarrow\uparrow} \ll 1$ even in the region of energies of the order of 1 MeV. Consequently, the contribution to L due to the the magnetic collisions with spin flip can be neglected.

As to the role of collisions with spin flip due to the Coulomb exchange scattering, we have in the case of sufficiently rapid electrons

$$\sigma^1_{\downarrow\uparrow} = \frac{16\pi}{3}\frac{me^2}{\hbar^2}\left(\frac{e^2}{mv^2}\right)^3.$$

As a result, $l/L = (4/3)\pi(4me^2/\hbar^2k)^3$. For electrons with energy $E \approx 10$ keV, the ratio $l/L \approx 0.03$ and decreases with further increase of energy.

Attention must be called to the fact that the beam electron spin precesses around the direction of the effective field $\mathbf{J}$ in the medium. Therefore, if the direcof the beam momentum does not coincide with the direction of $\mathbf{J}$, then the spin of the incident electrons will flip relative to the direction momentum. At electron energies ≈ 10 keV, the direction of $\mathbf{J}$ is determined by the direction of the field $\mathbf{G}'$, i.e., of the effective field $\mathbf{G}'$ connected with the exchange Coulomb scattering, so that in this case the direction of $\mathbf{J}$ coincides with the direction of the target electron polarization $\mathbf{P}$, and does not depend on the direction of the beam momentum $\mathbf{p}$. Let now the longitudinally polarized beam of electrons be incident on the target perpendicular to $\mathbf{P}$. As a result of the precession, the spin of the electrons will turn relative to the momentum $\mathbf{p}$ after going through the substance. Thus, after passing through the target, the electron beam has a spin directed at an angle to the momentum, depending on the path covered. For example, at a fixed energy, a longitudinal-polarized beam becomes transversely polarized after covering a path $x = l/4$.

To maintain the beam energy constant (with increasing path length), it is possible to place the target in an electric field. The target can then be constructed in the form of an aggregate of polarized thin plates spaced a certain distance apart. Since the electric field does not change the spin state of the particle, such a system of plates acts in fact like one thick plate with a thickness equal to the sum of the thicknesses of the thin plates.

We note that fields analogous to G' and G'' act not only on the free electrons but also on the electrons of the atoms (μ-mesic atoms, positronium) contained in a beam passing through the medium with the polarized electrons. These fields lead to a splitting of the ground and excited states of the atoms moving in the polarized paramagnetic gas (compare with the analysis of the line shift of the excited states of atoms in an unpolarized gas, given in[3,4]). Since the kinetic energy of the electrons in the excited states of the atom is small, a decisive role in the line splitting is played in this case by the field produced by the exchange Coulomb interaction.

It should also be noted that the foregoing pertains, of course, both to atoms passing through a gas consisting of atoms differing from the beam atoms, and to the case when we deal with identical atoms. The only difference is that in the former case the frequency of the change of the spin direction of the beam (and also the shift and splitting of the levels) is determined by the amplitudes for the scattering of the beam-atom electrons by the atoms of the medium, whereas in the latter case the frequency of the change of spin direction of the beam atoms (shift and splitting of levels) is determined by the much larger amplitude of atom-atom scattering.

We call attention, in conclusion, to the fact that the field G' considered above, due to the exchange Coulomb scattering, is similar in its nature to the effective molecular field responsible for the ferromagnetism, and to the effective field acting on the conduction electrons in the s-d exchange model (see, for example,[5-7]). The difference lies in the fact that in our case we can vary the energy of the incident beam freely, and therefore vary the magnitude of the field, whereas in the case of[5-7] the effective field is due to the interaction of the electrons belonging to the medium, and therefore its parameters are constant under the given conditions. Furthermore, the wave functions of the beam electrons and of the medium electrons overlap more strongly than the wave functions of the s- and d-electrons. Therefore, at beam velocities $\approx 10^8$ cm/sec (i.e., at velocities comparable with the velocities of the s-electrons), the field G' is stronger than the field introduced in[5,6]. In addition, the field G', of course, exists not only in ferromagnets, but also in paramagnets of any type (liquid, solid, gas).

The authors are deeply grateful to M. I, Podgoretskiĭ for a discussion and for valuable remarks.

[1] V. G. Baryshevskii and M. I. Podgoretskii, Zh. Eksp. Teor. Fiz. **47**, 1050 (1964) [Sov. Phys.-JETP **20**, 704 (1965)].

[2] M. Lax, Rev. Mod. Phys. **23**, 287 (1951).

[3] E. Fermi, Nuovo Cimento **11**, 157 (1934).

[4] V. A. Alekseev and I. I. Sobel'man, Zh. Eksp. Teor. Fiz. **49**, 1274 (1965) [Sov. Phys.-JETP **22**, 882 (1966)].

[5] S. V. Vonsovskiĭ and Yu. A. Izyumov, Usp. Fiz. Nauk **77**, 377 (1962) [Sov. Phys.-Usp. **5**, 547 (1963)].

[6] S. V. Vonsovskiĭ and Yu. A. Izyumov, ibid. **78**, 3 (1962) [**5**, 723 (1963)].

[7] D. C. Mattis, Theory of Magnetism, Harper, 1965.

Translated by J. G. Adashko
242