THE CRITICAL CURRENT IN SUPERCONDUCTING FILMS

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The dependence of the critical current on the magnetic field in a film of a type-II superconductor is investigated theoretically. The magnetic field is parallel to the film surface and perpendicular to the current. It is assumed that $\delta_0 \gg d \gg \xi(T)$ where δ_0 is the penetration depth of the magnetic field and d—the film thickness. It is shown that in weak fields the critical current is determined by the value for which vortices begin to form at one surface of the film, move, and are destroyed at the other end of the film. In fields close to $H_{c_1}(d)$ weak currents do not disturb the stability of the mixed state; the state is unstable for currents equal to the critical value. From the dependence of the critical current on the external magnetic field it follows that a peak effect should be observed in strong fields.

1. INTRODUCTION

THE destruction of superconductivity in films when a transport current due to some external source passes through them can occur for two reasons.

If the film thickness d is less than the coherence length $\xi(T)$, then an instability of the superconducting state appears when the velocity of the superfluid condensate reaches a certain critical magnitude for which intensive destruction of the electron pairs responsible for the superconductivity occurs. This is the only mechanism for the destruction of the superconductivity when $d < \xi(T)$. The critical current for this case has been calculated in the work of Ginzburg and Landau[1] and Bardeen.[2] This refers to type-I and type-II superconductors.

A completely different situation appears[3] in the case when $d > \xi(T)$ and when we are dealing with a type-II superconductor. Now two mechanisms for the destruction of superconductivity turn out to be possible. The first one is that which has been discussed above. Let us consider the second mechanism.

If a type-II superconductor [the penetration depth of a weak magnetic field $\delta_0(t) > \xi(T)$] is placed in an external magnetic field, then starting from some field H_{c_1} there will appear in such a superconductor stable superconducting vortices whose axes are parallel to the external field.[4] A mixed state appears. If the magnetic field is parallel to the surface of the film and $d > \xi(T)$, then a mixed state will also appear in such a film.[5] Let now a transport current pass through such a superconductor in a direction perpendicular to the external field and hence perpendicular to the axes of the superconducting vortices. The interaction between the transport current and the vortices will lead to the appearance of a Lorentz force[6-8] directed perpendicular to the field and to the current, and applied to the vortex cores. Under the influence of this force the vortices will move and energy will be dissipated,[9,10] i.e., the superconducting state will be destroyed. For this reason it is stated in all the work concerning the critical currents in hard superconductors that an ideally uniform type-II superconductor in the mixed state is absolutely unstable with respect to a transport current directed perpendicular to the external magnetic field. In other words, the critical current in this case is zero. It is a different matter if the superconductor is nonuniform, i.e., contains cavities, inclusions of another phase and similar macroscopic defects. The latter can become obstacles for the vortex filaments and the critical current will be determined by the force of adhesion of the vortices to these inhomogeneities (by the pinning force).

Such is the second mechanism of the destruction of superconductivity by a current. For the critical current there appears in this case an instability of the vortex system.

In this paper we consider the destruction of superconductivity by a transport current due to the development of a vortex instability in an ideally uniform type-II superconducting film [$d \gg \xi(T)$] placed in an external magnetic field parallel to its surface and perpendicular to the direction of the transport current. As will be seen below, the critical current in this case is not equal to zero since the surface of the film itself is the inhomogeneity which leads to a finite critical current.

2. STATEMENT OF THE PROBLEM

We are considering a type-II superconducting film. It is assumed that the constant of the Ginzburg-Landau theory $\kappa = \delta_0(T)/\xi(T) \gg 1$. The film thickness $d \gg \xi(T)$. The film surfaces coincide with the planes $x = \pm d/2$. The external magnetic field H_0 is directed parallel to the oz axis in the negative direction of this axis. The transport current is along the oy axis.

Our task is to calculate those external parameters (the field and the current) for which a vortex instability begins to develop in the film. This we understand to be the destruction of the stability of the vortex system in the film, if such existed, or the production of vortices on one surface of the film, their motion, and their annihilation on the opposite surface.

Abrikosov[5] found the first critical field of such a film $H_{c_1}(d)$ (in the absence of a transport current) and showed that for $H_0 \gtrsim H_{c_1}(d)$ the vortices form a linear chain, i.e., the axes of the vortex filaments parallel to the external magnetic field are in the plane x = 0 and are removed some distance $a(H_0)$ from one another.

Our treatment will also be limited to the case of such fields H_0 when a linear vortex chain is stable. In addition, we assume that $T_c - T \ll T_c$ and apply the Ginzburg-Landau theory. Since $d \gg \xi(T)$ and $\kappa \gg 1$, one can assume that the order parameter (the wave function) of the Ginzburg-Landau theory is constant in the entire film with the exception of the vortex axes (as was done in [5]). The Ginzburg-Landau equations reduce then to an equation for the magnetic field inside the film:[4]

$$\nabla^2 H - H = -\frac{2\pi}{\varkappa}\sum_m \delta(\mathbf{r}-\mathbf{r}_m), \tag{1}$$

$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, and $\mathbf{r}_m$ is a vector in the plane z = 0 which gives the coordinate of the center of the m-th vortex. Here and below we shall use the relative units introduced in [1]: as the unit of length we take the penetration depth δ_0, and as the unit of magnetic field intensity we take the quantity $\sqrt{2}\,H_{cm}$ where H_{cm} is the thermodynamic critical field.

3. FREE ENERGY OF A FILM WITHOUT A TRANSPORT CURRENT WITH A LINEAR CHAIN OF VORTICES

As a first stage we shall find the free energy of a film with a vortex chain in the absence of a transport current but placed in an external magnetic field H_0. Unlike in [5], where the chain was located at the equilibrium position in the center of the film, we shall here consider the case when the vortex chain is displaced by some distance x_0 from the center (Fig. 1) and we shall find the dependence of the free energy on x_0.

Let us solve Eq. (1) with the boundary condition $H(\pm d/2) = H_0$; the coordinates of the vortices will be taken in the form $\mathbf{r}_m = \{x_0, ma\}$, m = 0; ±1; ±2; a is the distance between the vortices.

Solving this equation in exactly the same way as in [5], we obtain the following solution:

$$H = H_0\frac{\operatorname{ch} x}{\operatorname{ch}{}^1/_2 d} + \sum_m \int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{ik(y-ma)}H_k(x), \tag{2}$$

$$H_k(x) = \begin{cases} \dfrac{2\pi}{\varkappa u}\dfrac{\operatorname{sh}[u({}^1/_2 d - x_0)]\operatorname{sh}[u({}^1/_2 d + x)]}{\operatorname{sh} ud} & x \leqslant x_0 \\[2ex] \dfrac{2\pi}{\varkappa u}\dfrac{\operatorname{sh}[u({}^1/_2 d + x_0)]\operatorname{sh}[u({}^1/_2 d - x)]}{\operatorname{sh} ud} & x \geqslant x_0 \end{cases} \tag{3}$$

$u = (k^2+1)^{1/2}$, m = 0, ±1, ±2, ...

The first term in (2) depends on the external magnetic field H_0 and represents the field which would exist in the film in the absence of the vortex chain. The second term in (2) gives, on the contrary, the field which would exist in the film if it contained the vortex chain in the absence of an external field. It is therefore natural that the second term does not depend on the external field.

Let us now find F_H—the free energy density of a film in an external magnetic field with the condition that the linear vortex chain is displaced by a distance x_0 from the center (Fig. 1).

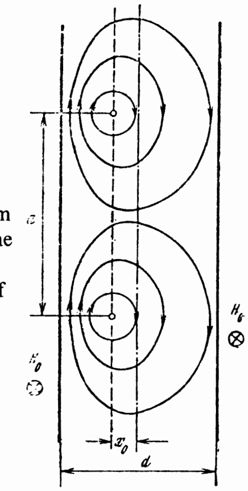

FIG. 1. A chain of vortices displaced from the equilibrium position by a distance x_0. The external magnetic field H_0 is directed away from the reader perpendicular to the plane of the drawing. The distance between vortices along the film is a.

According to [5] we have

$$F_H = \frac{\varepsilon(\infty)}{ad} - \frac{1}{ad}\int[H_\infty^2 + (\nabla H_\infty)^2]\,dV + \frac{1}{d}\lim_{Y\to\infty}\left\{\frac{1}{Y}\int_{-Y/2}^{Y/2}dy\int_{-d/2}^{d/2}[H^2+(\nabla H)^2 - 2H_0H]\,dx\right\}. \tag{4}$$

Here $\epsilon(\infty)$ is the total energy per unit length of an isolated vortex[4] in an infinite superconductor, and $H_\infty(\mathbf{r})$ is the magnetic field produced by such a vortex. The integration in the second term extends over all space. Formula (4) is valid under the assumption that the distance between the vortices $a \gg \kappa^{-1}$ [i.e., $a \gg \xi(T)$] and that the vortices are sufficiently far (compared with κ^{-1}) from the edges of the film.

Substituting (2) in (4), we have

$$F_H = \frac{\varepsilon(\infty)}{ad} - \frac{2H_0^2}{d}\operatorname{th}\frac{d}{2} - \frac{2\pi}{\varkappa^2 ad}I_1 - \frac{4\pi H_0}{\varkappa ad}\left(1 - \frac{\operatorname{ch} x_0}{\operatorname{ch}{}^1/_2 d}\right) + \frac{2\pi}{\varkappa^2 ad}I_2 \tag{5}$$

where

$$I_1 = \int_0^\infty\frac{dk}{\sqrt{k^2+1}}\left[1 - \frac{2\operatorname{sh}(\sqrt{k^2+1}({}^1/_2 d + x_0))\operatorname{sh}(\sqrt{k^2+1}({}^1/_2 d - x_0))}{\operatorname{sh}(d\sqrt{k^2+1})}\right]$$

$$I_2 = \sum_{m\neq 0}\int_{-\infty}^{\infty}\frac{dk}{\sqrt{k^2+1}}\frac{\operatorname{sh}(\sqrt{k^2+1}({}^1/_2 d + x_0))\operatorname{sh}(\sqrt{k^2+1}({}^1/_2 d - x_0))}{\operatorname{sh}(d\sqrt{k^2+1})},$$

The expression in the square brackets in the formula for I_1 is readily brought to the form

$$[\ldots] = -2\sum_{n=0}^{\infty}[e^{-2ud(n+1)} + e^{u(2x_0-d-2nd)} + e^{-u(2x_0+d+2nd)}],$$

if one makes use of the expansion

$$(1-e^{-2ud})^{-1} = \sum_{n=0}^{\infty}e^{-2nud}, \quad u = \sqrt{k^2+1}.$$

Integrating the series, we obtain

$$I_1 = \sum_{n=0}^{\infty}[K_0(2dn+d-2x_0) + K_0(2dn+d+2x_0) - 2K_0(2dn+2d)] \tag{6}$$

where K_0 is a Hankel function of imaginary argument.

In order to calculate I_2 we make use of an evaluation of the contour integral in the complex plane of the variable k. Assuming $d \ll 1$ and $a \gtrsim d$, we obtain,

$$I_2 \approx 4e^{-\pi a/d}\cos^2\frac{\pi x_0}{d}. \qquad (7)$$

Let us discuss formula (5). The sum of the first, third, and last term does not depend on the external field H_0 and gives the energy of the vortex chain itself in the film. The dependence of this quantity on x_0 characterizes the interaction of the chain with the edges of the film; the last term with I_2 gives the interaction between vortices. Since this term is small for $a > d$, we shall take no account below of the interaction of vortices.

The fourth term in (5) represents the energy of interaction of the vortex chain with the external magnetic field, or, what amounts to the same, the interaction energy of the vortices with the Meissner currents appearing in the film under the action of the field. The second term in (5) is simply the free energy of the superconducting film without vortices in a magnetic field H_0.

Let us introduce the definition

$$\Delta F_H \equiv F_H + \frac{2H_0^2}{d}\,\mathrm{th}\,\frac{d}{2}, \qquad (8)$$

i.e., we shall reckon the free energy of a film with vortices from its value in the absence of vortices. Let us discuss the dependence of ΔF_H on x_0 for various values of the magnetic field. We shall, first of all, show that $\Delta F_H(\pm \frac{1}{2}d) = 0$. A direct substitution of $x_0 = \pm\frac{1}{2}d$ in the formula for $I_1(x_0)$ leads to infinity. This occurs because the validity of our treatment is limited by the distances $\frac{1}{2}d - x_0 \gg \kappa^{-1}$. According to (6) for $x_0 \to \pm d/2$ the value of $I_1(x_0) \to K_0(d \mp 2x_0)$.

On the other hand, the field of a single filament in an infinite superconductor[4] is $K_0(r)/\kappa$, and the field at the center of such a filament is $2H_{c1}(\infty)$. We have, therefore, $I_1(\pm d/2) = 2\kappa H_{c1}(\infty)$. Taking into account the fact[4] that $\epsilon(\infty) = 4\pi H_{c1}(\infty)/\kappa$, we obtain $\Delta F_H(\pm d/2) = 0$. This is also understandable physically: a vortex filament on the surface of the film is completely extinguished by its image and therefore does not make any contribution to the free energy. The $\Delta F_H(x_0)$ dependence for different values of H_0 is shown in Fig. 2. For $H_0 = 0$ the vortices are absolutely unstable in the film, because they are attracted to the surface. According to Bean and Livingston[11] this can be explained by the interaction of a vortex with its image. The interaction of the vortices with Meissner currents which tend to repulse the vortex chain from the edge of the film becomes appreciable as the external field increases. As a result, starting from a certain field $H_0 = H'$ a potential well is formed for vortices and the chain becomes stable (curve 2, Fig. 2). Let us find the field H'.

The field H' is obviously a field for which

$$[\partial^2 \Delta F_H / \partial x_0^2]_{x_0=0} = 0.$$

Making use of Eq. (A.1) derived in the Appendix, we obtain

$$H' = \pi^2 / 2\kappa d^2, \quad d \ll 1. \qquad (9)$$

Thus, metastable vortex states can appear beginning with the field H', and this field can be referred to as the minimum supercooling field of the mixed state in the film.

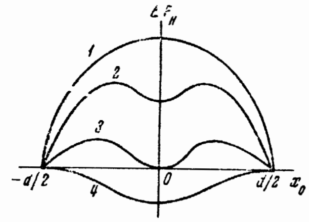

FIG. 2

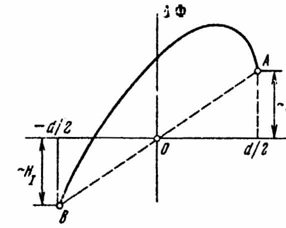

FIG. 3

FIG. 2. The magnitude of $\Delta F_H(x_0)$ for various values of the magnetic field H_0: curve 1—$H_0 = 0$, 2—$H_0 > H'$, 3—$H_0 = \tilde{H}_{c1}(d)$, 4—$H_0 = H_S$.

FIG. 3. The form of the dependence $\Delta\Phi(x_0)$ for $H_0 < H'$. A vortex cannot appear and start to move from point A to point B, since to do so it would have to overcome a potential barrier.

When the field reaches the value of the first critical field (Abrikosov calculated this field[5])

$$H_{c1}(d) = \frac{4}{\kappa d^2}\left(\ln\frac{\gamma\kappa d}{\pi} + 0.081\right), \quad d \ll 1, \quad \gamma = e^C \approx 1.78, \qquad (10)$$

the production of vortices becomes energetically advantageous, but the penetration of vortices into the film disturbs the Bean-Livingston barrier[11] which disappears for a field $H_0 = H_S$. This is obviously a field for which

$$[\partial \Delta F_H(x_0) / \partial x_0]_{x_0=d/2} = 0.$$

An exact calculation of H_S within the framework of our theory is impossible, since the theory is valid for $d/2 - x_0 \gg \kappa^{-1}$. One can, however, estimate the order of magnitude of H_S substituting in the final answer κ^{-1} for $d/2 - x_0$. Then, using Eq. (A.1) of the Appendix, we have $H_S \sim d^{-1}$ for $d \ll 1$. We note that for $d \gg 1$ the value of $H_S \sim 1$, i.e., it is of the order of H_{cm}—the thermodynamic critical field.[11] The quantity H_S can be referred to as the maximum superheating field of the Meissner state.

Thus it is now clear that in an external field equal and greater than $H_{c1}(d)$ the vortex chain in the film is stable and located at the center of the film. It is now also clear that the switching on of a sufficiently weak transport current will not lead to a disturbance of the stability of the vortex chain and will only cause a displacement of its equilibrium position. In other words, it follows from Fig. 2 that the edges of the film play precisely the role of inhomogeneities which stop the vortex motion.

Let us go over to the calculation of those values of the transport current for which a vortex instability of the system begins to develop.

4. THE FREE ENERGY OF A FILM WITH A LINEAR VORTEX CHAIN AND A TRANSPORT CURRENT

What thermodynamic potential reaches a minimum in the equilibrium state when in addition to an external uniform magnetic field parallel to the film surface a transport current also flows through it? When a transport current is passed through a film an interaction force between the transport current and the vortices—the Lorentz force—is added to the forces acting on the vortices. A calculation of the Lorentz force[8] $\tilde{f}_L$ shows that in the usual Gaussian units

$$\tilde{f}_L = \frac{1}{c}[j_T \varphi_0],$$

where j_T is the density of transport current at the point where the center of the vortex is located, $\varphi_0 = hc/2e$ is the magnetic flux quantum, and $\tilde{f}_L$ is the force acting per unit length of the vortex filament.

It is readily shown that in our relative units and for our geometry the Lorentz force density will be

$$f_L = \frac{\tilde{f}_L}{ad} = -\frac{4\pi}{\varkappa ad} j_T(x_0), \quad j_T(x_0) = H_I \frac{\operatorname{ch} x_0}{\operatorname{sh} {}^1/_2 d}, \tag{11}$$

where H_I is the field produced by the transport current on the surface of the film; f_L is directed along the negative direction of the ox axis if j_T is directed along the positive direction of the oy axis, and H_0—in the negative direction of the oz axis. The current density j_T in (11) is obtained from a solution of the equation $d^2H/dx^2 - H_j = 0$ with the boundary conditions $H_j(\pm \frac{1}{2}d) = \pm H_I$, and from the Maxwell equation $j_T = dH_j/dx$; H_j is the field in the superconductor produced by the transport current.

The Lorentz force can be considered to be the derivative of some potential U:

$$f_L = -\frac{\partial U}{\partial x_0}, \quad U = \frac{4\pi H_I}{\varkappa ad} \frac{\operatorname{sh} x_0}{\operatorname{sh} {}^1/_2 d}. \tag{12}$$

Therefore in the state of thermodynamic equilibrium the quantity $\Phi = F_H + U$ reaches a minimum; here F_H is given by formula (4) and H in this formula is the solution of Eq. (1) with boundary conditions $H(\pm d/2) = H_0 \pm H_I$.

Carrying out calculations entirely analogous to those in the preceding section, we find

$$\Phi = F_H + \frac{2H_I^2}{d} \operatorname{cth} \frac{d}{2} + \frac{4\pi H_I}{\varkappa ad} \frac{\operatorname{sh} x_0}{\operatorname{sh} {}^1/_2 d}, \tag{13}$$

with F_H determined in (5).

Let us introduce the definition

$$\Delta\Phi = \Phi + \frac{2H_0^2}{d} \operatorname{th} \frac{d}{2} - \frac{2H_I^2}{d} \operatorname{cth} \frac{d}{2}, \tag{14}$$

i.e., we shall reckon the free energy of a film with vortices and with a transport current from the free energy of a film without vortices but with the previous transport current. Thus, finally:

$$\Delta\Phi = \frac{\varepsilon(\infty)}{ad} - \frac{2\pi}{\varkappa^2 ad} I_1 - \frac{4\pi H_0}{\varkappa ad}\left(1 - \frac{\operatorname{ch} x_0}{\operatorname{ch} {}^1/_2 d}\right) + \frac{4\pi H_I}{\varkappa ad} \frac{\operatorname{sh} x_0}{\operatorname{sh} {}^1/_2 d}.$$

As has been noted above, the interaction between vortices is neglected.

5. CALCULATION OF THE CRITICAL CURRENT

It is seen from Fig. 2 that in weak fields ($H_0 < H'$) the vortices in the film are absolutely unstable. The $\Delta\Phi(x_0)$ dependence is in this case of the form shown in Fig. 3. It is seen that for sufficiently weak currents, so long as there exists a potential barrier, there will be no motion of vortices from point A to point B and the superconducting state will be stable. It is thus clear that the critical current in this case (of weak fields H_0) should be defined as the current for which the potential barrier in the dependence $\Delta\Phi(x_0)$ vanishes. In stronger fields H_0, as we already know, a minimum may appear on the $\Delta\Phi(x_0)$ curves which corresponds to the appearance of stability of the vortex chain, and the critical current should lead to a vanishing of this minimum.

One can, therefore, immediately formulate a general criterion for the critical current. The development of a vortex instability of our system will commence when the transport current reaches a (critical) value for which the function $\Delta\Phi(x_0)$ becomes monotonic.

Let us find this critical current. The requirement that the function $\Delta\Phi(x_0)$ be monotonic reduces to the requirement of the absence in it of extremum points which are determined by the equation $\partial\Delta\Phi/\partial x_0 = 0$, or

$$\frac{\partial F_H}{\partial x_0} = -\frac{4\pi H_I}{\varkappa ad} \frac{\operatorname{ch} x_0}{\operatorname{sh} {}^1/_2 d}. \tag{15}$$

Utilizing the condition $d \ll 1$ and formula (A.1) of the Appendix, we obtain in place of (15) the equation

$$P(x_0) = -2H_I/d, \tag{16}$$

where we have introduced the notation

$$P(x_0) = H_0 x_0 - \frac{\pi}{2\varkappa d} \operatorname{tg}\left(\frac{\pi x_0}{d}\right). \tag{17}$$

Let us note, first of all, that $P(x_0)$ is with an accuracy up to a constant factor the derivative of the quantity ΔF_H with respect to x_0. Since our treatment is valid for $d/2 - x_0 \gg \kappa^{-1}$, a direct substitution of $x_0 = d/2$ in (17) is not valid. However, making use of the definition of the field H_s as being the field for which $[\partial \Delta F_H/\partial x_0]_{x_0=d/2} = 0$, we immediately obtain

$$P(d/2) = {}^1/_2 d(H_0 - H_s).$$

The solutions of the algebraic equation (16) are the abscissas of points at which the horizontal straight line at a distance $(-2H_I/d)$ from the abscissa axis intersects the $P(x_0)$ curve. Figure 4 illustrates the graphical solution of Eq. (16). Let us first consider the case of relatively weak fields H_0, i.e., such fields for which the point $P(d/2)$ is located below the minimum of the $P(x_0)$ function (the points M). It is this case which is illustrated in Fig. 4.

For $H_I = 0$ the solutions of (16) will be points 1 and 2 corresponding to the maximum of the $\Delta\Phi$ curve, and the point 0 corresponding to the minimum on the $\Delta\Phi(x_0)$ curve. Switching on of a weak transport current will shift the maxima to the right [the points 1′ and 2′, and the minimum to the left (the point 0′)]. A further increase of the current will lead to an elimination of the minimum on the $\Delta\Phi(x_0)$ curve; however, so long as the maximum (point 2″) remains no vortex instability of the system will appear, and the superconducting state will be retained. A superconducting transport current passes through the film, but there are no vortices. The $\Delta\Phi(x_0)$ curve will become monotonic only when the current reaches such a value that

$$-\frac{2H_I}{d} = P\left(\frac{d}{2}\right) = \frac{d}{2}(H_0 - H_s).$$

Hence we obtain an expression for the critical current:

$$H_{Ic} = {}^1/_4 d^2 (H_s - H_0). \tag{18}$$

But this will only be so for such fields H_0 for which $M > P(d/2)$.

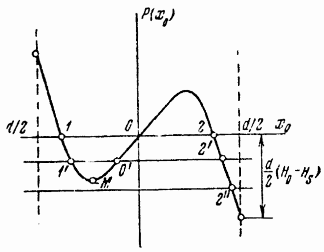

FIG. 4

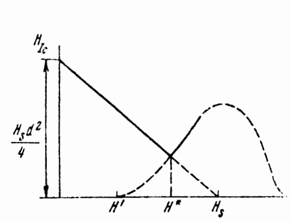

FIG. 5

FIG. 4. Graphical illustration of the solution of Eq. (16). The abscissas of the points 1, 0, 2; 1′, 0′, 2′ and of the point 2″ are the abscissas of the extremum points of the function $\Delta\Phi(x_0)$ for three different values of the transport current.

FIG. 5. The dependence of the critical current on the external magnetic field. For $H_0 < H^*$ the critical current decreases linearly with the field (18); for $H_0 > H^*$ the critical current increases with the field [(20) and (21)] (solid line). For $H_0 = H_{C3}(d)$ the critical current vanishes. The assumed variation of the critical current with the field in this region is shown by the dashed curve.

With increasing external field H_0 the $P(x_0)$ curve is deformed in such a way that starting with some field (we shall denote it by H^*) M becomes less than $P(d/2)$.

Let us consider this case. If now $M < -2H_I/d < P(d/2)$, then the straight line $(-2H_I/d)$ will intersect the curve $P(x_0)$ only in the two points 1′ and 0′ (in analogy with the notation in Fig. 4). The point 1′ corresponds to a maximum on the $\Delta\Phi(x_0)$ curve and the point 0′—to a minimum. At this minimum the vortex chain will exist stably. The critical current will occur when the points 1′ and 0′ coalesce, i.e., now there will be a new equation for the critical current:

$$-2H_{I_c}/d = M. \tag{19}$$

Let us find the quantity M. We determine for this purpose the abscissa x_{0C} of the minimum point of the $P(x_0)$ curve, i.e., we solve the equation $\partial P/\partial x_0 = 0$. Utilizing (9) and (17), we have

$$x_{0c} = -\frac{d}{\pi}\arccos\sqrt{\frac{H'}{H_0}}.$$

Substituting this expression in (17), we find $M \equiv P(x_{0C})$. The critical current is finally obtained from (19):

$$H_{I_c} = \frac{d^2}{2\pi}H_0\left[\arccos\sqrt{\frac{H'}{H_0}} - \sqrt{\frac{H'}{H_0}}\sqrt{1-\frac{H'}{H_0}}\right]. \tag{20}$$

It is readily seen that for $H_0 - H' \ll H'$ expression (20) reduces to the form[1)]

$$H_{I_c} = \frac{\sqrt{2}\sqrt{\varkappa}\,d^3}{3\pi^2}\left(H_0 - \frac{\pi^2}{2\varkappa d^2}\right)^{3/2}. \tag{21}$$

Thus the critical current in the film is determined by formulas (18), (19), and (21).

6. DISCUSSION OF THE RESULTS AND CONCLUSIONS

The dependence of the critical current of a film on the magnetic field is shown in Fig. 5. In fields $0 \le H_0 \le H^*$ the critical current decreases linearly with increasing external magnetic field [formula (18)]. In this range of fields (for a current below the critical current) we have a stable superconducting state of the film without vortices. When the current reaches the critical value vortices begin to appear on the surface $x = d/2$, move in the direction of the $x = -d/2$ surface, and are annihilated on this surface. The field H^* is the field for which $P(d/2) = M$ (Fig. 4). Starting from this field, the field dependence of the critical current is determined by Eqs. (20) and (21). The graph of the function determined by these formulas starts on the abscissa axis at the point $H_0 = H'$ and for the field H^* intersects the function (18). If the current is smaller than the critical current and the field $H_0 > H^*$, then the superconducting state with a chain of vortices located at the minimum of the function $\Delta\Phi(x_0)$ is stable. For the critical current this state ceases to be stable.

Thus we see that a peak effect should be observed on the graph of the dependence of the critical current of a film on the external magnetic field transverse to the current, i.e., one observes an increase of the critical current with increasing external field which is on first sight anomalous.

The essence of this phenomenon reduces to the fact that with increasing magnetic field there is an increase in the depth of the potential well in which the vortices are located. One must therefore increase the current required to destroy the stability of the vortex structure.

On the other hand, it is clear that at least for a field H_{C3} the superconductivity of the film will be completely destroyed and the critical current will therefore vanish. This is the cause of the existence of the critical current peak.

However, here we must make a very important reservation. So far we have neglected the interaction of vortices. A rearrangement of vortices will commence in strong fields and they will produce a two-dimensional lattice. In this case a weaker current is required to destroy the stability of the vortex lattice. The strongest peak effect should apparently occur in thin films of a type-II superconductor whose thickness $d \gtrsim \kappa^{-1}$. In this case one can expect that the vortex structure in the form of linear chain will be stable up to fields $H_0 \sim H_{C3}$, since according to the calculations of Kulik[12] for such films the vortices form a linear chain for $H_0 \sim H_{C3}$.

Let us estimate the magnitude of the critical current of our film for $H_0 = 0$. According to (18) and with allowance for the fact that $H_S \sim d^{-1}$ we have $H_{I_C}(0) \sim d/4$ which literally coincides with the critical unpairing current (according to Ginzburg-Landau theory $H_{I_C} = 2d/3\sqrt{3}$). However, (18) has a broader region of applicability if we understand H_S to be not the maximum superheating field of the Meissner state but the real field for which the penetration of vortices into the film without a transport current commences. If the film surface is rich in defects, then there are grounds for assuming that the Bean-Livingston barrier will be lowered appreciably, and the penetration of vortices into the film will begin in fields appreciably lower than H_S. When the barrier is fully destroyed by defects we have another limiting formula for the region of low fields:

$$H_{I_c} = \frac{d^2}{4}\left(H_{c1}(d) - H_0\right),$$

1) This expression coincides with an accuracy up to an unimportant numerical coefficient with the result of [3]. The discrepancies are due to the fact that the asymptotic behavior of $\partial I_1/\partial x_0$ has been estimated incorrectly in [3].

where $H_{c1}(d)$ is determined by (10); then

$$H_{I_c}(H_0=0) \sim \varkappa^{-1}\left(\ln\frac{\gamma\varkappa d}{\pi}+0.081\right).$$

This case was apparently observed in [13].

Let us summarize the results obtained.

We have calculated the maximum superheating field of the Meissner state $H_S \sim d^{-1}$ and the minimum supercooling field of the mixed state $H' = \pi^2/2\kappa d^2$ for a film in an external magnetic field parallel to its surface with $d \ll 1$, $\kappa d \gg 1$, and $\kappa \gg 1$. We have found the dependence of the critical current of such a film on the external magnetic field. It has been shown that the mixed state in an ideally uniform film is stable against a transport current flowing perpendicular to the magnetic field. An explanation of the peak effect is given.

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APPENDIX

In order to calculate $\partial I_1/\partial x_0$, we make use of the determination of I_1 in (6) and differentiate term by term the series:

$$\frac{\partial I_1}{\partial x_0}=2\sum_{n=0}^{\infty}[K_1(2dn+d-2x_0)-K_1(2dn+d+2x_0)].$$

In order to estimate this expression, we make use of the condition $|x_0| \le d \ll 1$. We estimate the sum

$$S^{(-)}=\sum_{n=0}^{\infty}K_1(2dn+d-2x_0).$$

Introducing the notation $\epsilon = 2d$ and $\beta = (d - 2x_0)/2d$ and dividing the sum into two parts, we have

$$S^{(-)}=\sum_{n=0}^{N-1}K_1[\varepsilon(n+\beta)]+\sum_{n=N}^{\infty}K_1[\varepsilon(n+\beta)],$$

where $\epsilon \ll 1$, $0 \le \beta \le 1$, and N is chosen such that $1 \ll N \ll \epsilon^{-1}$. Let us denote the first sum in this formula by S_1 and the second one by S_2. Since in S_1 the argument $\epsilon(n+\beta) \ll 1$, we make use of the asymptotic behavior of $K_1(z)$ for $z \ll 1$ and obtain

$$S_1=\varepsilon^{-1}\sum_{n=0}^{N-1}\frac{1}{n+\beta}=\varepsilon^{-1}[\psi(\beta+N)-\psi(\beta)],$$

where $\psi(z) = d \ln \Gamma(z)/dz$. The asymptote of $\psi(z)$ for $z \gg 1$ is: $\psi(z) \approx \ln z$; therefore,

$$S_1 \approx \varepsilon^{-1}[\ln N-\psi(\beta)].$$

Making use of the smallness of ϵ, we approximate the second sum S_2 by the integral

$$S_2 \approx \varepsilon^{-1}\int_{\varepsilon N}^{\infty}K_1(z)\,dz=\varepsilon^{-1}K_0(\varepsilon N).$$

Taking into account that $\epsilon N \ll 1$, we have

$$S_2 \simeq \varepsilon^{-1}\left[\ln\frac{2}{\gamma\varepsilon}-\ln N\right],$$

Here $\gamma = e^C \approx 1.78$. Combining S_1 and S_2, we obtain finally

$$S^{(-)}(\beta)=\varepsilon^{-1}\left[\ln\frac{2}{\gamma\varepsilon}-\psi(\beta)\right].$$

Estimating analogously the sum

$$S^{(+)}=\sum_{n=0}^{\infty}K_1(2dn+d+2x_0),$$

we have

$$S^{(+)}(\beta)=\varepsilon^{-1}\left[\ln\frac{2}{\gamma\varepsilon}-\psi(1-\beta)\right].$$

Finally,

$$\frac{\partial I_1}{\partial x_0}=2(S^{(-)}-S^{(+)})=\frac{\pi}{d}\operatorname{tg}\left(\pi\frac{x_0}{d}\right). \qquad \text{(A.1)}$$

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