RING LASER WITH A NONLINEAR ABSORBING CELL

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Generation conditions in a ring laser with a nonlinear absorbing cell within the cavity are considered. It is shown that under certain conditions narrow power resonances may appear in the laser radiation which are due to both line burning and to interaction between modes at generation frequencies close to the centers of the amplification and absorption lines. In the latter case, the power resonances may be narrower than the resonances of a laser with a Fabry-Perot resonator.

1. One of the effective methods of realizing highly stable frequency sources in the visible and infrared is the use of narrow resonances in the output of gas lasers. Depending on the generation regime, the output power of a laser is either a maximum at the center of the gain line, or there is a dip in the center of the line (the "Lamb dip"). However, the width of these resonances is relatively large (10^7–10^9 Hz). Narrower resonances ($\sim 10^5$ Hz) can be obtained in lasers using two-component media: absorbing and amplifying. Because of the Lamb dip in the absorbing gas the output power of the laser has a peak at the central absorption frequency.[1-5] This peak can be extremely narrow, since the pressure and radiative decay time of the levels of the gas in the absorbing cell can be rather low.

In this paper we shall investigate narrow resonances in the radiation of a ring laser. Physically, the presence of resonances in the output power of a ring laser is associated not only with the Lamb dip, but also with the interaction of modes at frequencies close to the centers of the gain curves[6-14] or of the absorption lines of the substance. Through the use of specific regimes in the operation of a ring laser, it is possible to obtain resonances in the output power of traveling waves which can be narrower than the resonances of a laser with a Fabry-Perot resonator.

2. Let us consider the generation regime of a ring laser with nonlinear amplifying and absorbing media. We represent the field of the laser E (x, t) in the form of a superposition of two traveling waves:

$$E(x,t) = E_1(t)\cos(\omega t + \varphi_1(t) - kx) + E_2(t)\cos(\omega t + \varphi_2(t) + kx), \quad (1)$$

where ω is the frequency and k is the wave vector.

In this case, if the parameters of the absorbing substance are given a minus sign and those of the amplifying substance a plus sign, the amplitudes $E_i(t)$ of the field will satisfy the following system of equations:

$$\dot{E}_i + \tfrac{1}{2}\Delta\omega E_i = \tfrac{1}{2}\varkappa_+\{1 - \alpha_+ s_+^2 E_i^2 - \beta_+ s_+^2 E_j^2\}E_i + \tfrac{1}{2}\varkappa_-\{1 - \alpha_- s_-^2 E_i^2 - \beta_- s_-^2 E_j^2\}E_i. \quad (2)$$

Here $\Delta\omega$ is the width of the resonator line, κ is the gain coefficient for a weak field, and αs, βs are saturation coefficients. We shall consider that the following conditions are fulfilled:

$$\left|\frac{\varkappa_-}{\varkappa_+}\right| < 1, \quad \mu = \left|\frac{\varkappa_- s_-^2}{\varkappa_+ s_+^2}\right| < 1. \quad (3)$$

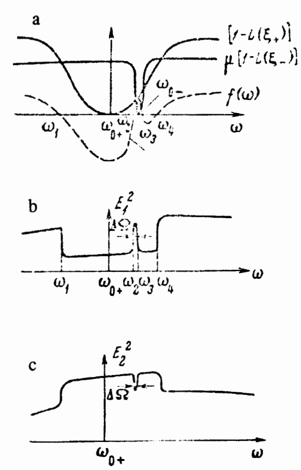

FIG. 1

Conditions (3) impose definite requirements on the parameters of the two-component medium. The first of these is that the weak-field gain coefficient be positive. In this case there are soft regimes. The second of the conditions (3) is the requirement that the nonlinear components of the polarization of the medium be negative (we assume that $\alpha \sim \beta \sim 1$). Only in this case, in the framework of the model (2), is there a saturation effect and the possibility of stable generation regimes.

When (3) is fulfilled, there are stationary solutions of (2) that describe generation both in one traveling wave (unidirectional laser emission) and in two traveling waves (standing wave generation).

It is easy to show that upon fulfillment of the condition

$$f(\omega) > 0,$$

where

$$f(\omega) = \tfrac{1}{2}[(\alpha_+ - \beta_+) - \mu(\alpha_- - \beta_-)], \quad (4)$$

the stable regime is a standing wave; otherwise generation occurs in a single traveling wave. The regions of qualitatively different regimes are divided by the roots of the equation $f(\omega) = 0$. In an approximation linear in γ/ku this equation has the form

$$[1 - L(\xi_+)] - \mu[1 - L(\xi_-)] = 0,$$

where

$$\xi = \frac{\omega - \omega_0}{ku}, \quad \eta = \frac{\gamma}{ku}, \quad L(\xi) = \left[1 + \left(\frac{\xi}{\eta}\right)^2\right]^{-1}, \quad (5)$$

and ω_0, γ, ku are respectively the central frequency of the Doppler line, the homogeneous line width, and the Doppler line width.

The function $f(\omega)$ vanishes at four points ω_i. In case all the roots of the equation $f(\omega) = 0$ are real, there are five regions with different regimes (Fig. 1a). The output power of the traveling waves of the laser for $\gamma_+ > \gamma_-$ corresponding to this division is shown in Figs. 1, b, c. The peak appearing between ω_2 and ω_3 in one wave (and the dip corresponding to it at ω_{0-} in the other) is determined by the width of the homogeneous line of the absorbing gas and the mismatch $|\omega_{0+} - \omega_{0-}|$ between the centers of the amplification and absorption lines. When $|(\omega_{0+} - \omega_{0-})/\gamma_+| \ll 1$. the width $\Delta\Omega$ of the given resonance

$$\Delta\Omega \sim \frac{\gamma_-}{\sqrt{\mu}} \left| \frac{\omega_{0+} - \omega_{0-}}{\gamma_+} \right| \tag{6}$$

can be much smaller than γ_-.[1)] If $|(\omega_{0+} - \omega_{0-})/\gamma_+| \sim 10^{-2}$ and $\mu^{1/2} \sim 1$, then $\Delta\Omega \sim 10^{-2}\gamma_-$. For mismatches $|\omega_{0+} - \omega_{0-}|$ comparable to γ_+, we have $\Delta\Omega \sim \gamma_-$.

It is interesting to note that although in the latter case the width of the resonance is the same as in a laser with a Fabry-Perot resonator and absorbing cell, its nature is not associated with Lamb burning of the Doppler line, but with the interaction of the traveling waves of a ring laser: at frequencies ω close to ω_{0-}, but lying outside the interval $\Delta\Omega$, the laser is unidirectional; at frequencies within $\Delta\Omega$, the laser generates in two waves.

Let us now consider in more detail the case when the central frequencies of the amplifying and absorbing media are close to each other: $\omega_{0+} \sim \omega_{0-} \sim \omega_0$. If we take terms of the order η^2 into account in the polarization expansion, the function $f(\omega)$ will have the form

$$f(\omega) = {}^1/_2 \left\{ [1 - L(\xi_+)] - \mu[1 - L(\xi_-)] - \frac{1}{2}\eta_+^2 + \mu\frac{1}{2}\eta_-^2 \right\}. \tag{7}$$

For small ξ

$$f(\omega) \approx {}^1/_2 \left\{ \left(\frac{\xi}{\eta}\right)_+^2 - \mu\left(\frac{\xi}{\eta}\right)_-^2 - \frac{1}{2}\eta_+^2 + \mu\frac{1}{2}\eta_-^2 \right\}.$$

If we assume that $\gamma_+ \gg \gamma_-$, the term $\frac{1}{2}\mu\eta_-^2$ may be neglected. The term $(\xi/\eta)_+^2$ may also be neglected in comparison with $\mu(\xi/\eta)_-^2$, since in a laser with an absorbing cell we have

$$\mu\frac{\eta_+^2}{\eta_-^2} = \left|\frac{\varkappa_- s_-^2}{\varkappa_+ s_+^2}\right| \left(\frac{\gamma_+}{\gamma_-}\right)^2 \gg 1.$$

Thus, for small ξ, the function $f(\omega)$ is given by:

$$f(\omega) \approx -\frac{1}{2}\left\{\mu\left(\frac{\xi}{\eta}\right)_-^2 + \frac{1}{2}\eta_+^2\right\} < 0.$$

For large ξ, the function $f(\omega)$ is close to

$$f(\omega) \approx {}^1/_2(1 - \mu)$$

and greater than zero. The graph of $f(\omega)$ is shown in Fig. 2.

In the frequency region

$$\omega_{0-} - \gamma_+\sqrt{\frac{\mu}{1-\mu}} \leqslant \omega \leqslant \omega_{0-} + \gamma_+\sqrt{\frac{\mu}{1-\mu}}$$

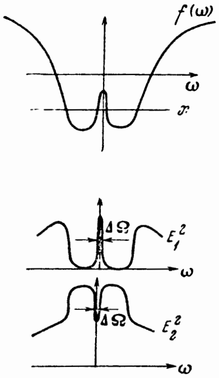

FIG. 2

laser generation is unidirectional; outside this region, $f(\omega) > 0$, and generation is in two traveling waves of different intensities. The width of the resonance in each of the waves is rather large: $\Delta\Omega \sim \gamma_+[\mu/(1-\mu)]^{1/2}$. However, even in the case $\omega_{0+} \sim \omega_{0-}$ it is possible under certain conditions to expect narrow resonances in the radiation of each of the traveling waves of a ring laser. Actually, the function $f(\omega)$, depending on its sign, determines the stability of the limiting cycle of solutions $E_i = E_j$ and $E_i \neq 0$, $E_j = 0$ of the system (2). As follows from (7), $f(\omega)$ has a sharp peak when $\omega \approx \omega_0$ (see Fig. 2). Thus, at frequency $\omega \approx \omega_{0-}$ the stability of the limiting cycle for a solution of the form $E_i \neq 0$, $E_j = 0$ (generation in one traveling wave) is low. The stability of the limiting cycle of this solution can be still less if the external mirrors, by inverse reflection of energy, create a coupling between the traveling waves. Mathematically, this coupling can be taken into account by introducing into the right hand side of (2) the term $\frac{1}{2}\Delta\nu E_j \cos(\varphi_1 - \varphi_2)$, where the parameter $\Delta\nu$ determines the so-called traveling-wave frequency trapping band of a ring laser.[14] With this change, condition (4) for the stability of the solutions of (2) is replaced by the following:

$$f(\omega) + x > 0, \quad x = -\Delta\nu / \varkappa_+ s_+^2(E_1^2 + E_2^2).$$

The intersection of the functions $f(\omega)$ and $x \approx -(\Delta\nu/\Delta\omega)(R-1)^{-1}$ (where R is the excess of pump power over threshold $R_0 = 1$) determines the regions of different radiation regimes. Three qualitatively different regimes can be realized with a change in ξ, depending on the magnitude of x. In the case of most interest to us, when the equation $f(\omega) + x = 0$ has four roots, the dependence of the output power of the traveling waves on mismatch $(\xi)_-$ is shown in Fig. 2. The width of the central peak is given by

$$\Delta\Omega \sim \gamma_-\left\{\frac{1}{\mu}\frac{\Delta\nu}{\Delta\omega}\frac{1}{R-1} - \frac{1}{2}\eta_+^2\right\}^{1/2}. \tag{8}$$

For a He-Ne laser the magnitude of η_+^2 is 10^{-1} to 10^{-2}. It is easy to make the quantity $(1/\mu)(\Delta\nu/\Delta\omega)(R-1)^{-1}$ of the same order, since it is determined by three controllable parameters (R, $\Delta\nu$, $\Delta\omega$). For example, $\Delta\nu$ can vary from 5×10^1 and higher.[14] Thus, in order of magnitude we find

$$\Delta\Omega \sim 10^{-1} - 10^{-2}\gamma_-. \tag{9}$$

With power stabilization (i.e., stabilization of R), the

[1)]When terms quadratic in η are taken into account, the right hand side of (6) is replaced by

$$\frac{\gamma_-}{\sqrt{\mu}}\left\{\left(\frac{\omega_{0+} - \omega_{0-}}{\gamma_+}\right)^2 - \frac{1}{2}\left(\frac{\gamma_+}{ku_-}\right)^2\right\}^{1/2}.$$

magnitude of $\Delta\Omega$ can undoubtedly be decreased by another order of magnitude. If methane ($\gamma_- < 10^5$ Hz) is used as an absorber, for example, $\Delta\Omega$ will be $\sim 10^3$ Hz).

It should be noted that even without an absorbing cell the output power of the laser has a narrow resonance at the center of the gain curve (Fig. 3). For the intensity difference of the traveling waves, for example, it is possible to obtain the following expression:

$$E_1^2 - E_2^2 \simeq (E_1^2 + E_2^2)\left\{1 - \left[\frac{\Delta\nu}{\Delta\omega}\frac{1}{R-1}\left(\xi^2 - \frac{\eta^2}{2}\right)^{-1}\right]^2\right\}^{1/2}. \quad (10)$$

The width of the resonance can be considerably less than the natural width of line γ:

$$\Delta\Omega \approx \gamma\left\{\frac{1}{2}\left(\frac{\gamma}{ku}\right)^2 - \frac{\Delta\nu}{\Delta\omega}\frac{1}{R-1}\right\}^{1/2}. \quad (11)$$

3. Experiments were performed with a He-Ne laser at $\lambda = 3.39\ \mu$. Three plane mirrors with reflection coefficients $R_1 = R_2 = 99\%$ and $R_3 = 96\%$ constituted the resonator in the form of an equilateral triangle with 100-cm sides. One of the branches of the resonator contained a gain tube of length 85 cm. Since the principal sources of noise are traveling striations and ionic plasma oscillations,[15] The discharge in the gain tube was excited by a special combination of high-frequency and dc currents. The absorbing cell, in another leg of the resonator, was a tube of length 100 cm and internal diameter 2 cm filled with gaseous methane, the pressure of which was varied within the limits 10^{-3} to 10^{-1} Torr. One of the resonator mirrors was affixed to a piezoceramic driven with a sawtooth voltage. The radiation of each of the laser waves was extracted from the resonator through one of the mirrors and registered by a Ge-Au photodetector cooled with liquid nitrogen. The signal from the photodetector was applied to the vertical input of an oscilloscope. The horizontal scan of the oscilloscope was provided by a voltage from the generator which drove the piezoceramic. As a result, the oscilloscope registers the laser output as a function of generation frequency.

On scanning the frequency over the Doppler line, one could observe on a dual-beam oscilloscope the regimes of laser radiation both in the traveling waves and in the standing waves. Since the gain of the He-Ne laser at $3.39\ \mu$ is rather high and the resonator length was about 3×10^2 cm, the radiation pattern was rather complex. This could have been due both to competition of adjacent axial modes and to the interaction of traveling waves with the same wave vector. Figure 4 shows an oscillogram of the power of one of the traveling waves as a

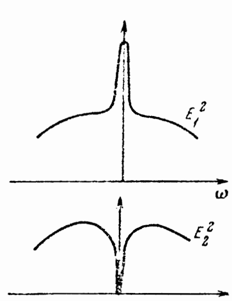

FIG. 3

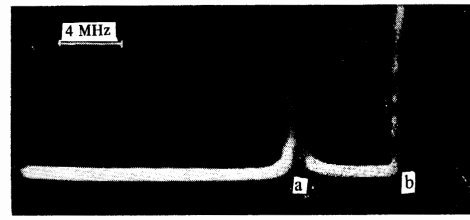

FIG. 4

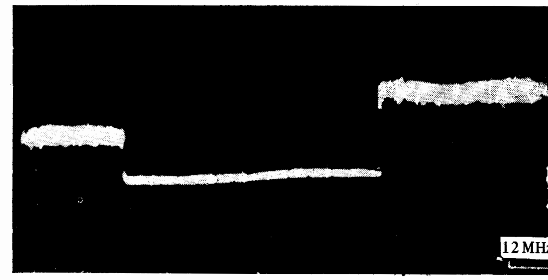

FIG. 5

function of resonator frequency, which includes a peak at the central frequency of CH_4. The other wave has a dip (cf. Fig. 1). The CH_4 pressure in the cell was 3×10^{-2} Torr, that of the He-Ne mixture was 4 Torr. The half-width of the peak was $\Delta\Omega \sim 300$ kHz. The separation between the maxima of the gain and absorption curves was rather great in this case. Hence the width of the peak, as expected (see Eq. (6)), was of the order of the width of the natural emission line of methane. Remember that in this case the nature of the resonance is not due to Lamb burning of the absorption line, but by the interaction of the traveling waves of the ring laser.

It is interesting that when the methane pressure was changed, the boundary of the region of stability of the standing wave (point b, Fig. 4) shifted relative to the generation peak (point a) until they coincided (at low methane pressures). The physics of this is the following. With decreasing CH_4 pressure, the quantity μ we introduced above decreases, and the root ω_4 of the equation $f(\omega) = 0$ approaches the root ω_3 (see Fig. 1). For sufficiently small μ, the roots come together. In this case the center of the absorption line falls in the region of stability of the standing wave and generation of the ring laser at $\omega \sim \omega_{0-}$ will not differ from that of a Fabry-Perot laser.

Note that the frequency regions in which is realized the regime of unidirectional generation for a ring laser with an absorbing cell and without it are essentially different. In the first case this region can be considerably greater than in the second.

Figure 5 is an oscillogram showing unidirectional generation of a ring laser with an absorbing cell and generation in two traveling waves. The region of frequencies in which unidirectional generation exists is shown in the center of the oscillogram and is 75 MHz wide in this case. The He-Ne pressure was ~ 4 Torr, that of the methane $\sim 9 \times 10^{-3}$ Torr. At these pressures the methane peak is gone, because of the interaction of laser modes. The reason for this was discussed above. At the same time, the emission peak associated with

Lamb burning of the absorbing gas is not observed, although in this case generation of the laser at $\omega \approx \omega_0$ occurs as a standing wave. The latter, evidently, is a consequence of the fact that at such low pressure of methane the saturation effect is small and the resolution of the apparatus does not permit registration of the peak in the output.

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