DESTRUCTION OF A LARGE-AMPLITUDE ION-ACOUSTIC WAVE

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A numerical experiment on the propagation of a powerful ion-acoustic wave in a one-dimensional plasma model was performed on an electronic computer. The nature of breaking of the wave front at large Mach numbers is investigated, and it is found that it is of a pulsation type with a frequency of the order of the plasma ion frequency.

In the physics of nonlinear plasma oscillations, the method of numerical experiment with a plasma model on a high-speed electronic computer has achieved widespread popularity.[1,2] The results are given below of the application of this method to the well-known problem of the destruction of a nonlinear ion-acoustic wave of supercritical amplitude (above the breaking threshold).

As is well known, an arbitrary initial profile of an ion-acoustic excitation in a plasma with $T_e \gg T_i$ breaks up in the course of time into a set of stationary waves—solitons. The properties of the ion-acoustic solitons have been investigated analytically[3] (see also the experimental observation[4]). However, at sufficiently large amplitudes (Mach numbers greater than 1.6), solitons cannot exist, since the breaking phenomenon takes place, leading to "multistream motion." By staying within the framework of the hydrodynamic approximation, it is not possible to consider the process of the development of the multistream motion. A kinetic consideration becomes inevitable when it is necessary to track the trajectory of each particle. Modern computers still do not let us carry out such "observations" for real plasma models;[5] however, for clarification of the process of the generation of the multistream motion, one can consider a very simple model which takes the basic effects into account.

We shall assume that the electrons has a Boltzmann distribution, and will follow only the motion of the ionic component. Such an approach materially simplifies the problem which, for the one-dimensional case, reduces to a solution of the following equations:

$$\frac{dv_j}{dt}=\frac{e}{M}E_j, \quad E=-\frac{d\varphi}{dx},$$
$$\frac{d^2\varphi}{dx^2}=-4\pi e\,[n_i-n_0e^{e\varphi/T}],$$
$$n_i=\int f(t,x,v)\,dv, \tag{1}$$

where the first equation describes the motion of the j-th particle, f(t, x, v) is the distribution function, M, v, and n_i the mass, velocity and density of the ions, respectively. For the numerical experiment, use is made of a model of ionic layers of finite width. Each layer carries a charge e and mass M; E_j and v_j in the first equation of (1) are referred to the center of the layer.

Fourier analysis of small oscillations of such a plasma leads to the dispersion relation

$$1=\frac{T}{M}\frac{\exp(-k^2a^2)}{1+k^2D^2}\int\frac{\partial f/\partial v}{v+\omega/k}dv, \tag{2}$$

where $D=\sqrt{T/4\pi e^2 n_0}$ is the Debye radius, and a the width of the ionic layer. For the case in which the ions have a Maxwellian distribution, it is not difficult to see that (2) transforms in the case of small k into the well-known dispersion relation for ionic oscillations of a cold plasma:

$$\omega=\sqrt{\frac{T}{M}}\frac{k}{(1+k^2D^2)^{1/2}}\exp\left(-\frac{k^2a^2}{2}\right),$$

with the exclusion of the factor $\exp(-k^2a^2/2)$, which is associated with the "laminated" description of the plasma. However, it can be assumed that the solution will be sufficiently accurate for sufficiently small thickness of the layer.

To test this method, we solved the problem of the decomposition of an excitation of subcritical amplitude, in which $\Delta n/n = 0.2$ at the initial instant. As was to be expected, the formation of solitons was observed.

In the solution of the boundary problem for the equation

$$\psi(\varphi)=\frac{d^2\varphi}{dx^2}+4\pi e\,(n_i-n_0e^{e\varphi/T})=0 \tag{3}$$

the latter is replaced by the equation[6]

$$\psi(\varphi)=-\psi_\varphi{}'\varphi_t{}',$$

the solution of which will be the solution of (3) as $t\to\infty$. The problem of the propagation of a simple wave was solved by this method. The profile of the wave at the initial instant was taken in the form

$$n_i(x)=N\exp[-(x-x_0)^2/l^2],$$

where N is the amplitude of the excitation, and l its half width. The space in which the wave could move was equal to 75 Debye lengths and 1570 ionic layers were located in this space, of which 30% were in the wave at the initial instant. The condition

$$\varphi_x{}'(0)=\varphi(L)=0$$

was imposed on the potential at the boundaries of the interval and the condition of reflection of the layers from the boundary was established. The problem could be solved up to the point at which the excitation reached the right-hand boundary.

For comparison, two cases of wave propagation were studied. In the first, the ratio of the maximum ampli-

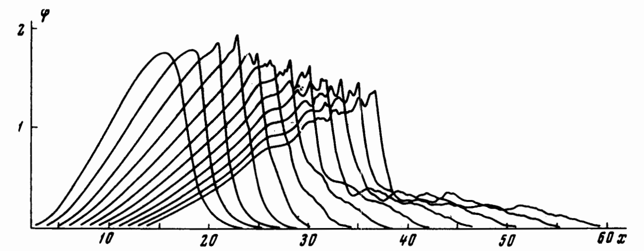

FIG. 1

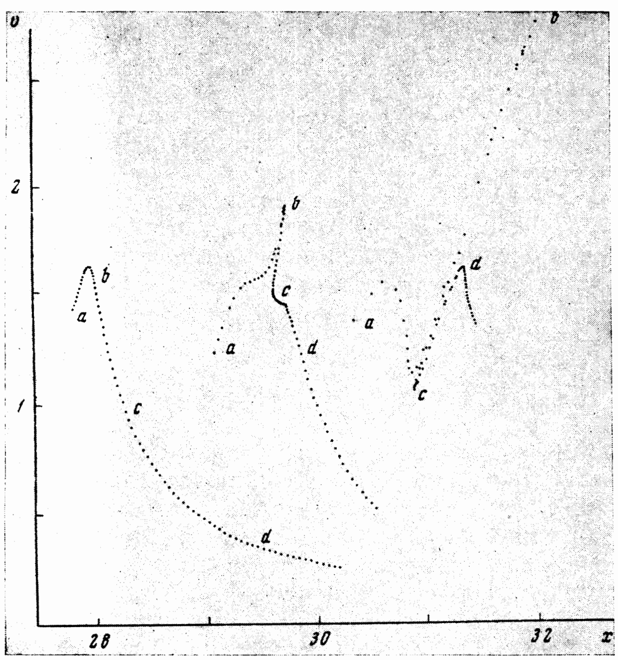

FIG. 2

tude of the density to the undisturbed state at the initial instant amounted to 3 : 1, in the second, to 6 : 1. In the first case, an enlargement of the profile of the wave and an acceleration of it to a velocity somewhat less than critical took place. At this moment, some mixing of the particles takes place on the crest, but reflection of the particles by the wave was not observed. In this case, the hydrodynamic consideration of the problem remained valid and oscillations of the soliton type were formed. Here and below, the velocity v, the potential φ and the coordinates x and t are expressed in units of $\sqrt{T/M}$, T/e, λ_D and ω_{pi}^{-1}, respectively.

We now consider the second case. After the steepening of the simple wave, the propagation velocity and the maximum potential turn out to be greater than critical, i.e., if we change over to the system of the wave, it turns out that incoming ions cannot surmount the potential "hump" and are reflected. Here the velocity of the reflected particles should be 2u (u is the velocity of the front). Inasmuch as we have an essentially nonstationary regime, all the considerations for the stationary case are satisfied approximately in the mean. It must be observed that the "breaking" begins at that moment when the incoming particles have just the energy equal to the height of the potential "hump," and in what follows the potential in the "hump" does not exceed this critical value. As the number of ions accelerated by the wave increases, the potential before the front of the wave increases appreciably, as a result, the condition for breaking is already determined by the quantity $\varphi_{max} \approx (u^2 + \varphi_0)/2$. Figure 1 shows the curves $\varphi(x)$ for successive instants of time from t = 2 to 2 = 13, at which it it seen that the characteristic width of the leading front is of the order of the Debye length.

Figure 2 shows the distribution v = f(x) for the chosen group of ionic layers at successive instants of time, which illustrates an interesting feature of a wave, which propagates with a speed exceeding the critical value. After breaking, a group of ions obtains the velocity $v > u$, as a result of which the total effective velocity of the wave of potential is increased and the maximum value of the potential is somewhat lowered. The incoming current of ions has an energy greater than the potential energy at the peak of the leading front of the wave

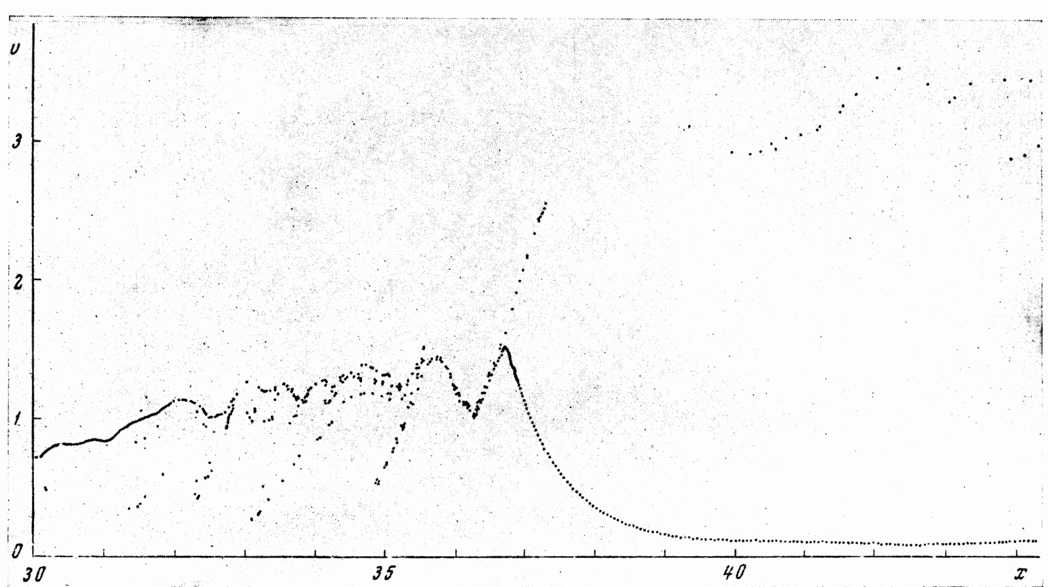

FIG. 3

and therefore crosses over it. Then the potential again increases and becomes sufficient for breaking of the incoming current and so forth.

Thus, the resultant breaking of the wave has a pulsation character, with a frequency of the order of ω_{pi}. Figure 3 gives the ionic distribution in the space v, x for t = 13, it is clearly seen that before the wave front, there are "tails" of retarded ions, formed after each cycle of reflection. The plasma before the front of the ion-acoustic wave is penetrated by fast ions which appear after the breaking. This leads to the formation of a "pedestal" in the profile of the potential wave (Fig. 1) that moves away from the front. The fact that this current of reflected ions does not excite any beam instabilities corresponds in the given machine experiment to the one-dimensional model assumed by us.

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