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MIXING OF THE LEVELS OF THE HYDROGEN ATOM DUE TO THE HYPERFINE INTERACTION

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The mixing of the levels of the hydrogen atom due to the interaction with the magnetic field created by the proton spin is considered. In contrast to previous work pertaining to other atoms, not only the case $\Delta l = 0$ but also the case $\Delta l = 2$ is analyzed. The change of the atom's magnetic moment and the removal of the forbiddenness of certain optical transitions are discussed.

If an atom is placed in an external magnetic field H, then those of its states which possess the same parity and the same projections M_F of the total angular momentum cease to be stationary and are intermixed with each other. It is well-known that the indicated mixing changes the magnetic moments of the atoms and affects the selection rules for optical transitions (see[1]).

If the nucleus of an atom possesses a magnetic moment, then one can assume that the atom is located in a magnetic field, and one should expect the appearance of similar effects. Since in the present case the total angular momentum F of the system is conserved, then for the mixing not only is coincidence of the values of P and M_F necessary but also the values of F must be the same. Since the amount of mixing is inversely proportional to the distance between the levels which are being mixed, it is advisable to consider states which are energetically close together, possessing the same value of the principal quantum number n.

States with both the same and with different values of the orbital momentum l may intermix. The situation in which levels having identical values of the orbital angular momentum are mixed together was considered in a number of articles.[2-5] The corrections to the magnetic moment of the system, which turned out to be linear in the mixing, are obtained in articles[2,3]; their magnitude is two orders of magnitude less than the magnetic moment of the nucleus. There it is shown that upon placing the atom in an external magnetic field which breaks the hyperfine coupling, the corrections to the magnetic moment of the system are not changed.

Garstang[4,5] showed that the mixing under consideration leads to the removal of the forbiddenness of certain optical transitions.

In the present article we study states with differing values of the orbital angular momentum. Specifically, we deal with the hydrogen atom since only in this case can states with different l be sufficiently close together. Levels with values of orbital momentum differing by unity will not be mixed by the magnetic field of the nucleus, since they possess different parities. Since the proton spin I and the electron spin s are each equal to 1/2, levels with $\Delta l = 0, 2$, whose parity is the same, may intermix. Therefore, the effects of interest to us can only occur for $n \geq 3$.

Let us compare the distances between the levels which are being mixed in the cases $\Delta l = 0$ and $\Delta l = 2$. In the first case we obtain

$$\delta_{l=l'} = \frac{\alpha^2}{l(l+1)n^3}\,[\mathrm{Ry}],$$

and in the second case[1)]

$$\delta_{l=l'+2} = \frac{\alpha^2}{(l'+1)(l'+2)n^3}\,[\mathrm{Ry}]$$

(α is the fine structure constant). The quantities $\delta_{l=l'}$ and $\delta_{l=l'+2}$ are close to one another in value; therefore the mixing in both cases will be of the same order of magnitude.

Since the distances between the intermixed levels is much larger than the hyperfine splitting, then in order to find new stationary states one can use perturbation theory, regarding the interaction W of the nucleus' magnetic moment with the angular momentum of the shell as a small perturbation.

Let us write an expression for the wave function of the system in terms of states with orbital momentum l:

$$\varphi_1 = \psi_1 + \beta\psi_2 - {}^1\!/_2\beta^2\psi_1. \tag{1}$$

Here $\psi_1 \equiv |s\,l\,JIFM\rangle$, $\psi_2 \equiv |s\,l'J'IFM\rangle$, and

$$\beta = \frac{\langle slJIFM|W|sl'J'IFM\rangle}{E_{lJ} - E_{l'J'}},$$

J denotes the total angular momentum of the shell. The term of second order in formula (1) is calculated from considerations which will be clear below. The wave function of the state with orbital momentum l' is written in the following way:

$$\varphi_2 = \psi_2 - \beta\psi_1 - {}^1\!/_2\beta^2\psi_2. \tag{2}$$

The matrix elements $\langle s\,l\,JIFM|W|s\,l'J'IFM\rangle$ are easily evaluated.[6]

Now let us find the projection of the atom's magnetic moment on the z axis in the states φ_1 and φ_2. Neglecting terms of order β^3, for the state φ_1 we obtain

$$\mu_z = \langle\psi_1|\hat{\mu}_z|\psi_1\rangle + \beta^2\{\langle\psi_2|\hat{\mu}_z|\psi_2\rangle - \langle\psi_1|\hat{\mu}_z|\psi_1\rangle\}. \tag{3}$$

The first term on the right hand side of formula (3) represents the magnetic moment of the system without taking mixing into consideration, the second term repre-

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[1)] The Lamb shift of the level with $l' = 0$ decreases the value $\delta_{l=l'+2}$ by approximately 10%.

sents the additional magnetic moment due to the appearance of the interaction of the magnetic field of the proton's spin with the electron's angular momentum:

$$\mu_{\text{add}} = \beta^2\{\langle\psi_2|\hat{\mu}_z|\psi_2\rangle - \langle\psi_1|\hat{\mu}_z|\psi_1\rangle\}. \tag{4}$$

For the state φ_2 we obtain

$$\mu_{\text{add}} = -\beta^2\{\langle\psi_2|\hat{\mu}_z|\psi_2\rangle - \langle\psi_1|\hat{\mu}_z|\psi_1\rangle\}. \tag{5}$$

The additional magnetic moment which appears in connection with the mixing of levels with identical l is proportional to the first power of β; however, in the case of mixing of levels possessing different orbital momenta, it turns out to be quadratic in β.

Using the well-known formulas, it is not difficult to calculate the values of the matrix elements appearing in expressions (4) and (5):

$$\langle slJIFM|\hat{\mu}_z|slJIFM\rangle = \\ = -\mu_0 M \frac{[3J(J+1)+l(l+1)-s(s+1)][F(F+1)+J(J+1)-I(I+1)]}{4F(F+1)J(J+1)}, \tag{6}$$

where μ_0 is the Bohr magneton.

In the case of a mixing of the levels $3d_{3/2}$ with total momentum F = 1 and $3s_{1/2}$ with F = 1, the additional magnetic moment in the d- and s-states has the form

$$\overset{d}{\mu}_{\text{add}} = -\overset{s}{\mu}_{\text{add}} = 1.15\,\mu_0\beta^2 = 6.44\,\mu_0 g_I (m/m_p)^2, \tag{7}$$

where g_I denotes the gyromagnetic ratio of the nucleus, and m_p is the proton mass.

The shift of the levels due to mixing is also determined by the correction of second order, and in the case of mixing of the levels $3d_{3/2}$ and $3s_{1/2}$ it is given by

$$\Delta E = 0{,}1 g_I{}^2 (m/m_p)^2 \alpha^2 \text{ [Ry]}. \tag{8}$$

Its value does not fall outside the limits of the natural width of the levels.

Now let us clarify the influence of the mixing of levels with different values of the orbital momentum on the selection rules for electric dipole transitions.

Let us consider three levels A, B, and C. Let level A be characterized by the quantum numbers $n = n'$, $l = l' + 1$, $J = l' + 1/2$, let level B be characterized by the quantum numbers $n = n''$, $l = l'$, $J = l' - 1/2$, and level C by the quantum numbers $n = n''$, $l = l' - 2$, $J = l' - 3/2$, where $n' \neq n''$ and $l' \geq 2$. The transition A → B is allowed since $\Delta l = 1$, $\Delta J = 1$. For the transition A → C, $\Delta l = 3$ and $\Delta J = 2$, i.e., it is forbidden both with respect to l and with respect to J. However, in practice the transition will occur in view of the fact that the levels B and C are mixed by the magnetic field of the nucleus. Let us denote the intensity of the transition A → C by I_{AC}. Then one can easily see that

$$I_{AC} = \beta^2 \frac{\omega_{AC}}{\omega_{AB}} I_{AB}, \tag{9}$$

where $\omega_{ik} = (E_i - E_k)/\hbar$, β denotes the coefficient of mixing of the levels B and C, and I_{AB} denotes the intensity of the transition A → B.

For example, if the mixing of the levels $3d_{3/2}$ and $3s_{1/2}$ is taken into account, then the transition $4f_{5/2} \to 3s_{1/2}$ turns out to be allowed. The square of the coefficient for the mixing of the levels $3d_{3/2}$ and $3s_{1/2}$ is given by

$$\beta^2 = 5{,}6 g_I (m/m_p)^2.$$

Thus, the intensity of the transition $4f_{5/2} \to 3s_{1/2}$ which is allowed by mixing is $[5.6 g_I (m/m_p)^2]^{-1}$ times smaller than the intensity of the transition $4f_{5/2} \to 3d_{3/2}$.

The appearance of additional magnetic moments due to mixing was considered above. In conclusion we note that if one takes mixing into consideration, then it turns out that the hydrogen atom in the state with $l = 0$ also possesses a nonvanishing quadrupole electric moment. From the fact that the operator for the quadrupole moment is not diagonal with respect to the quantum number l, one can conclude that the quadrupole moment is a quantity of first order in the mixing.

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