FINITE-AMPLITUDE WAVE BEAMS IN A MAGNETOACTIVE PLASMA

A. G. LITVAK

Radiophysics Institute, Gorkiĭ State University

Submitted March 15, 1969

Zh. Eksp. Teor. Fiz. 57, 629–636 (August, 1968)

The self-effect of electromagnetic wave beams in a magnetoactive plasma is stuided for the case of longitudinal propagation. In the weak-nonlinearity approximation a parabolic equation is obtained for the amplitude of the transverse component of the electric field of a wide (on the scale of the wavelength) wave beam. The nonlinearity mechanisms for the magnetoactive plasma are studied and expressions are obtained for the nonlinear corrections to the refractive index due to the striction, heating, and nonlinear motion of a single electron. A necessary condition for self-focusing is obtained for this case and the characteristic parameters for self-focusing are determined. Estimates are given which indicate the possibility of self-consistent nonlinear channelization of atmospheric whistlers in the ionisphere and decimeter electromagnetic waves in a laboratory plasma.

A number of papers in recent years have been devoted to the theoretical and experimental investigation of self-focusing of electromagnetic waves in nonlinear media (cf. for example the reviews in[1,2]). The overwhelming majority of these publications have been concerned with investigations of this effect in the optical region, in which it is possible to achieve extremely large electric fields (10^7–10^8 V/cm) by means of lasers. We note, however, that in early work[3-5] on the theory of self-focusing of waves discussions were given of the features of the dynamics of self-channelization of fields in a plasma because in a plasma the nonlinear effects become important at rather low electric fields. The results of the "optical" theory of self-focusing of electromagnetic waves also apply to an isotropic plasma under appropriate conditions and we need only consider the specific mechanisms for the nonlinearity and the relaxation processes[1)]; however, the self-effect of the waves in the plasma in a fixed magnetic field has a number of interesting features and merits special investigation. Various aspects of self-channelization (uniform in the direction of propagation) of beams in a magnetoactive plasma have been considered in[6-8]. In the present work certain features of the propagation dynamics of wave beams will be illustrated using the example of high-frequency waves propagating in the direction of the fixed magnetic field H_0 (longitudinal propagation).

1. PARABOLIC EQUATION

We consider waves in an anisotropic medium characterized by the dielectric tensor

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon & -i\varepsilon_a & 0 \\ i\varepsilon_a & \varepsilon & 0 \\ 0 & 0 & \varepsilon_{\parallel} \end{pmatrix}. \tag{1.1}$$

The components of the dielectric tensor are functions of the electric field. The nonlinearity is due to the dependence of the stationary distribution of electron density in the plasma on the field strength which arises as a consequence of heating, striction or ionization; the relativistic dependence of the electron mass on kinetic energy and the nonlinearity of the Lorentz force can also provide nonlinear behavior.

The amplitude of the electric field of the stationary wave beam $\vec{\mathscr{E}} = \vec{\mathscr{E}}(r)e^{i\omega t}$ is given by the equations

$$\Delta\vec{\mathscr{E}} - \operatorname{grad}\operatorname{div}\vec{\mathscr{E}} + k_0^2\hat{\varepsilon}\vec{\mathscr{E}} = 0, \tag{1.2}$$
$$\operatorname{div}\hat{\varepsilon}\vec{\mathscr{E}} = 0. \tag{1.3}$$

In what follows we shall limit ourselves to the weak-nonlinearity approximation in which case the dependence of the components of the tensor (1.1) on the field amplitude can be written in the form

$$\varepsilon = \varepsilon_0 + \varepsilon' f(\vec{\mathscr{E}}), \quad \varepsilon_a = \varepsilon_{a0} + \varepsilon_a' f(\vec{\mathscr{E}}), \quad \varepsilon_{\parallel} = \varepsilon_{\parallel 0} + \varepsilon_{\parallel}' f(\vec{\mathscr{E}}), \tag{1.4}$$

where

$$\frac{\varepsilon'}{\varepsilon_0} \ll 1, \quad \frac{\varepsilon_a'}{\varepsilon_{a0}'} \ll 1, \quad \frac{\varepsilon_{\parallel}'}{\varepsilon_{\parallel 0}} \ll 1, \quad \varepsilon_0 = 1 - \frac{\omega_L^2}{\omega^2 - \omega_H^2}$$
$$\varepsilon_{a0} = \frac{\omega_L^2\omega_H}{\omega(\omega^2-\omega_H^2)}, \quad \varepsilon_{\parallel 0} = 1 - \frac{\omega_L^2}{\omega^2}, \quad \omega_L^2 = \frac{4\pi e^2 N_0}{m}, \quad \omega_H = \frac{eH_0}{mc} \quad k_0 = \frac{\omega}{c}$$

and N_0 and m are the electron density and mass in the region in which the alternating field vanishes while e is the electron charge. The forms of the nonlinearity coefficients ϵ', ϵ_α' and $\epsilon_{\parallel}'$ and the function $f(\vec{\mathscr{E}})$ will be given in detail below.

We consider a plane (two-dimensional) beam of electromagnetic waves propagating in the direction of the fixed magnetic field:

$$\mathscr{E}_x = E_x(z, x)e^{-ikz}, \quad \mathscr{E}_y = E_y(z, x)e^{-ikz}, \quad \mathscr{E}_z = E_z(z, x)e^{-ikz}. \tag{1.5}$$

Let the characteristic scale size for the amplitude of the field in the longitudinal and transverse direction Λ_x and Λ_z be large compared with the wavelength

$$k\Lambda_x \gg 1, \quad k\Lambda_z \gg 1, \tag{1.6}$$

where $k = k_0 n$; n is the refractive index for the linear medium; $n^2 = \epsilon_0 \pm \epsilon_{a0} = 1 - \omega_L^2/\omega(\omega \pm \omega_H)$ where the upper sign corresponds to the ordinary wave and the lower sign to the extraordinary wave. Then, the longitudinal component of the field E_z is small compared with the transverse components ($E_z \ll E_x, E_y$) and can be expressed in terms of these by means of (1.3):

[1)]These problems have been considered in detail in [7].

$$E_z = -\frac{1}{ik\varepsilon_{\|0}}\frac{\partial}{\partial x}(\varepsilon_0 E_x - i\varepsilon_{a0}E_y). \tag{1.7}$$

If we assume that the normal waves in the linear medium are circularly polarized in longitudinal propagation, i.e., $E_x = \pm iE_y$ and $\mathbf{E}_\perp = E_\perp(\mathbf{x}_0 \pm i\mathbf{y}_0)/\sqrt{2}$; $\mathbf{x}_0$, $\mathbf{y}_0$ are unit vectors, substituting Eq. (1.7) in Eq. (1.2), after some simple manipulation we obtain an equation for the amplitude of the transverse component of the electric field

$$-2ik\frac{\partial E_\perp}{\partial z} + \alpha\Delta_\perp E_\perp + k_0^2\tilde{\varepsilon}' f(\mathbf{E})E_\perp = 0, \tag{1.8}$$

as well as the linear relation between E_z and $E_\perp$:

$$E_z = -\frac{n}{ik_0\varepsilon_{\|0}}\operatorname{div}_\perp \mathbf{E}_\perp. \tag{1.9}$$

Here, $\Delta_\perp$ and $\operatorname{div}_\perp$ denote vector operations of differentiation with respect to the transverse coordinates, $\tilde{\epsilon}' = \epsilon' \pm \epsilon'_\alpha$ and $\alpha = (1/2)(1 + n^2/\epsilon_{\|0})$. It is easy to show that Eqs. (1.8) and (1.9), which have been obtained for a two-dimensional beam, also hold in the three-dimensional case.

Equation (1.8) is a parabolic equation for the field amplitude in the wave beam propagating in the direction of the fixed magnetic field. In the limiting case $\omega_H \to 0$ we find $\epsilon_\alpha \to 0$ and $\epsilon \to \epsilon_\|$ and Eq. (1.8) becomes the usual parabolic equation for a weakly nonlinear isotropic medium. When $\omega_H \neq 0$ in the general case $\alpha \neq 1$ and transverse diffusion of the rays will occur either more rapidly or more slowly than in the isotropic medium[2)] depending on the relations between the parameters ω_L/ω and ω_H/ω. Furthermore, in certain frequency regions the diffusion coefficient α is negative. Since $\alpha_0 < 0$ for the ordinary wave in the frequency range $\bar{\omega}_0 < \omega < \omega_L$ where $\bar{\omega}_0$ is one root of the cubic equation

$$\bar{\omega}_0{}^3 + \bar{\omega}_0{}^2\omega_H - \bar{\omega}_0\omega_L{}^2 - {}^1\!/_2\,\omega_L{}^2\omega_H = 0.$$

If $\omega_L \gg \omega_H$ then $\bar{\omega}_0 \approx \omega_L - (1/4)\omega_H$ and for $\omega_L \ll \omega_H$ we find $\bar{\omega}_0 \approx \omega_L^2/2\omega_H$. For the extraordinary wave in a plasma with $\omega_L \gg \omega_H$ the coefficient $\alpha_e < 0$ for frequencies $\omega_H/2 < \omega < \omega_H$; however, if $\omega_H > \omega_L$ the diffusion coefficient is negative when $\bar{\omega}_e < \omega < \omega_L$

$$\bar{\omega}_e{}^3 - \bar{\omega}_e{}^2\omega_H - \bar{\omega}_e\omega_L{}^2 + {}^1\!/_2\,\omega_L{}^2\omega_H = 0.$$

In a strong magnetic field ($\omega_H \to \infty$) $\alpha_e = (2\omega^2 - \omega_L^2)/2(\omega^2 - \omega_L^2)$.

We now wish to examine the physical situation associated with the negative coefficient of transverse diffusion $\alpha < 0$. For this purpose in Eq. (1.8) we make the substitution of variables

$$E_\perp = E_0(\mathbf{r}_\perp, z)\exp[-i\varphi(\mathbf{r}_\perp, z)]; \quad \mathbf{r}' = k\mathbf{r},$$

and, omitting primes, write a system of equations for the real amplitude E_0 and the phase φ:

$$2\frac{\partial\varphi}{\partial z} + \alpha(\nabla_\perp\varphi)^2 = \frac{\tilde{\varepsilon}'}{n^2}f(\mathbf{E}_0) + \alpha\frac{\Delta_\perp E_0}{E_0}, \tag{1.10}$$
$$\frac{\partial E_0{}^2}{\partial z} + \alpha\operatorname{div}_\perp(E_0{}^2\nabla\varphi) = 0.$$

[2)]The difference between the coefficient α and unity can be due to the contribution of the longitudinal component of the field in the beam which need not be taken into account in the isotropic medium in the quasioptical approximation (1.6) (the so-called scalar problem). This means that in the case of an anisotropic medium the scalar approximation can lead to important errors.

If we neglect the last two terms in the first equation in (1.10) we obtain the equations of geometric optics for the case of quasi-longitudinal propagation in a linear magnetoactive plasma. These equations, in particular, are satisfied by spherical waves which, in the quasioptical approximation, can be written in the form

$$\varphi = \frac{r_\perp{}^2}{2(z+R_0)\alpha}, \quad E_0{}^2 = \frac{A_0{}^2}{(z/R_0+1)^2}F\left(\frac{r_\perp}{a(z/R_0+1)}\right). \tag{1.11}$$

Here, $R_{ph0} = R_0\alpha$ is the radius of curvature of the initial ($z = 0$) phase front in the isotropic medium $R_{\Phi 0} = R_0$; F is an arbitrary function which determines the initial distribution of amplitude across the beam.

It follows from Eq. (1.11) that when $z \to -R_0$, $E_0^2 \to \infty$, that is to say the beams intersect at the point $z = -R_0 = -R_{ph0}/\alpha$; when $\alpha > 0$, as in an isotropic medium, focusing of the beam corresponds to phase fronts with negative radius of curvature $R_{ph} < 0$ and when $\alpha < 0$ we have phase fronts with $R_{ph} > 0$.

Qualitatively this effect can be understood as follows. As a rule, in anisotropic media the directions of the phase and group velocities are not the same. In a magnetoactive plasma, for certain values of the parameters it is possible to have cases[3)] in which the angle between the wave vector and the direction of the fixed magnetic field $\mathbf{H}_0$ is opposite in sign to the angle between the ray vector and $\mathbf{H}_0$. It is precisely in these cases in which a converging beam corresponds to divergence of the phase fronts and vice versa.

In view of the considerations given above it is not difficult to show, for example, that the boundary between an isotropic medium and an anisotropic medium with $\alpha < 0$ will exhibit focusing properties.[9] Thus, if we locate a point source in an isotropic medium at a distance z_0 from the boundary ($z = 0$), in the anisotropic medium at a distance $z = |z_0/\alpha|$ from the boundary we will have a real image (caustic). Similarly, it can be shown that a plane uniform layer of linear magnetoactive plasma can be used as a lens.

2. NONLINEARITY OF THE MEDIUM

In order to investigate the self-effects of the electromagnetic waves we must take into account the actual form of the dependence of the refractice index on the amplitude of the electric field. The nonlinearity in the plasma is due to the modification of the distribution of the density of charged particles which occurs because of the inhomogeneous electromagnetic field (striction, heating, ionization); it can also be related to the nonlinearity in the motion of a single charged particle in the field of the electromagnetic wave. Without considering the details we shall present the pertinent expressions directly.

Electrostriction plays a dominant role in a collisionless plasma (more precisely, when the characteristic inhomogeneity in scale size for the field L_E is small compared with the mean free path: $L_E \ll l$). In this case

$$[\tilde{\varepsilon}'f(\mathbf{E})]_c = \frac{\omega_L{}^2e^2}{4Tm\omega(\omega\pm\omega_H)}\left(\frac{|\mathbf{E}_\perp|^2}{\omega(\omega\pm\omega_H)} + \frac{|E_z|^2}{\omega^2}\right), \tag{2.1}$$

[3)]These cases can easily be seen by tracing the surfaces of the refractive index.

where T is the sum of the electron and ion temperatures measured in energy units.

Taking account of Eq. (1.9), for a two-dimensional beam we find

$$[\tilde{\varepsilon}'f(E_0)]_c = \frac{\omega_L^2 e^2}{4Tm\omega(\omega \pm \omega_H)}\left[\frac{E_0^2}{\omega(\omega \pm \omega_H)} + \frac{n^4}{2\omega^2 \varepsilon_{\parallel 0}^2}\left(\frac{dE_0}{dx}\right)^2\right]. \quad (2.2)$$

Since the longitudinal component of the electric field E_z, which is given by Eq. (1.9), is small compared with the transverse component, under ordinary conditions the second term in Eq. (2.2) can be neglected. As a result, the expressions for the nonlinear corrections to the refractive index assume the following form, which is standard for a weakly nonlinear cubic medium:

$$\tilde{\varepsilon}'f(\mathbf{E}_0) = \beta |E_\perp|^2, \quad (2.3)$$

$$\beta_c = \frac{\omega_L^2 e^2}{4Tm\omega^2(\omega \pm \omega_H)^2}. \quad (2.4)$$

In the ordinary wave the striction interaction leads to an expulsion of plasma from a region of strong field while in the extraordinary wave, with $\omega < \omega_H$, this effect can lead to an increase in the plasma density in the region of strong field; nevertheless, the coefficient of cubic striction of the nonlinearity is positive $\beta_c > 0$ for both waves at all frequencies ω. In a sufficiently strong magnetic field $\omega_H \gg \omega$ it is necessary to take account of the dependence of the refractive index on the longitudinal amplitude E_z.

Heating always leads to an increase in the kinetic pressure in the field region and, consequently, to a reduction in the plasma density at the axis of the beam. For example, if the amplitude of the electric field in a weakly ionized plasma does not vary greatly over an electron mean free path $L_E \gg l/\sqrt{\delta}$, the correction to the refractive index due to heating can be written in the form

$$[\tilde{\varepsilon}'f(\mathbf{E})]_T = \frac{\omega_L^2 e^2}{3\omega(\omega \pm \omega_H)Tm\delta}\left(\frac{|E_\perp|^2}{(\omega \pm \omega_H)^2} + \frac{|E_\parallel|^2}{\omega^2}\right), \quad (2.5)$$

where δ is the fraction of the energy lost by an electron in one collision with a heavy particle. In particular, if the magnetic field is not too strong, in which case the cubic approximation to the nonlinearity (2.3) is valid, the appropriate coefficient β_T is given by

$$\beta_T = \frac{\omega_L^2 e^2}{3(\omega \pm \omega_H)^3 \omega Tm\delta}. \quad (2.6)$$

In the ordinary wave $\beta_T > 0$ regardless of the frequency; in the extraordinary wave $\beta_T < 0$ when $\omega < \omega_H$ and $\beta_T > 0$ when $\omega > \omega_H$.

The disturbance of the balance between recombination and ionization that occurs under the effect of the field usually leads to an increase in the density of charged particles. Since there is no universal expression for the correction $\tilde{\epsilon}'f(E)$ associated with ionization (methods of computing this expression for certain cases are given, for example, in[8,10]) we note here only that the coefficient for the cubic ionization nonlinearity $\beta_i < 0$ for the ordinary wave; for the extraordinary wave $\beta_i > 0$ when $\omega < \omega_H$ and $\beta_i < 0$ when $\omega > \omega_H$.

If the nonlinearity mechanisms considered above are characterized by long relaxation times[7] then the weaker nonlinearity due to the motion of a single charged particle becomes essentially inertialess and can play a dominant role in the self effect for short pulses of the high-frequency field. The appropriate cubic nonlinearity coefficient β_{el} can be written in the form

$$\beta_{el} = \frac{3\omega_L^2 e^2}{4m^2c^2(\omega \pm \omega_H)^3\omega}\left(1 - \frac{n^2}{18}\right). \quad (2.7)$$

The first term in Eq. (2.7) is related to relativistic effects while the second is due to the nonlinearity in the Lorentz force. The sign of the coefficient β_{el} depends on the magnitude of the refractive index n, that is to say, on the relations between ω_L and ω_H. In particular, in an isotropic plasma ($n^2 < 1$) the nonlinearity associated with the Lorentz force is negligibly small as compared with the relativistic contribution and $\beta_{el} > 0$ that is to say, in an isotropic plasma in principle it is possible to have inertialess self-focusing of electromagnetic waves.

3. SELF-FOCUSING OF ELECTROMAGNETIC WAVES

It follows from the considerations given above that in a modestly strong fixed magnetic field the self-effect of stationary wave beams in longitudinal propagation can be described by a scalar equation of the form

$$-2i\frac{\partial E_\perp}{\partial z} + \alpha\Delta_\perp E_\perp + \frac{\beta}{n^2}|E_\perp|^2 E_\perp = 0. \quad (3.1)$$

Mathematically, Eq. (3.1) is identical with the equation for one-dimensional wave packets in a weakly nonlinear dispersive medium.[11] Hence a number of the results given below can be based on the results of investigations of self-compression and self-modulation of wave packets.[11] In particular, the characteristic parameter that determines the possibility of self-focusing is the ratio β/α. If $\beta/\alpha > 0$, self-focusing of wave packets occurs and a plane wave is found to be unstable against perturbations of its spatial structure and so on.[2] If $\beta = \partial n^2/\partial|E_\perp|^2 > 0$ and $\alpha > 0$ the nonlinearity leads to a retardation of the wave and when $\beta < 0$ the self-channelization of the wave is found to be faster than that for waves in a linear medium[4)] while the self-focusing beam exhibits a phase front with a positive radius of curvature.

For example, we can consider the structure of a two-dimensional single beam

$$E_\perp = E_m(\operatorname{ch}\varkappa_m x)^{-1}e^{-i\gamma z} \quad \gamma = \frac{1}{2}\frac{\beta}{n^2}E_m^2, \ \varkappa_m^2 = \frac{1}{2}\frac{\beta}{n^2\alpha}E_m^2 \quad (3.2)$$

and the expression for the growth rate of the spatial instability of a plane wave $E_\perp = \overline{E}_0 e^{ipz}$, $p = 1 + (\beta/n^2)\overline{E}_0^2$ with respect to small perturbations of the form $e_\perp = e_\perp^0 \exp(-i\kappa_\perp r_\perp - \Gamma z)$

$$\Gamma^2 = \frac{\alpha^2\varkappa_\perp^2}{2}\left(2\frac{\beta}{n^2\alpha}\overline{E}_0^2 - \varkappa_\perp^2\right). \quad (3.3)$$

The maximum growth rate $\Gamma_{max} = (\beta/n^2)\overline{E}_0^2$ is exhibited by perturbations with transverse wave number $\kappa_\perp^2 = (\beta/\alpha n^2)\overline{E}_0^2$. As has been shown in[12] as a result of development of this instability a plane wave breaks up into a cluster of narrow beams; the power of the beams can be estimated from Eq. (3.3)

---

4) The possibility of self-focusing of fast waves in an anisotropic medium was first indicated in [6] for a plasma in an infinite magnetic field.

$$W = c\lambda_0^2 n(128\beta/\alpha)^{-1} \approx P_{cr} \quad (3.4)$$

The quantity W is of the same order of magnitude as the power of a three-dimensional single beam.[12]

It follows from Eqs. (3.2) and (3.3) that the necessary condition for self-focusing is the inequality $\beta/\alpha > 0$. This requirement determines the frequency region in which various actual nonlinearity mechanisms can produce self-focusing. For example, self-channelization of the extraordinary wave near the gyroresonance in a plasma with $\omega_L \gg \omega_H$ can be due to heating and the relativistic nonlinearity; in a plasma with $\omega_L \ll \omega_H$ the effect can be due to ionization and striction. When $\omega \ll \omega_H$ self-focusing is caused by ionization and striction while defocusing is caused by the relativistic effect. Self-channelization of the ordinary wave at frequencies $[\omega_L^2 + \omega_H^2/4]^{1/2} - \omega_H/2 < \omega < \omega_L$ is possible only in a plasma with an ionization nonlinearity.

It is also possible to show that the self effect of wide (on the wavelength scale) stationary wave packets $E_\perp = E_\perp(z, \mathbf{r}_\perp, t)e^{i\omega t}$ is described by a parabolic equation of the form

$$-2i\frac{\partial E_\perp}{\partial z} + \alpha\Delta_\perp E_\perp + kv_\omega' E_{\tau\tau}'' + \frac{\beta}{n^2}|E_\perp|^2 E_\perp = 0. \quad (3.5)$$

Here, $\tau = k(z - vt)$ and $v = \partial\omega/\partial k$ is the group velocity of the wave in the linear medium. In particular, we find from Eq. (3.5) that even if $\beta/\alpha < 0$ a plane wave in a nonlinear medium is unstable against small space-time perturbations in certain ranges of frequencies when $\beta/v_\omega' > 0$. Furthermore, it follows from Eqs. (3.5) that there are three-dimensional self-channelized wave packets in space which move with velocity v.

In conclusion we present numerical estimates that indicate the possibility of self-focusing of electromagnetic waves in a plasma in a magnetic field under laboratory conditions and astrophysical conditions.

As a first example we consider the instability of low-frequency waves (atmospheric whistlers) in the ionisphere at heights above h = 300 km. The frequency of these waves $\omega \ll \omega_H$; furthermore, at these heights the condition $\omega_L \gg \omega_H$ is satisfied while the mean free path for the electrons is large compared with the characteristic scale size of the inhomogeneity. Hence, the basic nonlinearity mechanism for the plasma in this region is electrostriction; the coefficients in (3.3) can be written in the form

$$n^2 \approx \omega_L^2/\omega\omega_H, \qquad \alpha = 1/2,$$
$$\beta = n^2/E_{cr}^2, \qquad E_{cr}^2 = 4\omega\omega_H Tm/e^2$$

we can simplify the expressions for the optimum dimenisonless parameters: $\overline{\kappa}_\perp^2 = 2\overline{E}_0^2/E_{cr}^2$, $\Gamma_{max} = \overline{E}_0^2/E_{cr}^2$. With $\omega_L = 10^7$ sec^{-1}, $\omega_H = 10^6$ sec^{-1}, $\omega = 10^4$ sec^{-1} and T = 4000°K the characteristic field $E_{cr} \approx 3 \times 10^{-3}$ V/cm and n = 10^2. Typical fields for atmospheric whistlers in the ionisphere[13] are found to be $\overline{E}^0 \approx 10^{-4}$ V/cm and the characteristic dimension corresponding to the developed instability of the beams $\Lambda_\perp = \pi/k\overline{\kappa}_\perp \approx 60$ km while the self-focusing length is $\Lambda_\parallel \approx 1/k\Gamma \approx 300$ km. The critical power for self-focusing

$$P = \frac{\pi^2 c^3 Tm}{16e^2}\left(\frac{\omega_H}{\omega}\right)^{1/2}\frac{\omega_H}{\omega_L}$$

for these values of the parameters is $P_{cr} \approx 4 \times 10^3$ W. This rather rough estimate indicates the possibility of self-consistent channelization of whistlers in the ionisphere.

On the basis of the considerations given above we can also estimate the parameters for self-focusing of helicons in a laboratory plasma. In a plasma with $N_0 = 10^{14}$ cm^{-3}, $H_0 = 3 \times 10^3$ g and T = 1 eV the critical power for self-focusing of electromagnetic waves at a frequency $\omega = 10^{10}$ sec^{-1} ($\lambda_0 \approx 20$ cm, $\lambda = \lambda_0/n \approx 1$ cm) is $P_{cr} \approx 2 \times 10^3$ W. This power is easily obtained under laboratory conditions with standard sources of electromagnetic energy.

[1] S. A. Akhmanov, A. P. Sukhorukov and R. V. Khokhlov, Usp. Fiz. Nauk 93, 19 (1967) [Sov. Phys.-Usp. 10, 609 (1968)].

[2] V. I. Bespalov, A. G. Litvak and V. I. Talanov, Proc. 2-nd All-union Symposium on Nonlinear Optics, Novosibirsk, Nauka, 1968, p. 428.

[3] G. A. Askar'yan, Zh. Eksp. Teor. Fiz. 42, 1567 (1962) [Sov. Phys.-JETP 15, 1088 (1962)].

[4] V. I. Talanov, Iz. vyssh. uch. zav., Radiofizika (News of the Higher Schools, Radiophysics) 7, 564 (1964).

[5] R. Y. Chiao, E. Garmire and C. H. Townes, Phys. Rev. Letters 13, 479 (1964).

[6] A. G. Litvak, Izv. vyssh. uch. zav. Radiofizika (News of the Higher Schools, Radiophysics) 8, 1148 (1965).

[7] A. G. Litvak, Dissertation, NIRFI, Gorkiĭ, 1967.

[8] A. G. Litvak, Izv. vyssh. uch. zav. Radiofizika (News of the Higher Schools, Radiophysics) 9, 628 (1966); ibid 9, 900 (1966).

[9] L. B. Felsen, IEEE Trans. Prof. Group on Antennas and Propagation, Vol. AP-12 624 (1964).

[10] A. V. Gurevich, Geomagnetizm i aéronomiya (Geomagnetism and Aeronomy) 7, 291 (1967).

[11] A. G. Litvak and V. I. Talanov, Izv. vyssh. uch. zav. Radiofizika (News of the Higher Schools, Radiophysics) 10, 539 (1967).

[12] V. I. Bespalov and V. I. Talanov, ZhETF Pis. Red. 3, 47 (1966) [JETP Lett. 3, 28 (1966)].

[13] B. N. Gershman and V. Yu. Trakhtengerts, Usp. Fiz. Nauk 89, 201 (1966) [Sov. Phys.-Usp. 9, 414 (1966)].

Translated by H. Lashinsky
71