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TEMPERATURE SCATTERING OF SOUND IN A SOLID

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The problem of the scattering of a sound wave by thermal fluctuations of deformations and temperature in an isotropic solid is solved. An expression is obtained and analyzed for the spectral intensity of scattered sound. Numerical estimates are given for the value of the sound intensity scattered by the temperature branch of combined temperature and sound oscillations. A comparison of this value with the value of the sound intensity scattered by the acoustical branch is given.

THE circle of phenomena which have been studied to date in nonlinear acoustics is limited basically to nonlinear interactions of acoustical waves with one another. Remaining unconsidered has been an entire class of acoustical phenomena, the characteristic feature of which is the nonlinear coupling of the sound waves with waves of another kind. Inasmuch as such waves can have a different dispersion dependence than that of sound waves, it is clear that even this fact can introduce new features in similar interactions, compared with purely acoustical interactions. For example, it is known that one of the reasons which inhibit the existence of interaction of a limited number of harmonic sound components in one direction is the weak dispersion of acoustical waves. In the consideration of the interaction of sound with excitations of a different nature, which have a strong dispersion, no such problem arises. If it proves possible to produce scattering of sound from some sort of excitations having a resonant character, then such scattering will obviously be analogous to Raman scattering of light.

1. The purpose of the present work is to consider sound scattering from the temperature branch of joint fluctuations of oscillations of temperature and deformations, i.e., to analyze the spectral composition of the scattered sound, to estimate its intensity and the possibility of its experimental observation, to compare the temperature scattering with scattering from the acoustical branch, and to ascertain what evidence one can obtain on material constants on the basis of the temperature scattering of sound.

As is known,[1] the temperature branch is characterized by a dispersion relation that differs essentially from that of the acoustical branch. In particular, this leads to the result that, in contrast with the scattering of sound by sound, where the scattered sound of a given frequency travels along a definite direction, the sound scattered from the temperature branch and having a definite frequency can travel along various directions. Here, as will be seen from Eq. (23), by comparing the intensity of the scattered sound in directions of, say, 90 and 180° with respect to the incident sound, one can deduce the value of some combination of the nonlinear moduli A, B, C of the substance from (1). The line width of the scattered sound allows one to determine that parameter of the material which is a criterion for the adiabatic character of the sound propagation.

2. In the solution of the problem, we start out from the nonlinear sound equation, supplemented by the nonlinear equation of heat conduction. Such an equation can be written down by using the form of the free energy $\mathcal{F}$ per unit volume of the material as a function of temperature T and of the deformation tensor u_{ik}. In this case, we take it into consideration that the deformations are accompanied by a temperature change (see[2]). Let the body be assumed to be undeformed in the absence of external forces at some temperature T_0. We shall assume that the temperature change $T - T_0$ accompanying the deformation and the deformation itself are small. Then, by expanding the energy $\mathcal{F}$ in a series in powers of $T - T_0$ and u_{ik} and retaining the first nonlinear terms, we have

$$\begin{aligned}\mathcal{F} = \mathcal{F}_0(T) - \tilde{k}\alpha(T-T_0)u_{ll} + {}^1/_2\tilde{k}u_{ll}^2 + \mu(u_{ik} - {}^1/_3\delta_{ik}u_{ll})^2 \\ - \tilde{k}\alpha\varepsilon(T-T_0)^2u_{ll} + {}^1/_2\tilde{k}\beta(T-T_0)u_{ll}^2 + \mu\gamma(T-T_0)(u_{ik} - {}^1/_3\delta_{ik}u_{ll})^2 \\ + {}^1/_3Au_{ik}u_{il}u_{kl} + Bu_{ik}^2u_{ll} + {}^1/_3Cu_{ll}^2.\end{aligned} \tag{1}$$

Here $\mathcal{F}_0$ is the free energy of the undeformed body, $\tilde{k}$ the isothermal compression modulus, μ the shear modulus, α the coefficient of thermal expansion, and ϵ, β, γ, A, B, C are nonlinear coefficients.

From (1) we have the nonlinear sound equation

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2} - \left(\tilde{k} + \frac{\mu}{3}\right)\nabla(\nabla\mathbf{u}) - \mu\Delta\mathbf{u} + \tilde{k}\alpha\nabla T = \mathbf{F}(\mathbf{r};t), \tag{2}$$

where

$$\begin{aligned}F_i = \left(\tilde{k}\beta - \frac{2}{3}\mu\gamma\right)\frac{\partial T}{\partial x_i}\frac{\partial u_l}{\partial x_l} + \left(\tilde{k}\beta + \frac{1}{3}\mu\gamma\right)T\frac{\partial^2 u_k}{\partial x_i\partial x_k} \\ - (\tilde{k}\alpha - \mu\gamma)\left(\frac{\partial T}{\partial x_k}\frac{\partial u_i}{\partial x_k} + T\frac{\partial^2 u_i}{\partial x_k^2}\right) - 2\alpha\varepsilon T\frac{\partial T}{\partial x_i} + \mu\gamma\frac{\partial T}{\partial x_k}\frac{\partial u_k}{\partial x_i} \\ + \left(\mu + \frac{A}{4}\right)\left(\frac{\partial^2 u_l}{\partial x_k^2}\frac{\partial u_l}{\partial x_i} + \frac{\partial^2 u_l}{\partial x_k^2}\frac{\partial u_i}{\partial x_l} + 2\frac{\partial^2 u_i}{\partial x_l\partial x_k}\frac{\partial u_l}{\partial x_k}\right) \\ + \left(\tilde{k} + \frac{\mu}{3} + \frac{A}{4} + B\right)\left(\frac{\partial^2 u_l}{\partial x_i\partial x_k}\frac{\partial u_l}{\partial x_k} + \frac{\partial^2 u_k}{\partial x_l\partial x_k}\frac{\partial u_i}{\partial x_l}\right) \\ + \left(\tilde{k} - \frac{2}{3}\mu + B\right)\frac{\partial^2 u_i}{\partial x_k^2}\frac{\partial u_l}{\partial x_l} + \left(\frac{A}{4} + B\right)\left(\frac{\partial^2 u_k}{\partial x_l\partial x_k}\frac{\partial u_l}{\partial x_i}\right. \\ \left. + \frac{\partial^2 u_l}{\partial x_i\partial x_k}\frac{\partial u_k}{\partial x_l}\right) + (B + 2C)\frac{\partial^2 u_k}{\partial x_i\partial x_k}\frac{\partial u_l}{\partial x_l}.\end{aligned}$$

Here and below, the difference $T - T_0$ is denoted by T.

The nonlinear contribution $Q(\mathbf{r}; t)$ to the equation of heat conduction can be obtained by using the following: the nonlinear part of the energy (1); the fact that the heat conduction coefficient is generally a function of temperature and pressure, the heat capacity is a function of temperature, the heat flow at sufficiently great temperature gradients ceases to be a linear function of the temperature gradient. As a result, we get

$$\begin{aligned}
&\rho c_v \frac{\partial T}{\partial t} + \rho \frac{c_p - c_v}{\alpha} \frac{\partial}{\partial t}\left(\frac{\partial u_l}{\partial x_l}\right) - \varkappa \Delta T = Q(\mathrm{r}, t) \\
&= -\alpha \tilde{k} T \frac{\partial}{\partial t}\left(\frac{\partial u_l}{\partial x_l}\right) - (\alpha \tilde{k} - \gamma\mu) T_0 \frac{\partial}{\partial t}\left(\frac{\partial u_i}{\partial x_k}\right)\frac{\partial u_i}{\partial x_k} \\
&+ \left(\tilde{k}\beta - \frac{2}{3}\mu\gamma\right) T_0 \frac{\partial}{\partial t}\left(\frac{\partial u_l}{\partial x_l}\right)\frac{\partial u_l}{\partial x_l} + \mu\gamma T_0 \frac{\partial}{\partial t}\left(\frac{\partial u_i}{\partial x_k}\right)\frac{\partial u_k}{\partial x_i} \\
&- 2\tilde{k}\varepsilon\alpha T_0 \left[\frac{\partial T}{\partial t}\frac{\partial u_l}{\partial x_l} + T \frac{\partial}{\partial t}\left(\frac{\partial u_l}{\partial x_l}\right)\right] \\
&- \rho c_v \lambda T \frac{\partial T}{\partial t} + \varkappa \nu' \tilde{k}\left[\left(\nabla \frac{\partial u_l}{\partial x_l}\right)\nabla T + \frac{\partial u_l}{\partial x_l}\Delta T\right] \\
&+ \varkappa\delta\{(\nabla T)^2 + T\Delta T\} + \varkappa\varphi(\nabla|\nabla T|\nabla T + |\nabla T|\Delta T).
\end{aligned} \tag{4}$$

Here λ, ν', δ, φ are nonlinear coefficients, c_p and c_V are the heat capacities.

3. We shall assume that a longitudinal wave is incident on some volume V (the scattering volume), in which there exist thermal fluctuations of sound and temperature.

$$\mathbf{u}_0 = \frac{\mathbf{k}_0}{|\mathbf{k}_0|} u_0 \exp\{i(\mathbf{k}_0\mathbf{r} - \omega_0 t)\}$$

As a consequence of the nonlinearity which the medium possesses, the incident wave interacts with the fluctuations in the given volume, being scattered by them. Since the nonlinearity of the medium is usually small, we use the method of perturbation theory to find the scattered wave. We construct first the Green's function of the linear set of equations corresponding to the set (24) and (4). We change over in (2) and (4) to the Fourier transforms $\mathbf{u}(\mathbf{r}; \omega)$, $\mathbf{T}(\mathbf{r}; \omega)$, $\mathbf{F}(\mathbf{r}; \omega)$, and $\mathbf{Q}(\mathbf{r}; \omega)$:

$$\begin{aligned}
&\rho\omega^2 \mathbf{u}(\mathbf{r}; \omega) + \left(\tilde{k} + \frac{\mu}{3}\right)\nabla(\nabla \mathbf{u}(\mathbf{r}; \omega)) + \mu\Delta\mathbf{u}(\mathbf{r}; \omega) - \tilde{k}\alpha\nabla T(\mathbf{r}; \omega) \\
&= -\mathbf{F}(\mathbf{r}; \omega),
\end{aligned} \tag{5}$$

$$i\omega\rho c_v T(\mathbf{r}; \omega) + \varkappa\Delta T(\mathbf{r}; \omega) + i\omega\rho\frac{c_p - c_v}{\alpha}\nabla\mathbf{u}(\mathbf{r}; \omega) = -Q(\mathbf{r}; \omega).$$

It is convenient to represent the set (5) in matrix form:

$$\hat{L}_{\mathrm{r}}\begin{pmatrix}\mathbf{u}\\ T\end{pmatrix} = \begin{pmatrix}-\mathbf{F}\\ -Q\end{pmatrix}. \tag{6}$$

The meaning of the operator $\hat{\mathrm{L}}_{\mathrm{r}}$ is clear from (5).

We now define the Green's function (the Green's matrix) in the following way:

$$\hat{L}_{\mathrm{r}} G_1(\mathbf{r} - \mathbf{r}'; \mathbf{m}; \omega) = \hat{L}_{\mathrm{r}}\begin{pmatrix}\mathbf{p}_1(\mathbf{r} - \mathbf{r}'; \mathbf{m}; \omega)\\ q_1(\mathbf{r} - \mathbf{r}'; \mathbf{m}; \omega)\end{pmatrix} = \begin{pmatrix}\mathbf{m}\delta(\mathbf{r} - \mathbf{r}')\\ 0\end{pmatrix}, \tag{7}$$

$$\hat{L}_{\mathrm{r}} G_2(\mathbf{r} - \mathbf{r}'; \omega) = \hat{L}_{\mathrm{r}}\begin{pmatrix}\mathbf{p}_2(\mathbf{r} - \mathbf{r}'; \omega)\\ q_2(\mathbf{r} - \mathbf{r}'; \omega)\end{pmatrix} = \begin{pmatrix}0\\ \delta(\mathbf{r} - \mathbf{r}')\end{pmatrix}. \tag{8}$$

Here $\mathbf{m}$ is an arbitrary vector. Then the solution of the set (5) is represented in the form

$$\begin{pmatrix}\mathbf{u}\\ T\end{pmatrix} = -\int_V d^3\mathbf{r}'\left\{\begin{pmatrix}\mathbf{p}_1[\mathbf{r} - \mathbf{r}'; \mathbf{F}(\mathbf{r}'; \omega); \omega]\\ q_1[\mathbf{r} - \mathbf{r}'; \mathbf{F}(\mathbf{r}'; \omega); \omega]\end{pmatrix} + \begin{pmatrix}\mathbf{p}_2(\mathbf{r} - \mathbf{r}'; \omega)\\ q_2(\mathbf{r} - \mathbf{r}', \omega)\end{pmatrix} Q(\mathbf{r}'; \omega)\right\}. \tag{9}$$

Integration in (9) is carried out over the scattering volume V. In what follows, we shall write down the expression for $\mathbf{u}(\mathbf{r}; \omega)$ only. We transform it to the Fourier transforms of the Green's functions $\mathbf{p}_1(\mathbf{k}; \mathbf{m}; \omega)$ and $\mathbf{p}_2(\mathbf{k}; \omega)$ and the functions $\mathbf{F}(\mathbf{k}; \omega)$ and $\mathbf{Q}(\mathbf{k}; \omega)$:

$$\mathbf{u}(\mathbf{r}; \omega) = -\frac{V}{(2\pi)^3}\int d^3\mathbf{k}\, e^{i\mathbf{kr}}\{\mathbf{p}_1[\mathbf{k}; \mathbf{F}(\mathbf{k}; \omega); \omega] + \mathbf{p}_2(\mathbf{k}; \omega) Q(\mathbf{k}; \omega)\}. \tag{10}$$

(The expression (10) can be obtained by carrying out the Fourier transformation directly in (9).)

We find the Fourier transforms $\mathbf{p}_1(\mathbf{k}; \mathbf{m}; \omega)$ and $\mathbf{p}_2(\mathbf{k}; \omega)$ from (7) and (8) after the corresponding Fourier transformations. Here it is convenient to represent $\mathbf{p}_{1,2}$ in the form of a sum of two vectors: $\mathbf{p}_{1,2} = \mathbf{p}_{1,2\parallel} + \mathbf{p}_{1,2\perp}$, where $\mathbf{p}_{1,2\parallel} \parallel \mathbf{k}$ and $\mathbf{p}_{1,2\perp} \perp \mathbf{k}$. As a result, we have from (7) and (8),

$$\begin{aligned}
\mathbf{p}_1 &= \mathbf{p}_{1\parallel} + \mathbf{p}_{1\perp} \\
&= -\frac{\mathbf{k}(\mathbf{km})(k^2\varkappa - i\omega\rho c_v)}{k^2(\tilde{k} + 4\mu/3)[k^2 - k_1^2(\omega)][k^2 - k_2^2(\omega)]\varkappa} + \frac{\mathbf{k}(\mathbf{km}) - k^2\mathbf{m}}{k^2\mu(k^2 - \rho\omega^2/\mu)}.
\end{aligned} \tag{11}$$

Here k_1 and k_2 are the roots of the dispersion equation:

$$(k^2\varkappa - i\omega\rho c_v)\left(k^2 - \frac{\rho}{\tilde{k} + 4\mu/3}\omega^2\right) - \tilde{k}i\omega\rho(c_p - c_v)\frac{1}{\tilde{k} + 4\mu/3} = 0; \tag{12}$$

$$k_1^2 \approx \frac{\rho}{k_{\mathrm{ad}}}\omega^2\left[1 - i\frac{\varkappa\omega}{k_{\mathrm{ad}}}\left(\frac{1}{c_v} - \frac{1}{c_p}\right)\right] \tag{13}$$

(acoustical branch),

$$k_2^2 \approx i\omega\rho c_p / \varkappa \tag{14}$$

(temperature branch).

In (13) k_{ad} is the adiabatic compressional modulus.

Formulas (13) and (14) were obtained under the assumption

$$\varkappa\omega / k_{\mathrm{ad}} c_v \ll 1. \tag{15}$$

The condition (15) is the condition for adiabatic propagation of a sound wave.[1] We note here that for most solids the following inequality holds and will be used from now on:

$$(c_p - c_v) / c_v \ll 1. \tag{16}$$

Further, we have

$$\mathbf{p}_2 = \mathbf{p}_{2\parallel} = -\frac{\mathbf{k}i\tilde{k}\alpha}{(\tilde{k} + 4\mu/3)[k^2 - k_1^2(\omega)][k^2 - k_2^2(\omega)]\varkappa}, \tag{17}$$

$$\mathbf{p}_{2\perp} = 0.$$

By computing the integral (10) and using (11), (17), we finally obtain the expression for $\mathbf{u}(\mathbf{r}; \omega)$ in the wave zone: $\mathbf{u}(\mathbf{r}; \omega) = \mathbf{u}_\parallel(\mathbf{r}; \omega) + \mathbf{u}_\perp(\mathbf{r}; \omega)$;

$$\mathbf{u}_\parallel(\mathbf{r}; \omega) = \frac{V}{4\pi}\frac{e^{i\mathbf{kr}}}{r}\left\{\frac{\mathbf{k}(\mathbf{kF}(\mathbf{k}; \omega))}{k^2(\tilde{k} + 4\mu/3)} + \frac{\mathbf{k}i\tilde{k}\alpha Q(\mathbf{k}; \omega)}{(\tilde{k} + 4\mu/3)(k^2\varkappa - i\omega\rho c_v)}\right\}, \tag{18}$$

$$\mathbf{u}_\perp(\mathbf{r}; \omega) = \frac{V}{4\pi}\frac{e^{i\mathbf{kr}}}{r}\frac{1}{\mu}\left\{\mathbf{F}(\mathbf{k}; \omega) - \frac{\mathbf{k}(\mathbf{kF}(\mathbf{k}; \omega))}{k^2}\right\}. \tag{18a}$$

In (18), $\mathbf{u}_\parallel(\mathbf{r}; \omega)$ is the longitudinal wave, and $\mathrm{k} = \mathrm{k}_1$ from (13). (The pole at $\mathrm{k} = \mathrm{k}_2$ does not contribute to the integral (10) in the wave zone.) In (18a), $\mathbf{u}_\perp(\mathbf{r}; \omega)$ is the transverse wave, and $\mathrm{k} = \omega\sqrt{\rho/\mu}$. In the following we shall, for simplicity, consider only the longitudinal wave $\mathbf{u}_\parallel(\mathbf{r}; \omega)$.

We shall compute the Fourier transforms $\mathbf{F}(\mathbf{k}; \omega)$ and $\mathbf{Q}(\mathbf{k}; \omega)$ from (18), using (3) and (4). Substituting them in (18), we get

$$\begin{aligned}
\mathbf{u}_\parallel(\mathbf{r}; \omega) &= u_0\frac{V}{4\pi}\frac{e^{ikr}}{r}\frac{k_0\mathbf{k}}{\tilde{k} + 4\mu/3} \\
&\times\left\{T(\mathbf{q}; \nu)\left[\tilde{k}\beta - \frac{2}{3}\mu\gamma - (\tilde{k}\alpha - 2\mu\gamma)\cos^2\theta + \tilde{k}\alpha\frac{\tilde{k}\alpha}{\rho c_v}\frac{\omega_0}{\omega}\right]\right. \\
&\left.+ E(\mathbf{q}; \nu)\left[\frac{1}{q^2}2a\cos\theta\{\cos\theta(k^2 + k_0^2) - k_0 k(1 + \cos^2\theta)\} + e\right]\right\}.
\end{aligned} \tag{19}$$

In (19), we have introduced the following notation $\mathbf{q} = \mathbf{k} - \mathbf{k}_0$, $\nu = \omega - \omega_0$; θ is the angle between the wave vectors of the incident and scattered waves, $T(\mathbf{q}; \nu)$ is the Fourier transform of the fluctuating temperature, $E(\mathbf{q}; \nu) \equiv \partial u_n(\mathbf{q}; \nu)/\partial n$ is the Fourier transform of the fluctuation deformation, where n is a coordinate whose unit vector is directed along $\mathbf{q}$;

$$a = \tilde{k} + {}^7/_3\mu + A + 2B,$$
$$e = \tilde{k} - {}^2/_3\mu + 2B + 2C.$$

(Since transverse sound waves are not connected with a change in temperature, only the longitudinal wave fluctuations are kept in (19), for simplicity.)

Equation (19) is obtained with accuracy to terms containing the small parameters $(c_p - c_V)/c_V$ and $\varkappa\omega/\tilde{k}c_V$. It is assumed here that the coefficients β; γ; λ; δ; $\tilde{k}\nu'\alpha$; $\varphi k_0\alpha$ do not exceed α in order of magnitude. If, in certain cases, this is not valid for all the coefficients, then the corresponding terms must remain.

We now find the time average of the intensity of the scattered sound, denoting by M and N the coefficients for T and E in (19). We have

$$I(\mathbf{r}) = \overline{|\mathbf{u}(\mathbf{r}; t)|^2} = u_0^2\frac{V^2}{(4\pi)^2}\frac{(k_0k)^2}{(\tilde{k}+4\mu/3)^2}\frac{1}{r^2}$$
$$\times\int_{-\infty}^{\infty}\int d\omega\, d\omega'\, \{M^2\overline{T(\mathbf{q}; \omega-\omega_0)T^*(\mathbf{q}; \omega'-\omega_0)}$$
$$+ N^2\overline{E(\mathbf{q}; \omega-\omega_0)E^*(\mathbf{q}; \omega'-\omega_0)} + MN[\overline{T(\mathbf{q}; \omega-\omega_0)E^*(\mathbf{q}; \omega'-\omega_0)}$$
$$+ \overline{T^*(\mathbf{q}; \omega'-\omega_0)E(\mathbf{q}; \omega-\omega_0)}]\}\, e^{-i(\omega-\omega')t}. \qquad (20)$$

From (20), we can obtain the following expression for the spectral intensity of the scattered wave:

$$I(\mathbf{r}; \omega) = u_0^2\frac{V^2}{(4\pi)^2}\frac{1}{r^2}\frac{(k_0k)^2}{(\tilde{k}+4\mu/3)^2} \qquad (21)$$
$$\times\{M^2|T|_\omega^2 + N^2|E|_\omega^2 + MN[(TE^*)_\omega + (T^*E)_\omega]\},$$

where the quantities denoted by $|T|_\omega^2$, $|E|_\omega^2$, $(TE^*)_\omega$, and $(T^*E)_\omega$ are computed in correspondence with the theory developed, for example, in[3,4].

As a result, the expression for the spectral intensity of the scattered sound $I(\omega)$ takes the form

$$I(\omega) = I(\omega_0+\nu) = u_0^2\frac{V}{(4\pi)^2}\frac{1}{r^2}\frac{(k_0k)^2}{(\tilde{k}+4\mu/3)^2}\frac{1}{\pi}$$
$$\times\Big\{M^2\frac{(\nu^2-\omega_L^2)^2\tau k'T_0^2/\rho c_v}{(\nu^2-\omega_L^2)^2+\nu^2\tau^2(\nu^2-\omega_S^2)^2}$$
$$+ N^2\frac{\omega_L^2(\omega_S^2-\omega_L^2)\tau}{(\nu^2-\omega_L^2)^2+\nu^2\tau^2(\nu^2-\omega_S^2)^2}\frac{k'T_0}{\tilde{k}+4\mu/3}$$
$$+ 2MN\frac{(\omega_L^2-\nu^2)\tau\tilde{k}aq^2/\rho}{(\nu^2-\omega_L^2)^2+\nu^2\tau^2(\nu^2-\omega_S^2)^2}\frac{k'T_0^2}{\rho c_v}\Big\}. \qquad (22)$$

The following notation is introduced in (22):

$$\tau = \left(\frac{\varkappa q^2}{\rho c_v}\right)^{-1}, \quad \omega_L^2 = \frac{\tilde{k}+4\mu/3}{\rho}q^2,$$
$$\omega_S^2 = \frac{k_{\text{ad}}+4\mu/3}{\rho}q^2;$$

k′ is Boltzmann's constant.

4. We now investigate the spectral composition of the scattered sound. The essential feature of the spectral distribution (22) is that it has two maxima: for $\nu = 0$ and for $\nu \approx \omega_S$.

We first consider the spectrum close to the first maximum. In this spectral region, Eq. (22) is materially simplified:

$$I(\omega_0+\nu) = u_0^2\frac{V}{(4\pi)^2}\frac{1}{r^2}\frac{(k_0k)^2}{(\tilde{k}+4\mu/3)^2}$$
$$\times\frac{\tau}{\pi(1+\nu^2\tau^2)}\frac{k'T_0^2}{\rho c_v}(M^2+N^2a^2+MNa). \qquad (23)$$

We note here that the adiabatic condition (15) is equivalent to the condition $1/\tau\ (\omega_0) \ll \omega_0$. Taking this condition into account, we can see that the spectral distribution (23) has the form of a Lorentz distribution with half-width Δ, approximately equal to $1/\tau\ (\omega_0)$,

$$\Delta \approx 2\varkappa\omega_0^2(1-\cos\theta)\,/\,\tilde{k}c_v. \qquad (24)$$

It also follows from condition $1/\tau(\omega_0) \ll \omega_0$ that $\nu \ll \omega_S$, ω_L in the limits of the resonance curve (23), which also allows us to obtain (23) from (22).

The given case is essentially scattering of sound from the temperature branch of the combined oscillations of temperature and sound. Thus the nonlinear interaction of sound with the temperature branch leads to sound scattering in all directions, with the maximum intensity at the undisplaced frequency ω_0, while the half-width of the line increases with increase in the scattering angle, as is seen from (24).

Let us make numerical estimates. The half-width Δ for many materials that are of interest for observation of the effect, can be rather large, for example $\sim 10^{-2}\omega_0$. So far as the absolute intensity of the scattered sound is concerned, it is determined by the coefficients α, β, γ, A, B, and C, as is seen from (23). Of the three terms in the curly brackets in (23), the largest contribution for most materials is clearly made by the second term. We shall also estimate it. The coefficients A, B, and C are measured for a small number of substances, and with great variation.[5] Therefore, we shall assume for estimate that, roughly, $A \sim B \sim C \sim \tilde{k}$. We set $\alpha \sim 5\times10^{-5}\,\text{deg}^{-1}$, $T_0 \sim 300°$K, $c_V \sim 0.5$ cal/g-deg, and $V \sim 1\ \text{cm}^3$. Since the interaction depends strongly on the energy of the incident wave, it is appropriate to take as the incident wave hypersound of high frequency, $f \sim 10^{10}$ Hz. We then obtain the following estimate for the ratio χ of sound power scattered in all directions and integrated over all frequencies (in the frequency range considered) to the intensity of the incident sound:

$$\chi \sim 10^{-5}. \qquad (25)$$

Account of damping decreases the value of χ. One must therefore choose a substance where the damping of hypersound is sufficiently small. It must be remarked that the estimate (25) is an average, made for parameters that are typical for solid materials. It can therefore be supposed that materials will be found for which χ is larger. In particular, χ depends strongly on the value of the moduli A, B, and C. The estimate (25) was made for $A \sim B \sim C \sim \tilde{k}$. However, in a number of materials, they are approximately one order of magnitude greater than $\tilde{k}$,[5,6] and allowance for this fact increases the estimated value considerably.

5. The second maximum of Eq. (22), $\nu \approx \omega_S$, represents the case of scattering of sound by sound, well

known in nonlinear acoustics. The results of the investigation of this maximum correspond to results obtained in previous researches on nonlinear acoustics, in which the interaction of the sound wave was considered not with fluctuations of the medium, as in the present work, but with another given sound wave (see, for example,[5,6]).

Let us compare in magnitude the intensity of the sound scattered by temperature waves with the sound intensity scattered by the acoustic branch (in the latter case, we limit ourselves to frequencies of the scattered sound equal to ω_0 in order of magnitude). The calculations show that the scattering by the temperature branch is less effective, by a factor $(c_P - c_V)/c_V$, than scattering by sound. Such a result is quite natural if we take the following into account. Since it was assumed that the maximum contribution is made by the second term in the brackets in (23), then the scattering by the temperature wave in the given case constitutes scattering not from the temperature fluctuations but from deformation fluctuations belonging to the temperature branch; however, it is clear that the deformation wave will belong to the temperature branch together with the acoustic branch only in the case $\alpha \neq 0$, or, what amounts to the same thing, $(c_P - c_V)/c_V \neq 0$.

6. Thus the scattering of sound by the temperature branch has its own features in comparison with the purely acoustical interactions. The intensity of the sound scattered in this way is a measureable quantity, as is seen from (25). In addition to this part of the scattered sound, there will also be sound scattered by the transverse sound waves, in the various directions. (The corresponding formulas have not been written down in this paper; the picture of scattering is analogous to scattering of a longitudinal sound wave by a given transverse sound wave for all possible angles of interaction.[5,6]). Sound scattered by transverse sound waves has an essentially different spectral composition in comparison with the sound scattered by temperature waves and therefore cannot mix up the recording of the latter.

Measurement of the width Δ (24) of the line scattered by the temperature waves allows us to draw conclusions on the contribution which the thermal conduction makes to the damping of the sound, inasmuch as this contribution is determined by the value of Δ[1,2] Moreover, it can be supposed that for sufficiently small wavelengths, the coefficient of thermal conductivity κ ceases to be a constant; by comparing the values of κ computed from the line width and by the usual method, we can decide whether the coefficient remains constant right down to wavelengths $\sim 10^{-5}$ cm with which we have to deal in the present work. By comparing the intensity of the sound scattered in different directions, we can obtain evidence on the nonlinear moduli A, B, and C. Moreover, the data on the spontaneous scattering of sound by temperature waves allows us to estimate the possibility of observation of stimulated sound scattering by temperature waves for sound intensities actually obtainable at the present time.

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[1] I. G. Mikhaĭlov, V. A. Solov'ev and Yu. P. Syrnikov, Osnovy molekulyarnoi akustiki (Fundamental of Molecular Acoustics) (Nauka, 1964).

[2] L. D. Landau and E. M. Lifshitz, Teoriya uprugosti (Theory of Elasticity) (Nauka, 1965).

[3] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics) (Nauka, 1964).

[4] M. A. Leontovich, J. Phys. 4, 499 (1941).

[5] L. K. Zarembo and V. A. Krasil'nikov, Vvedenie v nelineĭnuyu akustiku (Introduction to Nonlinear Acoustics) (Nauka, 1966).

[6] G. L. Jones and d. R. Kobbet, J. Acoust. Soc. Amer. 35, 5 (1963).

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