ABSORPTION OF SOUND IN AN $He^3 - He^4$ *SOLUTION*

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Absorption of first, second, and fourth sounds in superfluid $He^3 - He^4$ solutions is studied at arbitrary He^3 concentrations. It is shown that the absorption coefficients of the first and second sounds depend on all the second-viscosity coefficients, whereas the fourth sound absorption coefficient depends only on a single second-viscosity coefficient. The contribution of impurity diffusion to the sound absorptions is calculated.

SOUND propagation in a superfluid $He^3 - He^4$ solution has a number of peculiarities connected with the oscillation of the He^3 concentration in the acoustic wave. Whereas in pure helium II only the pressure oscillates in the first-sound wave, and only the temperature oscillates in the second-sound wave (neglecting the coefficient of thermal expansion, which is anomalously small for helium), in a solution there are pressure, temperature, and concentration oscillations in both waves. In the first-sound wave the oscillation of the temperature is proportional to the coefficient $\beta = (c/\rho)\partial\rho/\partial c$, and in the second-sound wave the same coefficient is proportional to the pressure oscillation (c—maximum He^3 concentration, ρ—density of the solution), and at low He^3 concentrations the quantities proportional to β cannot be neglected ($\beta \approx -0.3$—0.4 for highly-concentrated solutions). Unlike pure He^4, the first-sound wave in solutions contains a relative oscillation of the normal and superfluid liquids, the magnitude of which is proportional to β. In pure He^4 there are no oscillations of the total flux $\mathbf{J} = \rho_n\mathbf{v}_n + \rho_s\mathbf{v}_s$ in the second-sound wave, whereas in the solution the deviation from the equilibrium value of J is also proportional to β.

The propagation velocities of the first and second sounds at arbitrary He^3 concentration were calculated in[1]. A strong dependence of the sound propagation velocity on the He^3 concentration is observed and is due to the fact that the pressure and temperature oscillations in the sound wave cause also concentration oscillations. The sound absorption coefficients were calculated[1] at small He^3 concentrations, neglecting the effects connected with the presence of temperature (pressure) oscillations in the wave of the first (second) sound and the relative oscillation of the normal and superfluid parts of the liquid. As will be shown in the present note, at high He^3 concentrations these effects make a definite contribution to the absorption coefficients of both the first and second sounds.

To calculate the sound absorption coefficient, it is convenient to use the dissipation function R of the liquid. The dissipation function for the superfluid He^3—He^4 solution has the following form[2]:

$$R = \zeta_3[\operatorname{div}(\mathbf{J} - \rho\mathbf{v}_n)]^2 + \zeta_2[\operatorname{div}\mathbf{v}_n]^2 + 2\zeta_1 \operatorname{div}(\mathbf{J} - \rho\mathbf{v}_n)\operatorname{div}\mathbf{v}_n + \frac{1}{2}\eta\left(\frac{\partial v_{ni}}{\partial r_k} + \frac{\partial v_{nk}}{\partial r_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_{ne}}{\partial r_e}\right)^2 + \varkappa\frac{(\nabla T)^2}{T} + \rho D\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\left[\nabla c + \frac{k_T}{T}\nabla T + \frac{k_P}{P}\nabla P\right]^2, \quad (1)$$

where η is the viscosity coefficient, ζ_1, ζ_2, and ζ_3 are the second-viscosity coefficients, $\varkappa$ is the thermal-conductivity coefficient, and D, k_PD, and k_TD are respectively the coefficients of diffusion, barodiffusion, and thermodiffusion; the quantity $z = \rho(\mu_3 - \mu_4)$ is defined in terms of the chemical potentials μ_3 and μ_4 of the He^3 and He^4 in the solution.

The sound absorption coefficient is expressed in terms of the dissipation function R of the liquid in the following manner

$$\gamma = \bar{R}/2u\bar{E}, \quad (2)$$

where u is the speed of sound and E the energy of the sound wave. The bar denotes averaging over the time in the sound wave.

Using the connection between the oscillations of T, c, v_n, and v_s in the sound wave[3,4], obtained from the linearized hydrodynamic equations for He^3—He^4 solutions without allowance for the dissipation processes, we can easily calculate the sound-absorption coefficients by means of (1) and (2).

For the first-sound absorption coefficient, connected with the viscosity and the thermal conductivity, we obtain

$$\gamma_{\eta,\zeta,\varkappa}^{(1)} = \frac{\omega^2}{2\rho u_1^3}\frac{1}{1+(\rho_s/\rho_n)\beta^2}\left[\left(\frac{4}{3}\eta + \zeta_2\right)\left(1 - \frac{\rho_s}{\rho_n}\beta\right)^2 + \frac{2\rho\rho_s}{\rho_n}\zeta_1\beta\left(1 - \frac{\rho_s}{\rho_n}\beta\right) + \frac{\rho^2\rho_s^2}{\rho_n^2}\beta^2\zeta_3 + \frac{\varkappa}{C_{He}}\frac{\rho\rho_s}{\rho_n}\beta^2\bar{\sigma}^2\frac{1}{u_1^2}\frac{\partial T}{\partial\sigma}\right], \quad (3)$$

where c_{He} is the specific heat of the solution, σ is the entropy, and $\bar{\sigma} = \sigma - c\partial\sigma/\partial c$.

In pure He^4, the absorption of first sound is connected only with η and ζ_2, since in the first-sound wave there are no temperature oscillations (neglecting the coefficient of expansion of helium) and no relative oscillation of the normal and superfluid components of the liquid. In a solution, on the other hand, owing to the presence of the relative oscillation of the normal and superfluid parts of the liquid, the second-viscosity coefficients ζ_1 and ζ_3 contribute to the sound absorption. In view of the fact that the absorption connected with ζ_1 is proportional to $\operatorname{div}\rho_s(\mathbf{v}_n - \mathbf{v}_s)\operatorname{div}\mathbf{v}_n$, and that connected with ζ_3 is proportional to $[\operatorname{div}\rho_s(\mathbf{v}_n - \mathbf{v}_s)]^2$, the absorption connected with these viscosity coefficients is proportional to β and β^2, respectively. Since in the first-sound wave the temperature gradient is

proportional to β, the corresponding absorption is proportional to β^2. It should be noted that the absorption connected with η and ζ_2 changes in the solution also because of the presence of a factor $[1 + (\rho_S/\rho_N)\beta^2]^{-1}$, which varies rapidly with the temperature.

A contribution to the sound absorption is made also by the diffusion of the impurities

$$\gamma_D^{(1)} = \frac{1}{2}\frac{\omega^2}{u_1^5}\rho\beta^2 D\left\{c^2\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)\right\}^{-1} \times\left[\frac{\rho_s}{\rho_n}c\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\left(c+\frac{k_T\bar{\sigma}}{C_{He}}\right)+u_1^2\right]^2. \quad (4)$$

The diffusion of the impurities is due both to the presence of a concentration gradient and to pressure and temperature gradients. The first term in the square brackets is connected with the diffusion and with the thermodiffusion, and the second is connected with barodiffusion.

At small concentrations it is possible to neglect in (3) the terms proportional to β, and in (4) one can retain only the term connected with the barodiffusion[1].

For second sound, the absorption coefficient connected with the viscosity and thermal-conductivity coefficients is equal to

$$\gamma_{\eta,\zeta,\varkappa}^{(2)} = \frac{\omega^2}{2u_2^3}\frac{\rho_s}{\rho_n\rho}\frac{1}{1+(\rho_s/\rho_n)\beta^2}\left[\left(\frac{4}{3\eta}+\zeta_2\right)(1+\beta)^2 - 2\rho\zeta_4(1+\beta)+\rho^2\zeta_3+\frac{\varkappa}{C_{He}}\bar{\sigma}^2\frac{1}{u_2^2}\frac{\partial T}{\partial\sigma}\right]. \quad (5)$$

The coefficient of second-sound absorption in the solution, just as in pure He^4, includes all the second-viscosity coefficients, but a dependence of γ on β appears, and there is also a factor $[1 + (\rho_S/\rho_N)\beta^2]^{-1}$, which changes rapidly with temperature.

The sound-absorption coefficient connected with diffusion has the following form:

$$\gamma_D^{(2)} = \frac{\omega^2}{2u_2^5}\frac{\rho_s}{\rho_n}D\left\{c^2\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)\right\}^{-1} \times\left[c\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\left(c+\frac{k_T\bar{\sigma}}{C_{He}}\right)-\beta^2u_2^2\right]^2. \quad (6)$$

The first term in the square bracket is connected with the diffusion and thermodiffusion, and the second with barodiffusion.

At small He^3 concentrations, neglecting in (6) the terms proportional to β, and retaining in (6) only the terms connected with the diffusion and thermodiffusion of the impurities, we obtain the well known results[1].

Under conditions of complete standstill of the normal component of the liquid, a special type of oscillation, called fourth sound, propagates in the superfluid helium. The velocity of fourth sound, owing to the presence of concentration oscillations in the sound wave, depends strongly on the He^3 concentration[5]. The absorption of fourth sound is connected both with surface dissipation effects and with volume dissipation effects. The surface dissipation[6] is connected with the slippage of the normal component of the liquid and with thermal conductivity through the walls of the channel through which the fourth sound propagates. On the other hand, owing to the fact that in the fourth-sound wave there are no oscillations of the normal components of the liquid, the volume absorption is connected only with the second-viscosity coefficient ζ_2, and with the thermal conductivity and diffusion of the impurities. Using (1) and (2), we obtain for the coefficients of the volume absorption of fourth sound the following expressions:

$$\gamma_{\zeta,\varkappa}^{(4)} = \frac{1}{2}\frac{\omega^2}{u_4^3}\frac{\rho_s}{\rho^2}\left[\rho^2\zeta_3+\frac{\varkappa}{C_{He}}\bar{\sigma}^2\frac{1}{u_4^2}\frac{\partial T}{\partial\sigma}\right], \quad (7)$$

$$\gamma_D^{(4)} = \frac{1}{2}\frac{\omega^2}{u_4^5}D\left\{c^2\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\right\}^{-1}\frac{\rho_s}{\rho}\left[\left(c+\frac{k_T}{C_{He}}\bar{\sigma}\;c\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)+\frac{u_1^2\beta(1+\beta)}{1+(\rho_s/\rho_n)\beta^2}\right]^2 \quad (8)$$

where the first term in formula (8) in the square brackets is connected with diffusion and thermodiffusion, and the second is connected with barodiffusion of the impurities.

The coefficients of second viscosity, which enter in the sound absorption, can be calculated theoretically in two limiting cases: at small He^3 concentrations, on the basis of the model of an impurity gas of excitations[7], and in the direct vicinity of the λ point, where the phenomenological approach is valid[8-10]. By independently measuring the coefficient of the first viscosity η and the coefficients of thermal viscosity and diffusion, we can determine the coefficients of the second viscosity of the solution He^3—He^4 from the experimental values of the absorption coefficients of the first, second, and fourth sounds.

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