CASCADE IONIZATION OF A GAS DURING OPTICAL BREAKDOWN IN A WIDE RANGE OF RADIATION FLUXES

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The problem of cascade ionization of gases during optical breakdown is studied theoretically in a broad range of radiation flux densities. A universal dependence of the cascade-development parameter on flux density is derived. It is shown that the cascade-development constant goes through a maximum. A family of energy distribution functions for the cascade electrons is found by numerical integration.

1. The theory of optical breakdown of gas under the influence of laser radiation was considered in[1-3]. It was shown that in the case of light fluxes $q = cE_0^2/8\pi \leq 10^{15}$ W/cm^2 (c—velocity of light, E_0—field amplitude) the breakdown mechanism is cascade ionization.

As shown in[2,3], the characteristic parameter of the theory of cascade ionization is the quantity $\beta_0 \approx \gamma_{in} I_i/\alpha$, where γ_{in} is the frequency of the inelastic collisions of the electron with the neutral atom, I_i is the ionization potential of the gas atoms, $\alpha = \frac{1}{3}\epsilon_0 \nu_{eff}$, ν_{eff} is the frequency of the elastic collisions of the electrons with the atoms, and $\epsilon_0 = e^2E_0^2/2m\omega^2$ is the effective energy of the oscillations of an electron with charge and mass m in the light-wave field with frequency ω. The case $\beta_0 = \infty$, which is considered in[1], corresponds to the condition of relatively small rate of acquisition of energy by the electron in the field wave, with subsequent instantaneous ionization of the atom by an electron, reaching the energy $\sim I_i$, and is realized in experiments with giant laser pulses of duration $\sim 10^{-8}$ sec. According to[1], the number of electrons n in the cascade grows exponentially, $n \sim e^{\gamma t}$, with a cascade-development constant γ proportional to the radiation flux density q.

In another limiting case $\beta_0 \to 0$, which is considered in[2,3], γ is a decreasing function of the flux density q, and the electron concentration at a fixed value of the time decreases with increasing field intensity.

In this paper we develop a theory of optical breakdown of gas for an arbitrary value of β_0. By solving the kinetic equation, we obtain a family of cascade-electron distribution functions with respect to the energies, $f_{\beta_0}(\epsilon, t)$, and construct a universal plot of the cascade development constant $\gamma = \gamma(\beta_0)$. The function $\gamma = \gamma(\beta_0)$ in the limiting cases $\beta_0 \to \infty$ and $\beta_0 \to 0$ coincides, apart from numerical coefficients of the order of unity, with the relations obtained in[1-3], and reaches a maximum at the point $\beta_0 = 0.25$.

2. As usual, we seek the electron energy distribution function $n(\epsilon, t)$ in the form

$$n(\varepsilon, t) = e^{\gamma t} f(\varepsilon). \tag{1}$$

Then the kinetic equation for the function $f(\epsilon)$ is given by[1,3]

$$\frac{d}{d\varepsilon}\left(\alpha f - 2\alpha\varepsilon \frac{df}{d\varepsilon}\right) + \gamma f = \left(\frac{\partial f}{\partial t}\right)_{\text{col}}. \tag{2}$$

The collision term $(\partial f/\partial t)_{col}$, connected with the inelastic losses, can be represented in the general case in the form

$$\left(\frac{\partial f}{\partial t}\right)_{\text{col}} = N_0 \int d\sigma_{in}(\varepsilon, \varepsilon')\left[f(\varepsilon + \varepsilon') v(\varepsilon + \varepsilon') \times \frac{\partial \sigma_{in}(\varepsilon + \varepsilon', \varepsilon')}{d\sigma_{in}(\varepsilon, \varepsilon')} - f(\varepsilon) v(\varepsilon) \right], \tag{3}$$

where N_0 is the density of the neutral atoms, $d\sigma_{in}$ is the differential cross section for excitation in ionization of the atom by an electron having a velocity $v = (2\epsilon/m)^{1/2}$, and ϵ' is the energy transferred to the atomic electron.

We assume further that in each inelastic collision between the electron and the atom a constant energy I is transferred to the atomic electron; this energy is different for each atom. Such an assumption for primary electrons with energy exceeding about two ionization potentials I_i is certainly valid[4] (in this case $I \approx I_i$). Therefore, at least in the indicated energy region, we can put

$$d\sigma_{in}(\varepsilon, \varepsilon') = \frac{\varkappa(\varepsilon)}{I} \delta(\varepsilon' - I)\, d\varepsilon', \tag{4}$$

where $\kappa(\epsilon)$ is the effective deceleration[5]. The quantity is calculated in the Born approximation, which is almost exact at these energies; on the other hand, at low energies ($\epsilon \leq 2I_i$) the Born approximation is reasonable for the cross sections of the inelastic processes, since the error resulting from this approximation is $\sim 50\%$ [4]. In addition, the cross sections obtained in the Born approximation are universal, unlike other corresponding approximations, which give higher accuracy only for individual concrete cases. We therefore extend formula (4) to the entire energy range, assuming for $\kappa(\epsilon)$ an expression in the form

$$\varkappa(\varepsilon) = \frac{C}{\varepsilon} \ln \frac{\varepsilon}{I}, \tag{5}$$

where C is a constant.

Expression (5) differs somewhat from the known formula[5] by a factor on the order of unity preceding the logarithm sign; the assumption that the energy lost by the primary electron is constant in the region $\epsilon \sim 2I_i$ introduces the same error as is produced by the Born approximation (5) for $\kappa(\epsilon)$. On the other hand, from (4) and (5) we get for the total cross section $\sigma_{in}(\epsilon)$ of the inelastic collisions of the electron with the atom

$$\sigma_{in}(\varepsilon)=\int d\sigma_{in}(\varepsilon,\varepsilon')=\frac{\varkappa(\varepsilon)}{I}\sim\frac{1}{\varepsilon}\ln\frac{\varepsilon}{I}, \tag{6}$$

i.e., expression (6) yields for $\sigma_{in}(\epsilon)$ the correct energy dependence in the Born region, and vanishes when $\epsilon = I$ ($\sigma_{in}(\epsilon) \sim (\epsilon - I)$ when $\epsilon \sim I$); the quantity I now has the meaning of the effective threshold for the inelastic processes. Substituting then (4) and (3), we get

$$\left(\frac{\partial f}{\partial t}\right)_{\rm col}=\frac{N_0}{I}\left[f(\varepsilon+I)v(\varepsilon+I)\varkappa(\varepsilon+I)-f(\varepsilon)v(\varepsilon)\varkappa(\varepsilon)\right]. \tag{7}$$

It is easy to show that when $\epsilon \gg I$ expression (7) goes over into the formula for the collision term obtained in[3] by expanding the integral (3) in powers of ϵ'/ϵ.

For the subsequent calculations, it is convenient to represent the collision term in the form

$$\left(\frac{\partial f}{\partial t}\right)_{\rm col}=\frac{e\gamma_m}{2}\left[f(\varepsilon+I)\left(\frac{I}{\varepsilon+I}\right)^{1/2}\ln\frac{\varepsilon+I}{I}-f(\varepsilon)\left(\frac{I}{\varepsilon}\right)^{1/2}\ln\frac{\varepsilon}{I}\right], \tag{8}$$

where $\gamma_m = 2CN_0/eI^{3/2}$ is the maximum frequency of the inelastic collisions, i.e., the maximum value of the function $\gamma_{in} = N_0 I^{-1} v(\epsilon)\kappa(\epsilon)$. Thus, the kinetic equation at $\nu_{\rm eff} = N_0\sigma_{tr}(\epsilon)v(\epsilon) = {\rm const}$, where $\sigma_{tr}(\epsilon)$ is the transport cross section for the elastic scattering of an electron by an atom, is given by

$$\alpha\varepsilon\frac{d^2f}{d\varepsilon^2}+\alpha\frac{df}{d\varepsilon}-\gamma f+\frac{e\gamma_m}{2}\left[f(\varepsilon+I)\left(\frac{I}{\varepsilon+I}\right)^{1/2}\times\ln\frac{\varepsilon+I}{I}-f(\varepsilon)\left(\frac{I}{\varepsilon}\right)^{1/2}\ln\frac{\varepsilon}{I}\right]=0. \tag{9}$$

The undetermined constants contained in the solution of (9) are determined from the condition $f(\infty) = 0$ and the normalization condition. The cascade development constant γ is determined from the relation[3]

$$\gamma=\int_I^\infty f(\varepsilon)\gamma_i(\varepsilon)\,d\varepsilon\Big/\int_0^\infty f(\varepsilon)\,d\varepsilon, \tag{10}$$

where $\gamma_i(\epsilon)$ is the frequency of the ionization-producing inelastic collisions of the electron with the atoms. It is physically clear that $\gamma_i(\epsilon) = \gamma_{in}(\epsilon)$ in the region of large flux densities, for in this case the atoms excited by electron impact are instantaneously ionized by the radiation field. On the other hand, in the flux region at which the multiquantum photoeffect from the excited levels is negligible, we have $\gamma_i(\epsilon) = N_0\sigma_i(\epsilon)v(\epsilon)$, where $\sigma_i(\epsilon)$ is the cross section for the ionization of the atom by electron impact. For the quantity $\sigma_i(\epsilon)$ we can use in this case the expression for the Thomson ionization cross section

$$\sigma_i(\varepsilon)=\frac{C_1}{\varepsilon}\left(\frac{1}{I}-\frac{1}{\varepsilon}\right), \tag{11}$$

$$\gamma_i(\varepsilon)=\frac{2.6\gamma_{mi}}{\sqrt{\varepsilon/I}}\left(1-\frac{I}{\varepsilon}\right), \tag{12}$$

where γ_{mi} is the maximum value of the function $\gamma_i(\epsilon)$. We note that the physical relation (10) denotes equality of the electron flux in energy space of $\epsilon = 0$ to the number of secondary electrons produced in the entire energy space.

Equation (9) becomes universal if one goes over to the dimensionless variable $y = \sqrt{\epsilon/I}$:

$$\frac{d^2f}{dy^2}-\beta f+\beta_0\left[f(y^2+1)\frac{\ln(y^2+1)}{(y^2+1)^{1/2}}-f(y^2)\frac{\ln y^2}{y}\right]=0,$$
$$\beta_0=\frac{e\gamma_m I}{\alpha},\quad \beta=\frac{2I\gamma}{\alpha}. \tag{13}$$

In the case of large and small fluxes, respectively, relation (10) assumes the form

$$\beta=2\beta_0\int_1^\infty f(y)\ln y\,dy\Big/\int_0^\infty f(y)\,y\,dy, \tag{14}$$

$$\beta=2\Delta\beta_0\int_1^\infty f(y)\,(1-y^{-2})\,dy\Big/\int_0^\infty f(y)\,y\,dy, \tag{15}$$

where $\Delta = \gamma_{mi}/\gamma_m$.

Equation (13) was integrated numerically together with relation (14) and separately in conjunction with relations (15) at $\Delta = 0.5$[5]. As the result, two universal curves of $\beta(\beta_0)$ were plotted, determining the cascade development constant γ as a function of the radiation field and of the parameters of the medium (Fig. 1), and two families of curves were obtained for the distribution functions $f_{\beta_0}(\epsilon)$ (Fig. 2) corresponding to relations (14) and (15).

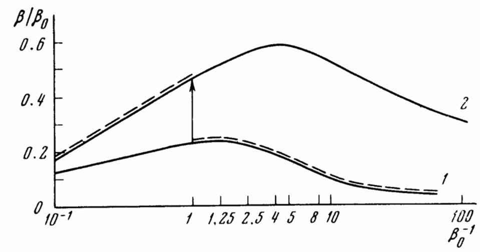

FIG. 1

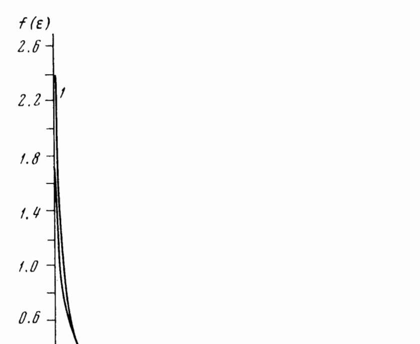

FIG. 2. Plots of the electron energy distribution functions, normalized to unity, for three values of the parameter β_0: 1 – $\beta_0 = 100$, $\langle\epsilon/I\rangle = 0.51$; 2 – $\beta_0 = 20$, $\langle\epsilon/I\rangle = 0.87$; 3 – $\beta_0 = 1$, $\langle\epsilon/I\rangle = 7.27$.

Figure 1 shows plots of $\beta/\beta_0 = f(\beta_0)$ (we note that $\gamma = \gamma_m\beta/\beta_0$), plotted formally in the entire flux region, i.e., in the entire region of the parameter β_0^{-1}. At small radiation fluxes (larger than β_0), the function $\gamma = \gamma_m\beta/\beta_0 = f(\beta_0)$ is determined by curve 1.

As follows from the foregoing, at a certain value of the parameter β_0^* ($\beta_0^* = 1$ in Fig. 1; generally speaking, the value of β_0^* depends on the character of the energy levels of the concrete medium) the function $\gamma = \gamma(\beta_0)$

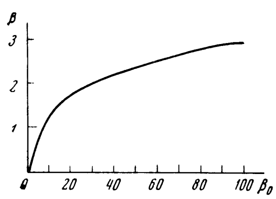

FIG. 3

is already determined by the curve 2. The indicated "transition" (shown dashed in Fig. 1) is quite abrupt, since multiphoton ionization is a threshold process.

As follows from Fig. 1, when $\beta_0 = 0.25$, corresponding to fluxes $q = 2.3(\gamma_m/\gamma_{eff}) \times 10^{15}$ W/cm^2, the cascade-development constant reaches a maximum, $\gamma_{max} = 0.58\,\gamma_m$.

Figure 3 shows a plot of $\beta(\beta_0)$ in the region $\beta_0 > 1$. It is seen from Fig. 3 that when $\beta_0 \to \infty$ we get $\beta \to 2.75$. Consequently, in the region of small fluxes we obtain

$$\gamma = 1.37\alpha / I,$$

i.e., the cascade-development constant γ turns out to be proportional to the flux density of the instant radiation. Formula (3), apart from a constant factor of the order of unity, coincides with the result obtained in[1]. In the opposite limiting case, $\beta_0 \to 0$, the cascade development constant decreases, corresponding to the results of[3].

Figure 2 shows plots of the electron energy distribution functions, normalized to unity, for three values of the parameter β_0, and the respective values of the average electron energy are indicated.

The region of applicability of the theory developed here is limited on the high radiation-flux side by the multiphoton processes, and has been estimated in[3].

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