AMPLITUDE AND FREQUENCY CHARACTERISTICS OF A RING LASER

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The characteristics of a gas laser with optical feedback between the opposed waves strengthened by means of return mirrors are investigated. The dependence of the difference of the opposed beam intensities and frequencies on the magnitude and relative phase of the coupling coefficient are derived.

THE ring laser first studied by Rosenthal[1] was used to investigate certain problems of electrodynamics in noninertial systems,[2] the interaction of light waves in nonlinear media,[3] and other subtle effects. However the precision of such experiments was significantly limited by physical phenomena due to the internal properties of the ring laser itself and mainly to the difference between the real and ideal optical resonator.

In the idealized ring laser (without back scattering) the natural oscillations are represented by waves traveling in opposite directions around the ring. The frequency difference between these waves is due to the phase independence of the optical ring resonator that can be caused in turn by rotation of the laser or by non-reciprocal effects in its elements (Faraday effect, Fizeau–Fresnel effect, and Langmuir effect in the laser tube with dc excitation of the discharge).[4] The ring resonator can have different Q factors for the opposed waves. The effect of the Q-factor difference caused by an amplitude-sensitive non-reciprocal element on ring laser characteristics was studied earlier in [5,6]. However the results of these efforts are only qualitative because the true value of the loss difference cannot be determined independently and because such a difference is unstable.

Detailed analysis shows that the instability of Q-factor and the opposed wave frequency differences, usually observed in the ring laser, is mainly due to the coupling between the opposed waves via scattering on the optical inhomogeneities of the resonator.

The present work deals with the theoretical and experimental investigation of the effect of the coupling on the ring laser characteristics. We have studied experimentally the behavior of the ring laser with coupling specially enhanced by means of "return" mirrors. The coupling parameters (amplitude and phase) were controllable. The results of the experiments are compared with theory.

1. THEORY OF A RING LASER WITH COUPLED OPPOSED WAVES

The presence of coupling between the opposed waves in a ring laser (linear in amplitude of the electric field) results in a situation in which traveling waves are no longer the natural oscillation of the laser. Each opposed wave is a combination of two waves with different frequencies associated with different natural oscillations of the ring laser similar to the normal oscillations of two coupled circuits. Although the ring laser, just as any oscillator, is essentially a nonlinear system, the concept of normal oscillations in such a laser is nevertheless a fairly useful way of interpreting the basic phenomena of coupling through scattering.

We consider a ring laser with linear polarization of radiation, working in a single-mode regime. The electric field in its resonator can be represented in the form

$$E = {}^1/_2\{E_+e^{ikr} + E_-e^{-ikr}\}e^{-i\nu t} + \text{к.с.} \qquad (1)$$

The equations for slow complex amplitudes $E_\pm$ of the opposed waves are obtained as usual by substituting the above expression into the wave equation and separating the harmonics

$$\dot{E}_\pm = \left(-\frac{\Delta\nu_\pm}{2} \pm \frac{if_0}{2}\varkappa - \alpha|E_\pm|^2 - \beta|E_\mp|^2\right)E_\pm - i\sigma_\pm E_\mp. \qquad (2)$$

Here $\Delta\nu_\pm$ and f_0 are pass bands and the frequency difference of the opposed waves for an optical resonator, without taking back-scatter coupling of the opposed waves into account. The complex coefficients κ, α, and β determine the polarizability of the active medium and depend on the working transition parameters, the isotopic composition of the active gas, and the deviation of the mean resonator frequency with respect to the gain maximum. The specific form of these coefficients that depends on the order of approximation chosen in the computation of the nonlinear polarizability of the active medium is not significant in this work. Therefore it is not given here and the reader is referred to the sources that contain these computations.[7,3] The coupling coefficients of the opposed waves are expressed in terms of complex (taking phase shift into account) back-scatter coefficients $r_\pm$, the length of the ring resonator perimeter L, and the velocity of light c:

$$\sigma_\pm = r_\pm c/L. \qquad (3)$$

Considering that the opposed wave amplitudes $E_\pm$ are not natural functions of the ring laser in the presence of back scatter, we introduce new functions $\mathscr{E}_\pm$ that convert into the amplitudes of normal oscillations of the corresponding linear system when the sum of opposed-wave intensities I tends to zero:

$$E_\pm = \mathscr{E}_\pm \mp \frac{i\sigma_\pm}{D(f_0,\Delta I)}\mathscr{E}_\mp. \qquad (4)$$

The denominator

$$D(f_0,\Delta I) = \frac{1}{2}\left[if_0 - \frac{\Delta\nu_+ - \Delta\nu_-}{2} - (\alpha-\beta)\Delta I + \sqrt{\left[if_0 - \frac{\Delta\nu_+ - \Delta\nu_-}{2} - (\alpha-\beta)\Delta I\right]^2 - 4\sigma_+\sigma_-}\right] \qquad (5)$$

in contrast with the usual conversion to normal oscillations in a linear system, is a function of the intensity difference of opposed waves $\Delta I = |E_+|^2 - |E_-|^2$, in addition to being a function of the difference frequency f_0, so that the relation between the opposed-wave amplitudes and the normal oscillation amplitudes turns out to be nonlinear.

Substituting (4) into (2) we find the equations for the normal oscillation amplitudes in the ring laser:

$$\dot{\mathscr{E}}_\pm = \lambda_\pm(f_0, I, \Delta I)\mathscr{E}_\pm - \frac{\sigma_+\sigma_-\dot{D}}{D^3(1+|\sigma_+\sigma_-/D|^2)}\mathscr{E}_\pm \mp \frac{i\sigma_\pm\dot{D}}{D^2(1+|\sigma_+\sigma_-/D|^2)}\mathscr{E}_\mp. \quad (6)$$

In the case of a linear system the transformation into normal oscillations leads to the diagonalization of the matrix of the system in which the constant natural frequency values determine the frequencies and attenuation of the normal oscillations. In the case of a nonlinear system an analogous transformation in general does not yield a separation of equations, since the function $D(f_0, \Delta I)$ depends on the opposed-wave intensities and consequently on the normal oscillation amplitudes. However we show below that, given sufficiently large values of the difference frequency f_0 relative to the coupling coefficients, the terms containing derivatives of the function $D(f_0, \Delta I)$ in the right-hand side of (6) turn out to be small. Neglecting these terms, the equations for normal oscillations in the ring laser can be written in the form

$$\dot{\mathscr{E}}_\pm = \lambda_\pm(f_0, I, \Delta I)\mathscr{E}_\pm. \quad (7)$$

These equations differ from the usual equations of normal oscillations in a linear system by the fact that the natural values

$$\lambda_\pm = \frac{1}{2}\Big[-\Delta\nu + 2\varkappa - (\alpha+\beta)I \pm \pm\sqrt{\Big[if_0 - \frac{\Delta\nu_+ - \Delta\nu_-}{2} - (\alpha-\beta)\Delta I\Big]^2 - 4\sigma_+\sigma_-}\Big] \quad (8)$$

are not constant but are functions of opposed-wave intensities and thus of normal oscillation amplitudes $\mathscr{E}_\pm$.

We consider the stationary two-frequency regime, in which both normal oscillations are excited. The solution of (7) can be written formally in the form

$$\mathscr{E}_\pm = \mathscr{E}^0_\pm \exp\Big\{\int \lambda_\pm\, dt\Big\}, \quad (9)$$

where $\mathscr{E}^0_\pm$ are complex constants that coincide in the zeroth approximation with the normal oscillation amplitudes. Since the opposed waves represent a mixture of normal oscillations with a frequency difference f, their intensities are periodic functions. Accordingly, the natural values $\lambda_\pm$ are also periodic functions. Expanding them in a series of harmonics of the difference frequency f and substituting into (9), we find

$$\mathscr{E}_\pm = \mathscr{E}_\pm{}^0 \exp\Big\{\lambda_\pm{}^0 t + \frac{1}{f}\sum_{n=-\infty}^{+\infty}{}' \frac{1}{in}\lambda_\pm^{(n)} e^{inft}\Big\}. \quad (10)$$

It is obvious that the necessary condition of stability of the two-frequency regime is represented by the equations

$$\mathrm{Re}\,\lambda_\pm^{(0)} = 0, \quad (11)$$

which lead to equations for the determination of the constant components of the sum and difference of the intensities

$$\mathrm{Re}\,(\alpha+\beta)I_0 = 2\varkappa - \Delta\nu, \quad (12)$$

$$\mathrm{Re}\Big\{\sqrt{\Big[if_0 - \frac{\Delta\nu_+ - \Delta\nu_-}{2} - (\alpha-\beta)\Delta I\Big]^2 - 4\sigma_+\sigma_-}\Big\}_0 = 0; \quad (13)$$

The imaginary parts of the constant components of the natural values determine the frequencies of normal oscillations and consequently the difference frequency

$$f = \mathrm{Im}\Big\{\sqrt{\Big[if_0 - \frac{\Delta\nu_+ - \Delta\nu_-}{2} - (\alpha-\beta)\Delta I\Big]^2 - 4\sigma_+\sigma_-}\Big\}_0. \quad (14)$$

Considering that the normal oscillation equations in form (7) are valid only for sufficiently large f we expand (13) and (14) into a series of reciprocal powers of the difference frequency, retaining terms of the order of f_0^{-1}. Then we find the constant component of the intensity difference from (13)

$$\Delta I_0 = -\frac{1}{\mathrm{Re}(\alpha-\beta)}\Big[\frac{\Delta\nu_+ - \Delta\nu_-}{2} + \frac{2\mathrm{Im}\{\sigma_+\sigma_-\}}{f_0}\Big]. \quad (15)$$

In the same approximation (14) yields the difference frequency

$$f = f_0 - \mathrm{Im}(\alpha-\beta)\Delta I_0 + \frac{2\mathrm{Re}\{\sigma_+\sigma_-\}}{f_0}. \quad (16)$$

To estimate the magnitude of neglected terms when going over from the exact equations (6) to the normal-oscillation equations (7) we must analyze the variable components of the sum and difference of the opposed-wave intensities. It is sufficient to consider only the first-order correction to the intensities that is linear with respect to the coupling coefficients. We substitute the expressions for normal oscillations (9) into (4) and write the sum and difference of the intensities in terms of normal oscillation amplitudes and of natural values. We then expand all expressions in reciprocal powers of the difference frequency leaving only terms of the order of f^{-1}. Considering (12)–(14) we find

$$I = |\mathscr{E}^0_+|^2 + |\mathscr{E}^0_-|^2 - (|\mathscr{E}^0_+|^2 + |\mathscr{E}^0_-|^2)\mathrm{Re}(\alpha+\beta)\int I_1 dt - (|\mathscr{E}^0_+|^2 - |\mathscr{E}^0_-|^2)\mathrm{Re}(\alpha-\beta)\int \Delta I_1\, dt - \frac{2|\mathscr{E}^0_+\mathscr{E}^0_-|}{f}\{\sigma_r^- \cos ft + \sigma_i^+ \sin ft\}, \quad (17)$$

$$\Delta I = |\mathscr{E}^0_+|^2 - |\mathscr{E}^0_-|^2 - (|\mathscr{E}^0_+|^2 - |\mathscr{E}^0_-|^2)\mathrm{Re}(\alpha+\beta)\int I_1\, dt + (|\mathscr{E}^0_+|^2 + |\mathscr{E}^0_-|^2)\mathrm{Re}(\alpha-\beta)\int \Delta I_1\, dt - \frac{2|\mathscr{E}^0_+\mathscr{E}^0_-|}{f}\{\sigma_r^+ \cos ft + \sigma_i^- \sin ft\}.$$

Here I_1 and ΔI_1 are variable components of the sum and difference of the intensities, and a notation is introduced for the combinations of coupling coefficients $\sigma_r^\pm = \mathrm{Re}\{\sigma_+ \pm \sigma_-\}$ and $\sigma_i^\pm = \mathrm{Im}\{\sigma_+ \pm \sigma_-\}$. Substituting the expansions of the sum and difference of the intensities in terms of difference frequency harmonics into these equations and equating the coefficients, we find the expressions for normal oscillation amplitudes in the zeroth approximation

$$|\mathscr{E}_\pm{}^0| = (I_0 \pm \Delta I_0)/2 \quad (18)$$

Taking (18) into account we then find the first harmonics of the variable intensity components

$$I_1 = I_0\Big[\frac{(\sigma_r^-)^2 + (\sigma_i^+)^2}{f^2 + [\mathrm{Re}(\alpha+\beta)I_0]^2}\Big]^{1/2} \cos(ft + \varphi_1), \quad (19)$$

$$\Delta I_1 = I_0\Big[\frac{(\sigma_r^+)^2 + (\sigma_i^-)^2}{f^2 + [\mathrm{Re}(\alpha-\beta)I_0]^2}\Big]^{1/2} \cos(ft + \varphi_2). \quad (20)$$

The beat phases φ_1 and φ_2 depend on the amplitudes and phases of the coupling coefficients and on the relations between the difference frequency and the parameters Re $(\alpha \pm \beta)I_0$. For the sake of simplicity, (19) and (20) were derived on the assumption that the constant component of intensity difference is small in comparison with that of the sum.

The resulting expressions are used to estimate the range of applicability of the developed theory and of the concept of normal oscillations in a ring laser. For this purpose we consider the neglected terms in (6)

$$\left(1+\frac{\sigma_+\sigma_-}{D}\right)^{-1}\frac{\sigma_\pm}{D}\frac{\dot{D}}{D}\approx\frac{\sigma_\pm}{f}\frac{(\alpha-\beta)\,\Delta I}{f}\approx$$

$$\approx\frac{\sigma_\pm}{f}\frac{(\alpha-\beta)I_0\sqrt{(\sigma_r^+)^2+(\sigma_i^-)^2}}{\sqrt{f^2+[\mathrm{Re}\,(\alpha-\beta)\,I_0]^2}}\cos(ft+\varphi_2). \qquad (21)$$

In the case when $f \gg \mathrm{Re}\ (\alpha - \beta)I_0$ this expression assumes the form

$$\sigma_\pm\left[\frac{(\alpha-\beta)I_0}{f}\frac{\sqrt{(\sigma_r^+)^2+(\sigma_i^-)^2}}{f}\right]\mathrm{ccs}\,(ft+\varphi_2), \qquad (22)$$

i.e., the coupling coefficients between the normal oscillations turn out to be small, of the order of f^{-2}, in comparison to those between the opposed waves. If the strong coupling condition $|\sigma| \gg \mathrm{Re}\ (\alpha - \beta)I_0$ is satisfied in this case, the normal oscillation equations remain valid with this accuracy up to $f \sim \sigma$, i.e., up to the stability limit of the two-frequency regime, and the corrections to the natural values compensating for the neglected terms are of the order of f^{-4}. In the converse case of a weak coupling $|\sigma| \ll \mathrm{Re}\,(\alpha - \beta)I_0$ when the difference frequency lies in the region $|\sigma| \ll f \ll \mathrm{Re}\,(\alpha - \beta)I_0$ the neglected terms result in corrections to the natural values of the order of f^{-1}. Therefore the expression for the amplitude and frequency characteristics derived above is not applicable to this frequency range.

These limitations of the difference frequency have a simple physical sense. Indeed when the beats period is small in comparison with the relaxation times of the sum and difference of intensities $[\mathrm{Re}\,(\alpha + \beta)I_0]^{-1}$ and $[\mathrm{Re}\,(\alpha - \beta)I_0]^{-1}$, normal oscillations become independent and behave just as in the case of a linear system. Conversely, when the opposed-wave coupling is weak, the two-frequency regime is stable at low frequencies when the beats period is longer than the relaxation times and normal oscillations are found to interact strongly throughout the nonlinear medium. In this case the normal oscillation concept itself loses its meaning.

2. THE EXPERIMENTAL SETUP AND MEASUREMENT METHOD

The experimental setup is shown in Fig. 1. The optical ring resonator consists of three mirrors 1 situated at the vertices of an equilateral triangle about 1 m on a side. Two mirrors of the resonator are flat while the third is spherical with a radius of curvature of 16 m. Two gas discharge tubes 2 with Brewster windows are filled with neon (natural isotopic mixture) and helium (He^3 isotope) in the ratio 1: 5 at the total pressure of about 1 Torr; the tubes generate at the wavelength of 0.63 μ. The tubes are excited with an RF oscillator. The opposed-wave frequencies are separated by "non-

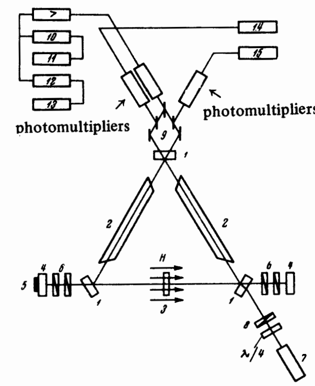

FIG. 1

reciprocal" element 3 based on the Faraday effect.[4] The free regions of the optical resonator are filled with dust-free air and sealed. The perimeter of the optical resonator is adjusted with an electrostriction device that controls the position of one of the resonator mirrors.

Two "return" mirrors[8] are used to produce between the opposed waves an optical coupling that is controllable in magnitude and phase. These mirrors are installed outside the optical resonator to reflect precisely backwards (toward the opposed waves) the beams emerging from the resonator. One return mirror 4 is mounted on the electrostriction device 5 to vary its distance from the resonator. Polarizing filters 6 are placed in front of the return mirrors to control the magnitude of the optical coupling coefficients.

The generation regime of the ring laser is monitored with Fabry–Perot scanning interferometer 7. The effect of the interferometer on the laser is eliminated by a decoupling system consisting of polaroid 8 and a $\lambda/4$ plate.

The beats between the opposed waves of the ring laser are picked up with symmetric photomixer 9 and a photoelectron multiplier. The beat frequency is measured precisely with electronic frequency meter 10, whose output is connected to printer 11. Low-precision beat frequency measurements are performed with pointer-type frequency meter 12 connected to recorder 13. The constant components of the opposed-beam intensities are recorded with a photomultiplier and recorders 14 and 15.

An air-core electromagnetic is used to generate the magnetic field at the non-reciprocal element. The field can be reversed periodically in time with the operation of the electronic frequency meter which periodically measures the beats frequency.

All measurements were performed in single-mode generation regime; the mode frequency before each measurement cycle of several minutes was set to the center of the Doppler curve with the scanning interferometer, using the following method: We determined two neighboring values of the voltage on the electrostriction device of the optical resonator, at which two modes with equal emission intensity are generated. A voltage equal to the mean of these two values is then applied to the electrostriction device. The variation of the perimeter of the optical resonator does not exceed 0.1λ per cycle of measurements.

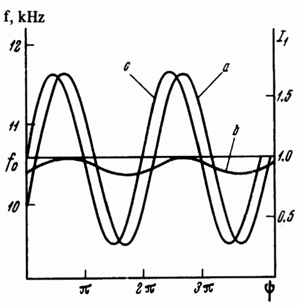

FIG. 2

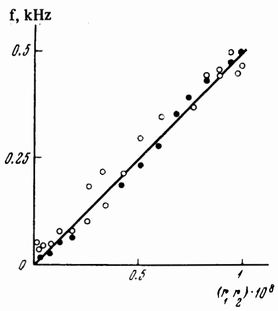

FIG. 3

3. RESULTS OF MEASUREMENTS

The values of coefficients $r_\pm$ of the external coupling can be varied with polarization filters within the limits from 0 to $\sim 10^{-4}$. The maximum values of $r_\pm$ obtainable without polarization filters and with precise alignment of the return mirrors are

$$r_{+\,ext} = r_{-\,ext} = (2 \pm 1)\cdot 10^{-4},$$

This was determined by direct measurements of the transmission of the optical resonator mirrors. The back-scattering coefficients r_{+int} and r_{-int} due to the scattering of light on the optical resonator inhomogeneities are considerably smaller than these values.

Figure 2 shows the experimental values of the beats frequency f (curve a) and intensity I_+ (in relative units) of one of the laser beams (curve c) for a maximum external optical coupling between the opposed waves as functions of relative phase ψ of waves reflected from the return mirrors. The sinusoidal function $f(\psi)$ (curve a) is almost symmetrical with respect to the straight line drawn through the value of beat frequency f_0 assumed in the absence of external optical coupling (i.e., when both return mirrors are covered). The period of the sinusoid corresponds to a change in the position of the movable return mirror of $\lambda/2$. The amplitude of the sinusoid $\Delta f = (f_{max} - f_{min})/2$ gives the maximum deviation of the beat frequency that can be observed for a given coupling coefficient. The frequency deviation Δf is a parameter that is independent of the phase of the reflected waves. The frequency deviation for curve a in Fig. 2 equals

$$\Delta f_1 = 1.1 \text{ kHz}$$

An analogous curve b was obtained with asymmetric optical coupling between the opposed waves (the fixed return mirror was completely covered). In this case the maximum beat frequency deviation is

$$\Delta f_2 = 0.1 \text{ kHz}$$

It is of interest to note that the sinusoid (curve b) is shifted towards the low frequency range when the coupling is strongly asymmetric ($r_+ = r_{+\,int} \ll r_- = r_{-ext}$). The sinusoid c representing the variation of the intensity component $I_+(\psi)$ of one of the beams of the ring laser with symmetric coupling was obtained simultaneously with curve a and is shifted relative to the latter by $\Delta\psi \approx \pi/5$.

Figure 3 shows the maximum beat frequency deviation as a function of the product $r_+ r_-$ of the external optical coupling obtained in our experiments. The value of Δf was obtained for each value of the product by scanning one of the return mirrors with the electrostriction device driven by a sawtooth voltage with an amplitude of 400 V and a frequency of about 0.03 Hz at a beat frequency 10 Hz. The filled circles were obtained by varying r_+ (scanned return mirror), and the light circles by varying r_- (fixed return mirror). As we see from Fig. 3, the maximum deviation of the beat frequency for high values of $r_+ r_-$ is directly proportional to their product (both filled and light circles lie on the same straight line). Figure 3 also shows that the circles obtained with varying r_- due to reflection from the fixed return mirror at low values of the product lie above the straight line through the origin of coordinates. This is explained by the presence of internal optical coupling between the opposed waves. This did not appear in the series of dots because the observed beat frequency deviation with fully covered scanning return mirror is by definition equal to zero for any values of $r_+ r_-$.

The obtained data can be used to obtain the magnitude of the coefficient of internal coupling between the opposed waves assuming that with a fully covered fixed return mirror

$$\Delta f_0 \approx a r_{+\,int} \quad r_{-\,ext} \quad \approx 0.06 \text{ kHz}$$

where r_{-ext} is the maximum coefficient of external coupling, equal to 10^{-4} in this case (the transmission of a fully open polarizing filter is 0.5). Hence for the back-scatter coefficient we find

$$r_{-\,int} \approx 0.8\cdot 10^{-5}.$$

A close value of the internal coupling coefficient can also be obtained, taking (2) into account, from data of the above experiment performed without polarization filters in front of the return mirrors

$$r_{-\,int} = r_{-\,ext}\ \Delta f_2 / \Delta f_1 \approx 0.5\cdot 10^{-5}.$$

The difference of frequency of the opposed waves (beat frequency) can be measured by varying the magnetic field at the non-reciprocal element from the limit of the capture region to 60 kHz which is by an order greater than the capture region for maximum coupling obtained with the return mirrors.

We investigated the beat-frequency dependence of the beat frequency deviation Δf for maximum (symmetric) external coupling between opposed waves. The results of the measurements are given in Fig. 4. The beat frequency f_0 measured in the absence of external coupling (with closed return mirrors) is laid off on the abscissa axis. The experimental points show a good fit with the hyperbolic function shown in Fig. 4 by a solid line.

An analogous dependence was obtained for the constant component of the opposed wave intensity (Fig. 5) (ΔI_0 is the constant component of the difference and I_0 = const is the sum of intensities of the opposed waves).

Figure 6 shows the values of f_{max} and f_{min} obtained for various values of f_0 in the case of a strong symmetric coupling between the opposed waves. It is obvious that the frequency characteristic (f as a function of f_0) of the ring laser with a coupling between the

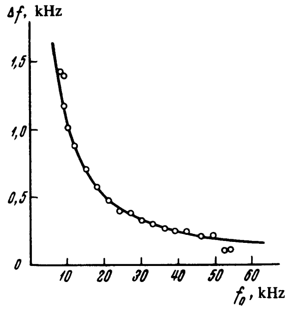

FIG. 4

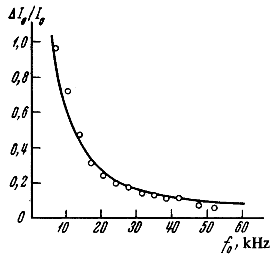

FIG. 5

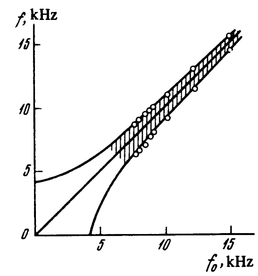

FIG. 6

opposed waves corresponding to an arbitrary fixed value of the relative phase ψ of reflected or back-scattered waves can pass through any point of the shaded region. The difference of values corresponding to the boundaries of the region comprises π.

4. DISCUSSION OF RESULTS

In a large-perimeter ring laser the single-mode regime is inevitably associated with a small pump excess over threshold and consequently with low intensity of emission. Therefore the condition of strong coupling of the opposed waves is usually satisfied even in the case of internal coupling through scattering on inhomogeneities of the optical resonator, and all the more so in the case of enhanced coupling as used in our experiments. This consideration allows us to propose a simple physical interpretation of the experimental results from the viewpoint of normal oscillations.

The characteristic feature of all the effects connected with back scattering is that their attenuation is inversely proportional to the difference frequency (see (15) and (16)). Such a relationship is clearly apparent from Fig. 5 for the constant component of the intensity difference and from Fig. 4 for the frequency deviation. This effect is due to the fact that the scattered wave incident on a "foreign" system is amplified to a magnitude of the order of $|\sigma|/f$, then again scattered and added in its "own" system to the primary wave. This changes the resonance frequency and the Q-factor of the primary wave. The double scattering is also associated with the dependence of the coupling effects on the product of the coefficients of scattering into opposed waves.

The inversely proportional dependence on the difference frequency enables us to separate readily the coupling effects from those caused by the difference in Q-factors of the opposed waves as discussed in [5,6]. We note that in [5] the instability of the results seems to be attributed to the coupling of opposed waves, although this work was marked by a large frequency difference and an artificial introduction of a considerable loss difference. At the same time, the measurements in [6] were carried out with a comparatively small difference frequency, so that the entire observe opposed-wave intensity difference and frequency deviation could have been explained by opposed-wave coupling via scattering rather than by the difference in losses of unknown origin, as assumed by the authors.

According to the formula for the constant components of intensity difference (15) the opposed wave coupling via scattering causes an additional difference in the Q-factors of normal oscillations and correspondingly to the difference in opposed-wave intensities. This results in a phase non-reciprocity of the active medium which in turn leads to an additional difference in the opposed wave frequency. This effect is shown in Fig. 2, where we see a phase shift between frequency deviation and a change in the constant component of intensity difference depending on the relative scattering phase. The magnitude of this shift can be readily evaluated by comparing (15) and (16) which can be rewritten in the form

$$\Delta I_0 = -\frac{1}{\mathrm{Re}(\alpha-\beta)}\frac{2|\sigma_+\sigma_-|}{f_0}\sin\psi, \qquad (15')$$

$$\Delta f = \frac{\mathrm{Im}(\alpha-\beta)}{\mathrm{Re}(\alpha-\beta)}\frac{2|\sigma_+\sigma_-|}{f_0}\sin\psi + \frac{2|\sigma_+\sigma_-|}{f_0}\cos\psi, \qquad (16')$$

where ψ is the sum of the scattering phases. Hence we see that the observed phase shift depends on the coefficients that describe the nonlinear polarizability of the active medium

$$\tan\chi = \mathrm{Im}\,(\alpha-\beta)\,/\,\mathrm{Re}\,(\alpha-\beta). \qquad (23)$$

In the natural mixture of neon isotopes used by us, $\tan\chi \approx 1.5$ near the gain maximum, which is in good agreement with the observed phase difference $\Delta\psi \approx \pi/5$.

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