CERENKOV RADIATION IN A MEDIUM WITH AN INVERTED POPULATION LEVEL

V. B. KRASOVITSKIĬ and V. I. KURILKO

Khar'kov State University

Submitted to JETP editor Feb. 7, 1969

Zh. Eksp. Teor. Fiz. 57, 864–869 (September, 1969)

A nonlinear theory is given for Cerenkov excitation of longitudinal waves and transverse waves in a transparent "inverted" dielectric, the waves being excited by a modulated current. The peak amplitudes and characteristic times for the radiation induced by the current are computed. The physical mechanism responsible for stabilization of the growth of the field amplitude is discussed.

At the present time, Cerenkov radiation in equilibrium media has been studied extensively and used widely in various fields of physics. On the other hand, it is of interest to investigate the features and potentialities, for practical use, of Cerenkov radiation in nonequilibrium media since the existence of a nonequilibrium condition provides the possibility for conversion of the force of the Cerenkov field that acts on the radiating particle. This effect was first discussed and investigated by I. E. Tamm for the case of a charge interacting with a moving dielectric. It was pointed out by V. I. Veksler that it would be possible to realize a moving dielectric by means of electron beams. Veksler proposed to use the conversion of the sign of the Cerenkov field for coherent acceleration of ions.

A similar effect obtains in a medium at rest in which the nonequilibrium is due to the inversion of the population of its energy levels. The possibility of conversion of the sign of the Cerenkov field in such a medium was noted by V. P. Silin (cf.[1]) and the corresponding problem for the case of an oscillator was solved by V. L. Ginzburg and V. Ya. Éĭdman.[1] It is of interest to investigate the possibility of using this effect for the acceleration of charged particles. The first suggestion for acceleration in an active dielectric medium by means of direct conversion of energy of the medium into kinetic energy of a beam of accelerated particles was given by Ya. B. Faĭnberg while acceleration in inverted paramagnetic medium was proposed by E. K. Zavoĭskiĭ.

The present work is devoted to a theoretical analysis of Cerenkov acceleration of a beam of charged particles in a transparent dielectric with an inverted population level, the inversion being due to the field induced by the beam. The field is comprised of longitudinal (polarization) oscillations and transverse waves (Cerenkov radiation).

In the general case the accelerating field induced by the beam due to the interaction of this beam with the inverted level population of the medium is comparable with the retarding field that acts on the beam due to the noninverted levels. The efficiency of the interaction of the beam with the active atoms of the medium can be enhanced by modulating the beam at an appropriate frequency. In this case the intensity of the emission of the inverted level population is enhanced by virtue of the coherent addition of the fields of individual bunches in the beam, as a consequence of which the amplitude of the accelerating field is increased substantially. In view of this circumstance we will consider the excitation of longitudinal and transverse waves in the medium by a modulated current.

The complete system of equations which, in the nonlinear approximation, describes the collective interaction of a beam of charged particles with the dielectric medium consists of Maxwell's equations with a forcing term due to the current j together with the equations of motion for the medium:

$$\mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \tag{1}$$

$$\mathrm{rot}\,\mathbf{H} = \frac{1}{c}\frac{\partial}{\partial t}(\mathbf{E} + 4\pi\mathbf{P}) + \frac{4\pi}{c}\mathbf{j};$$

$$\frac{\partial^2 \mathbf{P}}{\partial t^2} + \Omega^2\mathbf{P} = \pm\frac{2d}{\hbar}[(Nd\Omega)^2 - \dot{\mathbf{P}}^2 - \Omega^2\mathbf{P}^2]^{1/2}\mathbf{E}. \tag{2}$$

Here, $\mathbf{P}$ indicates the dielectric polarization of the medium, N is the density of active centers, d is the electric dipole moment per center, and $\Omega \equiv (E_2 - E_1)/\hbar$ is the appropriate resonance frequency. In the general case the right side of Eq. (2) is proportional to $N_1 - N_2$, the difference in the populations of the lower (E_1) and upper (E_2) energy levels;[2] hence the upper sign in front of the radical in (2) corresponds to the normal condition while the lower sign corresponds to an inverted population level at the initial time ($t = \mathbf{E} = \mathbf{H} = \mathbf{P} = \dot{\mathbf{P}} = 0$).

The two-level approximation (2) applies for the description of the interaction of the medium with the field when the characteristic frequencies of the electromagnetic fields are close to the resonance frequency of the medium Ω.[2-5] In the case of Cerenkov radiation and the excitation of polarization oscillations these conditions are satisfied for modest densities of active centers ($8\pi Nd^2 \ll \hbar\Omega$) and also for the excitation of a medium by a modulated current if the modulation frequency ω_m is reasonably close to the appropriate resonance frequency ($|\omega_m - \Omega| \ll \Omega$).

In the linear approximation [$\dot{\mathbf{P}}^2 + \Omega^2\mathbf{P}^2 \ll (Nd\Omega)^2$], which corresponds to neglecting the change in the level of population in the medium under the effect of the radiation, using Eq. (2) we can find an effective dielectric constant for the medium:

$$\varepsilon_\pm(\omega) \equiv 1 \pm \omega_r^2 / (\Omega^2 - \omega^2), \quad \omega_r^2 \equiv 8\pi Nd^2\Omega\hbar^{-1}. \tag{3}$$

Thus, in an active dielectric the frequency region in which the Cerenkov condition is satisfied ($\beta^2\epsilon_- > 1$) is higher than the resonance frequency Ω while the refractive index $(\epsilon_-)^{1/2}$ falls off with frequency in the transparency region (negative dispersion). Under these conditions the frequency of the polarization oscillations is smaller than the resonance frequency $[\omega_p \equiv \Omega^2 - \omega_p^2)^{1/2}]$.

Substituting (3) into the general expressions[6] for the Cerenkov field and the polarization field that act on the particle and taking account of the change in the phase of the logarithm due to the change in the order of traversing the zeroes and poles of the argument of the logarithm in the active medium, we have (cf.[7-11])

$$E_C = \mp \frac{e}{2c^2} \int_{\beta^2\varepsilon_\pm > 1} \omega\, d\omega \left[1 - \frac{1}{\beta^2 \varepsilon_\pm(\omega)}\right],$$
$$E_p = \mp \frac{e}{2V^2} \omega_p^2 \ln \frac{k_m^2 V^2}{\Omega^2 \pm \omega_r^2}, \tag{4}$$

where e is the particle charge and V is the particle velocity.

The feasibility of practical use of this effect for the indicated purposes is determined by the amplitude of the accelerating field and the emission time. These are essentially nonlinear characteristics which can only be found from a nonlinear analysis, to which we now turn.

We start with the polarization oscillations, which can be considered in the one-dimensional approximation. Assume that polarization oscillations in the dielectric are excited by a sequence of charged plane sheets with surface charge density σ which move in the medium with velocity V in the direction of the normal to the plane surface. In this case the field in the medium is determined by the one-dimensional system of equations consisting of Eq. (2) and Poisson's equation:

$$\frac{\partial}{\partial z}(E + 4\pi P) = 4\pi\sigma \sum_{s=-\infty}^{+\infty} \delta(z - Vt - sl),$$
$$\frac{\partial^2 P}{\partial t^2} + \Omega^2 P = -\frac{2d}{\hbar}[(Nd\Omega)^2 - \dot{P}^2 - \Omega^2 P^2]^{1/2} E, \tag{5}$$

where l is the distance between charge sheets.

Assume that the modulation frequency $\omega_m \equiv 2\pi V/l$ is equal to the frequency $\omega_\pi \equiv (\Omega^2 - \omega_p^2)^{1/2}$ which corresponds to the linear polarization oscillations; we now seek the nonstationary solution of the system in (5) using the Bogolyubov method:

$$E(z, t) = Nd[A(t)\cos\Phi + B(t)\sin\Phi],$$
$$P(z, t) = Nd[a(t)\cos\Phi + b(t)\sin\Phi],$$
$$\Phi(z, t) \equiv \omega_p t - k_\parallel z, \quad k_\parallel = 2\pi/l. \tag{6}$$

The dimensionless coefficients a(t) and b(t), which vary slowly in time, are described by the following system of equations:

$$2\frac{da}{d\tau} - nb + n(b + I_p)(1 - a^2 - b^2)^{1/2} = 0,$$
$$2\frac{db}{d\tau} + na - na(1 - a^2 - b^2)^{1/2} = 0,$$
$$n \equiv \frac{8\pi N d^2}{\hbar\Omega} \equiv \frac{\omega^2_r}{\Omega^2}, \quad I_\pi \equiv \frac{\sigma}{4\pi^2 Nd}, \quad \tau \equiv \omega_p t. \tag{7}$$

Making use of the notation

$$a + ib = \sin\psi e^{i\varphi}, \quad \varphi(0) = \pi, \quad \psi(0) = 0$$

we have from Eq. (7)

$$\frac{d\psi}{d\tau} = \frac{1}{2} n I_p \left[1 - I_p^{-2} \sin^4\frac{\psi}{2} \operatorname{tg}^2\frac{\psi}{2}\right]^{1/2},$$

$$\sin\varphi = I_p^{-1} \sin^2\frac{\psi}{2} \operatorname{tg}\frac{\psi}{2}. \tag{8}$$

It follows from Eqs. (5)–(8) that the excitation of polarization oscillations alternates periodically with the absorption of the field of the oscillations in the medium.[4,5] For small values of the current ($I_p \ll 1$) the period T_p in units of Ω^{-1} is $4n^{-1} I_\pi^{-2/3}$. The peak amplitude of the accelerating field and is determined from the condition $\dot{A} = 0$ and for $I_p \ll 1$ is found to be

$$E_m = 2\sqrt{3}(\sigma\pi N^2 d^2)^{1/3}. \tag{9}$$

It is evident that in the cases being considered ($\sigma \ll Nd$) this field is large compared with the surface charge field $2\pi\sigma$. The increment of kinetic energy per unit surface of the layer referred to the density of energy stored in the medium (efficiency) is found to be proportional to $I_p^{-2/3}$.

Physically, the saturation of the amplitude of the field arises as follows. Excitation of polarization oscillations occurs by virtue of the energy stored in the atoms in the medium so that as the field amplitude increases there is a reduction in the difference of populations in the levels. The frequency of the longitudinal oscillations ω_p increases and violates the resonance between the frequency of the exciting force ω_M and the frequency of the characteristic oscillations of the medium. When $\tau \sim T_p$ the phase of the field changes sign and the acceleration becomes a retardation, even though the difference in the population levels ($\cos\psi$) does not change sign.

We now consider the problem of Cerenkov radiation due to a modulated current in a dielectric at a negative temperature. In this case, in order to estimate the peak amplitude of the Cerenkov field and the corresponding lifetime of the pulse we consider a model problem in which the Cerenkov radiation of the inverted medium is excited by a current with a density

$$\mathbf{j} = \mathbf{n} j_0 \cos\Psi, \quad \Psi = \omega_M t - k_\parallel z - k_\perp x, \tag{10}$$

where n is a unit vector along the z-axis while the wave numbers $k_\parallel$ and $k_\perp$ satisfy the dispersion equation obtained from the linear theory:

$$k_\parallel^2 + k_\perp^2 = \frac{\omega_M^2}{c^2} \varepsilon_-(\omega_M), \quad k_\parallel \equiv \omega_M / V. \tag{10a}$$

Substituting the current (10) in Maxwell's equations we seek a solution of these equations together with the equation of state of the medium (1), again using the Bogolyubov method:

$$\mathbf{E}(\mathbf{r}, t) = Nd[\mathbf{A}(t)\cos\Psi + \mathbf{B}(t)\sin\Psi],$$
$$\mathbf{P}(\mathbf{r}, t) = Nd[\mathbf{a}(t)\cos\Psi + \mathbf{b}(t)\sin\Psi].$$

We then obtain the following system of equations in total derivatives for the slowly varying amplitudes $a_\perp$ and $b_\perp$ of the transverse component of the polarization vector, assuming

$$2\frac{d}{d\tau}\left[\frac{\Delta u}{(1 - |u|^2)^{1/2}}\right] - \frac{2n}{\Delta}\frac{\dot{u}}{(1 - |u|^2)^{1/2}} -$$
$$- inu[(1 - |u|^2)^{-1/2} - 1] = -iK_\perp n I, \tag{11}$$

where

$$\tau \equiv \Omega t, \quad n \equiv 8\pi N d^2/\hbar\Omega, \quad I \equiv j_0/\Omega N d,$$
$$\Delta \equiv 1 - \omega_M^2/\Omega^2, \quad K_\perp \equiv k_\perp c/\Omega, \quad u \equiv a_\perp + i b_\perp.$$

In this case the amplitude of the longitudinal field component $E_\parallel$, which determines the effectiveness of the interaction of the beam with the medium, is given by the relation

$$A_{\parallel} = -K_{\perp}A_{\perp} = \frac{4\pi K_{\perp}}{n}\frac{2b_{\perp}+\Delta a_{\perp}}{1-a_{\perp}^2-b_{\perp}^2)^{1/2}}. \quad (12)$$

An investigation of linear solutions of the system in (11) shows that in the nonstationary regime acceleration of the beam by radiation from the medium can occur only when the condition $1 - n\Delta^{-2} < 0$ is satisfied. It is evident from Eq. (10a) that this condition is equivalent to the requirement that the group velocity of the excited wave be negative. Assuming that this inequality is satisfied with margin, making use of the substitution $u \equiv \sin\psi \exp(i\varphi)$ we have

$$\frac{d\psi}{d\tau} = \frac{\psi_m}{T}\left[1-\left(\frac{\psi}{\psi_m}\right)^6\right]^{1/2}, \quad \psi_m \equiv 2(K_{\perp}I)^{-2/3},$$

$$\sin\varphi = \left(\frac{\psi}{\psi_m}\right)^3, \quad T \equiv \frac{4\Delta}{n}\left(1-\frac{n}{\Delta^2}\right)(K_{\perp}I)^{2/3}. \quad (13)$$

It will be evident from this equation that if relaxation effects are neglected the process of emission of the Cerenkov field in the medium and the acceleration of the beam alternate periodically with the inverse process of absorption in the medium and retardation of the beam. The characteristic period for the radiation is of order T (in units of Ω^{-1}) while the peak amplitude of the longitudinal component of the field is given by the expression

$$(A_{\parallel})_{max} = 4\pi K_{\perp}\frac{|\Delta|}{n}(K_{\perp}I)^{1/3}. \quad (14)$$

Strictly speaking, this expression applies when $K_{\perp}^2 \approx n/|\Delta| \ll 1$ $(n > \Delta^2)$. However, for the purpose of making estimates we can take $K_{\perp} \sim 1$. Thus we find

$$(E_{\parallel})_{max} = 4\pi Nd(2j_0/\Omega Nd)^{1/3}. \quad (15)$$

Physically, the stabilization in the growth of the field amplitude is explained by the dependence of the characteristic frequency of the field in the medium on its amplitude: as the amplitude increases the synchronism between the wave and beam is disturbed and as a result the field radiation and the particle acceleration are converted to field absorption and beam retardation. When $\psi_m \ll 1$, in a time corresponding to one cycle only a small fraction of the energy stored in the medium $(\sim\psi_m^2)$ is converted into radiation. The energy of the field can be increased by programming a change in the frequency of modulation of the beam.

Above, in analyzing the Cerenkov interaction of a beam of charged particles with an active dielectric we have assumed that the beam current is specified. Inasmuch as it is difficult to provide modulation of a beam by external sources in the shortwave region, one could make use of the Cerenkov instability of an unmodulated beam.

For the case of a beam whose diameter a is large compared with the wavelength $(a \gg \lambda)$ the dispersion equation for longitudinal oscillations is of the form[12]

$$1 \pm \frac{\omega_r^2}{\Omega^2-\omega^2} - \frac{\omega_b^2}{(\omega-k_{\parallel}V)^2} = 0, \quad \omega_b^2 \equiv \frac{4\pi e^2 n_b}{m_{\parallel}}, \quad (16)$$

where n_b is the beam density.

Using this equation and assuming $\omega_b^2 \ll \omega_r^2 \ll \Omega^2$ we can obtain the growth rate for the longitudinal oscillations:

$$\alpha \equiv \operatorname{Im}\omega = \frac{\sqrt{3}}{2^{4/3}}\left(\frac{\omega_r^2\omega_b^2}{\Omega}\right)^{1/3}.$$

Thus, the Cerenkov instability of a beam with respect to longitudinal oscillations can actually be used to obtain the modulation of the beam. The authors are indebted to Ya. B. Faĭnberg for suggesting the topic and for useful discussion and also to V. P. Silin, B. M. Bolotovskiĭ and R. V. Khokhlov for useful comments.

[1] V. L. Ginzburg and V. Ya. Éĭdman, Zh. Eksp. Teor. Fiz. 36, 1823 (1959) [Sov. Phys. JETP 9, 1300 (1959)].
[2] V. M. Faĭn and Ya. I. Khanin, Kvantovaya radiofizika (Quantum Radiophysics) Sov. Radio, 1965.
[3] L. W. Davis, E. T. Jaynes and F. W. Cummings, Proc. IRE 51, 112 (1963); ibid 51, 126 (1963).
[4] A. P. Kazantsev and V. S. Smirnov, Zh. Eksp. Teor. Fiz. 46, 182 (1964) [Sov. Phys. JETP 19, 130 (1964)].
[5] B. B. Krasovitskiĭ and V. I. Kurilko, Zh. Eksp. Teor. Fiz. 48, 353 (1965) [Sov. Phys. JETP 21, 232 (1965)].
[6] L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media, Addison Wesley, Reading, Mass. 1959.
[7] A. I. Akhiezer and Ya. B. Faĭnberg, Coll., Theory and Design of Linear Accelerators, Gosatomizdat, 1962, p. 320.
[8] B. M. Bolotovskiĭ and A. A. Rukhadze, Zh. Eksp. Teor. Fiz. 37, 1346 (1959) [Sov. Phys. JETP 10, 958 (1960).]
[9] B. M. Bolotovskiĭ and S. N. Stolyarov, Izvestiya vuzov, Radiofizika (News of the Schools, Radiophysics) 7, 442 (1964).
[10] K. A. Barsukov and B. M. Bolotovskiĭ, Zh. Eksp. Teor. Fiz. 45, 303 (1964) [Sov. Phys. JETP 18, 211 (1964)].
[11] M. I. Ryazanov, Zh. Eksp. Teor. Fiz. 48, 1490 (1965) [Sov. Phys. JETP 21, 997 (1965)].
[12] V. D. Shapiro and V. N. Tsytovich, Zh. Tekh. Fiz. 34, 764 (1964) [Sov. Phys.-Tech. Phys. 9, 583 (1964)].

Translated by H. Lashinsky
98