CONTRIBUTION TO THE THEORY OF QUANTUM GALVANOMAGNETIC PHENOMENA IN SEMICONDUCTORS

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A nonlinear kinetic equation for the one-electron density matrix in stationary crossed **E** and **H** fields is obtained for the case of the electron-phonon and impurity scattering mechanisms. It is demonstrated that when scattering by impurities is predominant the transverse conductivity becomes dependent in the ultraquantum limit on the electric field strength even at such small values of **E** which cannot lead to heating of the electron system. An interpolation formula is derived for this dependence in the case of scattering by short range impurities.

1. It is known that in the initial variants of the quantum theory of galvanomagnetic phenomena the kinetic coefficients were expressed in terms of integrals that diverged logarithmically at low energies[1)][1]. To eliminate the divergence it was necessary to introduce into the theory, from the outside, additional assumptions. In the later investigations, this shortcoming of the theory was overcome. In the case of the electron-phonon mechanism of scattering, it was shown[2] that the divergent integrals can be cut off in a natural manner at the characteristic phonon energy. For impurity scattering, the divergence is eliminated by foregoing the Born approximation, which is not applicable at sufficiently low energies. As shown in[3], for short-range potentials, the effective cut off parameter is the quantity $\epsilon_0 = \hbar^2 f^2/2ma^4$, where f is the amplitude for the scattering of the electron with zero energy in the absence of a magnetic field, and $a = (c\hbar/eH)^{1/2}$ is the quantum Larmor radius.

In this communication we wish to call attention to one more factor that has a bearing on the elimination of the divergence from the quantum theory of galvanomagnetic phenomena. We show that there is likewise no divergence in the Born approximation if inelasticity is completely neglected, provided the theory is not linearized with respect to the electric field. The change of the position of the center of the cyclotron orbit, due to the collision, and the associated change in the "longitudinal" energy of the particle, proportional to the electric field, turn out to be equivalent to inelasticity of the collision. The role of the cutoff factor in this case is assumed by the quantity eEa. The point E = 0 is singular. When $E \to 0$, the dissipative current behaves like $E \ln(1/E)$. However, this non-analyticity in the behavior of the current means only that at sufficiently small E it is impossible to use the Born approximation for scattering. The lower limit of the values of the electric field, at which the logarithmic growth of the conductivity stops, is determined, in order of magnitude, from the relation $eEa = \epsilon_0$. Thus, if the decisive scattering mechanism is scattering by impurities, then the transverse conductivity will have a nonlinear behavior already in relatively weak electric fields, when the "heating" nonlinearity is still not noticeable ($eEa \ll \hbar s/a$,[4]); a study of this behavior may turn out to be useful, since the form of the $\sigma_{xx}(E)$ dependence is a source of additional information concerning the scattering center.

2. We start out in the derivation of the expression for the current from the nonlinear equation for the single-particle density matrix $f_{\beta'\beta} = \mathrm{Tr}(\hat{\rho} a^+_{\beta} a_{\beta'})$, where $\beta = (n, k_y, k_z)$ is the set of quantum numbers determining the state of the electron in crossed **E** and **H** fields. (We shall henceforth use the following orientation of the fields and gauge of the potential: **E** = (E, 00), **H** = (0, 0, H), **A** = (0, Hx, 0).) In the approximation linear in the electric field, an equation for $f_{\beta'\beta}$ was obtained by Gurevich and Nedin[5] with the aid of the Konstantinov and Perel' diagram technique, and by Argyres[6] by the method of uncoupling the Bogolyubov chain of equations for the correlation functions. The method used in[6] can be readily generalized to obtain a nonlinear kinetic equation. Without stopping to discuss the derivation, which is perfectly standard, we present the final result:

$$i\omega_{\beta'\beta} f_{\beta'\beta} = \frac{\pi}{\hbar^2} \sum_{\gamma,\gamma';\alpha;q} |c_q|^2 \{ J_{\beta'\gamma'} J^*_{\alpha\gamma} [\delta(\omega_{\gamma'\beta} - \omega_q)((1+N_q) f_{\gamma'\gamma} \times (\delta_{\alpha\beta} - f_{\alpha\beta}) - N_q f_{\alpha\beta}(\delta_{\gamma'\gamma} - f_{\gamma'\gamma})) + \delta(\omega_{\gamma'\beta} + \omega_q)(N_q f_{\gamma'\gamma}(\delta_{\alpha\beta} - f_{\alpha\beta}) - (1+N_q) f_{\alpha\beta}(\delta_{\gamma'\gamma} - f_{\gamma'\gamma}))] + J_{\gamma\beta} J^*_{\gamma'\alpha} [\delta(\omega_{\gamma\beta'} - \omega_q)((1+N_q) f_{\gamma'\gamma} \times (\delta_{\beta'\alpha} - f_{\beta'\alpha}) - N_q f_{\beta'\alpha}(\delta_{\gamma'\gamma} - f_{\gamma'\gamma})) + \delta(\omega_{\gamma\beta'} + \omega_q)(N_q f_{\gamma'\gamma}(\delta_{\beta'\alpha} - f_{\beta'\alpha}) - (1+N_q) f_{\beta'\alpha}(\delta_{\gamma'\gamma} - f_{\gamma'\gamma}))] \}. \quad (1)$$

We adhere here, in the main, to the notation of[5]. Certain differences are connected only with the fact that in our case $f_{\beta'\beta}$ is the total single-particle density matrix, and not an addition, linear in the field, to the equilibrium density matrix. In addition, Eq. (1) is written in the representation of a Hamiltonian in crossed **E** and **H** fields, and therefore the difference between the single-electron energies $\epsilon_\beta - \epsilon_{\beta'} = \hbar\omega_{\beta\beta'}$ differs from the corresponding quantity in[5] by an increment containing the electric field, $\omega_{\beta\beta'} = \omega^0_{\beta\beta'} - eEX_{\beta\beta}\hbar^{-1}$. In view of the complexity of the expressions, we have written out in (1) only that part of the collision integral which is due

[1)] We have in mind here the quantum limit when only the zeroth Landau level is filled, and the energy is taken to mean that part of the energy which is connected with the motion of the particle along the magnetic field.

to the electron-phonon interaction. The impurity collision integral is obtained from the right side of (1) by discarding the factors N_q and $1 + N_q$, discarding the phonon energy in the arguments of the δ functions, and also by replacing $|c_q|^2$ by $N_d|V_q|^2$, where V_q is the Fourier component of the scattering potential and N_d is the number of impurities. We do not present an equation for N_q, since the phonon system will henceforth be assumed to be in equilibrium.

It is easy to verify that linearization of Eq. (1) with simultaneous transition from the representation of crossed fields to the representation of the Hamiltonian in a magnetic field only leads to the linear kinetic equation obtained in[5]. Formally, the linearization is realized by substituting in (1)

$$f_{\beta'\beta} = n_\beta \delta_{\beta'\beta} + f^{(1)}_{\beta'\beta} - \frac{eEv^{(x)}_{\beta'\beta}(n_\beta - n_{\beta'})}{\omega^{02}_{\beta'\beta}\hbar}, \qquad (2)$$

where $f^{(1)}_{\beta'\beta}$ is the linear addition to the equilibrium density matrix; this addition was considered in[5]. The last term in (2) is due to the difference in the representations.

3. Just as in[5], the expression for the dissipative current j_x will be sought in the lowest approximation in $1/\omega\tau$ (ω—cyclotron frequency, τ—relaxation time). In the zeroth approximation in this parameter, the only elements of $f_{\beta\beta'}$ which do not vanish are the diagonal elements determined by the equation

$$\sum_{q,\gamma} |c_q|^2 |J_{\gamma\beta}|^2 \{\delta(\omega_{\gamma\beta} - \omega_q)[(1+N_q)f_\gamma(1-f_\beta) - N_q f_\beta(1-f_\gamma)] + \delta(\omega_{\gamma\beta} + \omega_q)[N_q f_\gamma(1-f_\beta) - (1+N_q)f_\beta(1-f_\gamma)]\} = 0. \qquad (3)$$

It can be shown that (3) is equivalent to the energy-balance equation $j_x E = Q$, where $Q = V^{-1}\Sigma\hbar\omega_q(N_q)_{eph}$ is the power radiated by the phonons as a result of the electron-phonon interaction. If we stipulate that at a distance equal to the quantum Larmor radius the energy acquired by the electron from the electric field be small compared with the characteristic average energy reckoned from the zeroth Landau level, i.e., if we stipulate satisfaction of the condition $eEa(\bar{\epsilon} - \hbar\omega/2)^{-1} \ll 1$, and if in addition we use the assumption of small inelasticity ($\hbar\omega_q(\bar{\epsilon} - \hbar\omega/2)^{-1} \ll 1$), then Eq. (3) goes over into the balance equation derived in the analysis[4] of the problem of nonlinear heating of an electron system in crossed E and H fields.

The nondiagonal elements of $f_{\beta'\beta}$, in the first order in $1/\omega\tau$, are expressed in terms of the diagonal elements as follows:

$$f_{n'n}(k_z) = \frac{\pi}{i\omega(n'-n)\hbar^2}\sum_{q,\gamma}|c_q|^2 J_{\gamma\beta}J^*_{\gamma\beta'}\{\delta(\omega_{\gamma\beta}-\omega_q)\times[(1+N_q)f_\gamma(1-f_\beta) - N_q f_\beta(1-f_\gamma)] + \delta(\omega_{\gamma\beta}+\omega_q)\times[N_q f_\gamma(1-f_\beta) - (1+N_q)f_\beta(1-f_\gamma)] + \text{the same with } \beta \to \beta'\}; \quad n' \neq n. \qquad (4)$$

The notation used in the left side of (4) emphasizes the fact that $f_{\beta'\beta}$ is diagonal in k_y and k_z, and does not depend on k_y, this being the consequence of the spatial homogeneity and the absence of spatial dispersion.

Substituting (4) in the expression for the current

$$j_x = \frac{2e}{V}\sum_{\beta',\beta} v^{(x)}_{\beta'\beta} f_{\beta\beta'}$$

and adding the term corresponding to the impurity scattering, we obtain after a number of transformations

$$j_x = \frac{4e\pi}{V}\sum_{q;\beta',\beta} X_{\beta\beta'}|J_{\beta'\beta}|^2\{|c_q|^2\delta(\omega_{\beta'\beta}-\omega_q)[(1+N_q)f_{\beta'}(1-f_\beta) - N_q f_\beta(1-f_{\beta'})] + N_d|V_q|^2\delta(\omega_{\beta'\beta})(f_{\beta'}-f_\beta)\}. \qquad (5)$$

4. Let the decisive factor be scattering by impurities with small action radii ($r_0 \ll a$, $V(r)V_0\sum_i(r - r_i)$). In the ultraquantum case, assuming Boltzmann's statistics, we obtain from the general expression (5) for the current

$$\sigma_{xx} = \left(\frac{m}{T^3}\right)^{1/2}\frac{e^2 n n_d V_0^2}{2\pi^2\hbar^2}F(\xi),$$

$$F(\xi) = \frac{1}{\xi^3}\int_0^\infty \frac{dx}{\sqrt{x}}\int_0^\infty \frac{dx'}{\sqrt{x'}}(x-x')\exp\left[-x' - \frac{(x'-x)^2}{2\xi^2}\right]. \qquad (6)$$

Here n is the electron density, which is assumed fixed, n_d is the scatterer density, and $\xi = eEa/T$. In the derivation of (6) we have neglected the heating, assuming that $eEa \ll \hbar s/a = \alpha T$. Making in (6) the change of variables $x' - x = t$, $x' + x = 2u$, and confining ourselves to the principal order in ξ, we transform the expression for $F(\xi)$ into

$$F(\xi) = \frac{2\pi}{\xi^3}\int_0^\infty u^2 e^{-u^2/\xi^2}\left[I_0\left(\frac{u^2}{\xi^2}\right) - I_1\left(\frac{u^2}{\xi^2}\right)\right]e^{-u}\,du, \qquad (7)$$

where $I_0(x)$ and $I_1(x)$ are Bessel functions of imaginary argument. It is seen from (7) that $F(\xi) \sim \ln\xi^{-1}$ when $\xi \to 0$. However, if we introduce formally a cutoff δ at the lower limit of the integration with respect to u, then the resultant expression

$$F_\delta(\xi) = \frac{2\pi}{\xi^3}\int_\delta^\infty u^2 e^{-u^2/\xi^2}\left[I_0\left(\frac{u^2}{\xi^2}\right) - I_1\left(\frac{u^2}{\xi^2}\right)\right]e^{-u}\,du \qquad (8)$$

will be analytic at the point $\xi = 0$ and when $\xi \ll \delta$ it will take the form

$$F_\delta(\xi) \approx \sqrt{\frac{\pi}{2}}\int_\delta^\infty \frac{e^{-u}}{u}du \sim -\sqrt{\frac{\pi}{2}}\ln\delta,$$

which is equivalent to the result of[1].

If the need for cutoff is connected with the nonapplicability of the Born approximation at low energies, then $\delta \approx \epsilon_0/T$, and the function $F_\delta(\xi)$ can be regarded as an interpolation expression, which gives the correct behavior of the conductivity when $\xi \ll \delta$ and $\xi \gg \delta$. To obtain an exact expression for the intermediate region ($\xi \sim \delta$), a separate analysis is necessary, where it is necessary both to take into account the nonlinearity and to forego the Born approximation.

The upper limit of the values of ξ, for which expressions (7) and (8) are valid, is determined by the condition that there be no heating: $\xi \ll \alpha$. If $\delta \ll \alpha$, then the region of the nonlinear behavior of the conductivity in the absence of heating turns out to be quite wide. Thus, for example, putting $f \approx 10^{-8}$ cm, $a \approx 10^{-6}$ cm ($H \sim 10^5$ Oe), $T \approx 10^{-15}$ erg, and $m \approx 10^{-28}$ g, we obtain $\delta \sim 10^{-3}$ and $\alpha \sim 10^{-1}$. The character of variation of the conductivity in the indicated region of values of the electric field is illustrated by the following results of a numerical calculation of the integral (8) for $\delta = 10^{-3}$:

ξ	10^{-5}	10^{-4}	$5\cdot10^{-4}$	10^{-3}	$5\cdot10^{-3}$	10^{-2}
$F_\delta(\xi)$	7.93	7.93	8.04	8.25	7.18	6.33

In the foregoing analysis it was assumed that the density of the impurity centers is sufficiently low, and consequently we neglected the collision broadening of the energy levels of the electrons. If the line broadening is sufficiently large, then this broadening will determine the cut off in the integration with respect to the energy. Then, as shown in[7], the cut off is effected at the value

$$\tilde{\epsilon}_0 = \frac{\hbar^2}{2m}\left(\frac{4\pi f^2 n_d}{a^2}\right)^{2/3}.$$

For this case, the lower limit in the integral (8) should be chosen to be $\tilde{\delta} = \tilde{\epsilon}_0/T$, and the aforementioned nonlinearity in the behavior of the conductivity will occur when $\tilde{\delta} \ll \alpha$. The question of which of the cutoff mechanisms is most effective can be solved by comparing the values of ϵ_0 and $\tilde{\epsilon}_0$, from which it follows that broadening of the levels can be neglected if the inequality $n_d a^4 < f/4\pi$ is satisfied.

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