ANGULAR DEPENDENCE OF THE AMPLITUDE OF GIANT SOUND-ABSORPTION OSCILLATIONS

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Submitted February 17, 1969

Zh. Eksp. Teor. Fiz. 57, 551–559 (August, 1969)

The dependence of the amplitude of giant oscillations of sound absorption of semimetals on the angle between the direction of propagation of sound and the magnetic field has been studied. For angles which are not close to right angles, the principal role is played by collision-free sound absorption by the conduction electrons. In this region, the amplitude of the oscillations changes inversely with the cosine of the angle between the direction of propagation of sound and the magnetic field. For angles that are close to right, the indeterminacy in the law of conservation of energy becomes significant. This is associated with the scattering of electrons, which leads to a decrease in the amplitude of the giant oscillations relative to the smooth part of the absorption. At a definite value of the angle, which depends on the frequency of collisions of the electrons with the scatterers, the amplitude of the giant oscillations reaches a sharp maximum, and this amplitude can significantly exceed the amplitude of oscillations in sound propagation of the magnetic field. A similar effect can be used conveniently for an estimate of the length of the electron free path in crystals.

GUREVICH, Skobov, and Firsov[1] investigated the deformation sound absorption in semimetals in the case of a strong magnetic field ($\hbar\Omega \gg T$) and a large free path length of the conduction electrons L ($\Omega = eH/mc$ is the cyclotron frequency, T the temperature in energy units). It has been shown that the coefficient of sound absorption can experience giant oscillations with change in the magnetic field. The present research is devoted to the study of the dependence of the amplitude of the giant oscillations on the angle e between the directions of the sound wave vector κ and the intensity of the magnetic field H.

The reason for the appearance of giant oscillations is the following.[1] For $\hbar\Omega \gg T$, in the region of diffuseness of the Fermi surface, which is responsible for kinetic phenomena, only a narrow range of values of k_z are allowed ($\hbar k_z$ is the projection of the quasimomentum of the electron in the direction of the magnetic field H), the direction of which changes with change in the magnetic field.

On the other hand, if the free path length $L = v_F/\nu$ (v_F is the velocity of the electron on the Fermi surface, ν the collision frequency of electrons with scatterers) of the conduction electrons is much greater than the sound wavelength ($\kappa L \gg 1$), then the principal role is played by direct, collision-free absorption of sound by the electrons. In this case the laws of conservation of energy and momentum,

$$E_{n+l}(k_z + \varkappa_z) - E_n(k_z) = \hbar\omega_q \qquad (1)$$

(ω_q is the sound frequency) distinguish the values of k_z^l of the electrons that can take part in sound absorption. If each of these values lies in the allowed range of values of k_z on the Fermi surface, then a strong sound absorption takes place—a giant oscillation, the amplitude of which may easily be seen to be proportional to

$$\{\partial[E_{n+l}(k_z + \varkappa_z) - E_n(k_z)]/\partial k_z\}^{-1} \propto (\cos\theta)^{-1}.$$

In the contrary case, direct absorption of sound is absent.

As the angle θ approaches $\pi/2$ ($\kappa_z \to 0$) the laws of conservation of energy and momentum (1) cannot be satisfied exactly. In this case, there is a considerable indeterminacy in the law of energy conservation, brought about by the electron scattering. This indeterminacy leads to the result that there are electrons responsible for the absorption which have all the values of quasimomentum from 0 to p_F, which leads at $\theta = \pi/2$ to the disappearance of the giant oscillations. Only small oscillations remain (of the de Haas–van Alphen type), and their amplitude is inversely proportional to the number of occupied Landau levels, i.e., $\sqrt{\zeta/\hbar\Omega}$. Evidently, the amplitude of the giant oscillations should reach a maximum for some value of θ. A similar characteristic dependence of the amplitude of the oscillations on the angle θ has been observed experimentally by Toxen and Tansal in Bi.[2]

In analogy with the research of Skobov,[3] who studied the effect of electron scattering on sound absorption for $\kappa \parallel H$, we consider the case in which the transverse electric fields that appear upon deformation of the crystal do not play significant roles and can be neglected. We shall assume that the scattering of the electrons is due to interaction with randomly distributed, fixed centers, the radius of action of which is small in comparison with the mean distance between them and with the wavelength of the electron. For simplicity, we limit ourselves to the case of isotropic quadratic spectrum of electrons and do not take their spin into account.[1)]

Let the Hamiltonian of the electrons in the absence of sound be $\mathcal{H} = \mathcal{H}_0 + V$, where

[1)] In the absence of transitions with reversal of spin, allowance for the spin reduces to replacing ζ by $\zeta \pm \mu H$, where μ is the effective magneton. Allowance for anisotropy in the case of a quadratic spectrum does not present any great difficulty.

$$\mathcal{H}_0=\frac{1}{2m}\left(\hat{\mathbf{p}}-\frac{e}{c}\mathbf{A}\right)^2, \quad V=\sum_j V_j\equiv\sum_j V(\mathbf{r}-\mathbf{r}_j). \tag{2}$$

Here **A** is the vector potential of the magnetic field, $\hat{\mathbf{p}}$ the electron quasimomentum operator, m the effective mass of the electron, and V_j the interaction potential of the electron with a scattering center located at the point $\mathbf{r}_j$.

The perturbation produced by the interaction of the electron with the sound is written in the form

$$\mathcal{H}'(t)=\tfrac{1}{2}(U_e^{-i\omega t}+U^+e^{i\omega t}), \qquad U=U_0e^{i\varkappa r}, \tag{3}$$

where U_0 is the Hermitian operator, which generally depends on the quasimomentum of the electron. In the case of deformation absorption,

$$\mathcal{H}'(t)=\Lambda_{ik}u_{ik}, \tag{4}$$

where $u_{ik}=\frac{1}{2}(\partial u_i/\partial x_k+\partial u_k/\partial x_i)$ is the deformation tensor of the crystal, and Λ_{ik} the effective tensor of the deformation potential. For semimetals of the type of bismuth, the constant-energy surface for which consists of non-intersecting ellipsoids, the tensor Λ_{ik} can be regarded as constant. We note that the induced absorption under the conditions considered is small in comparison with the deformation absorption.

The energy absorbed by an electron per unit time is determined, in first approximation in $\mathcal{H}'$, by the expression[4]

$$Q=\frac{\pi\omega}{2}\sum_{a,b}\langle|U_{ba}|^2\delta(\hbar\omega_{ab}+\hbar\omega)[f_a(1-f_b)-f_b(1-f_a)]\rangle. \tag{5}$$

Here a and b characterize the stationary states of the Hamiltonian $\mathcal{H}$, $\hbar\omega_{ab}=E_a-E_b$ is the energy difference and f_a the distribution function of the electrons. Since the scattering of the electrons by the fixed centers is elastic, f_a is simply the Fermi function of argument $(E_a-\zeta)/T$. The angle brackets here and below denote averaging over the position of the scattering centers $\mathbf{r}_j$.

Proceeding as in[3], we represent Eq. (5) in the form

$$Q=\frac{\pi\omega}{2}\int_{\hbar\Omega/2}^{\infty}dE\,[f(E)-f(E+\hbar\omega)]\,I(E), \tag{6}$$

$$I(E)=\sum_{a,b}\langle|U_{ba}|^2\delta(E-E_b+\hbar\omega)\,\delta(E-E_a)\rangle, \tag{7}$$

and then, with the help of the identity

$$\delta(x)=\frac{1}{2\pi i}\left(\frac{1}{x-i\varepsilon}-\frac{1}{x+i\varepsilon}\right)_{\varepsilon\to+0}$$

in the form of a combination of four terms of the type

$$\frac{1}{(2\pi)^2}\mathrm{Sp}\left\langle\left\{\frac{1}{E-\mathcal{H}\pm i\varepsilon}U\frac{1}{E+\hbar\omega-\mathcal{H}\pm i\varepsilon}U\right\}\right\rangle, \tag{8}$$

which, just as in[3], we shall compute in the representation of the unperturbed Hamiltonian $\mathcal{H}_0$, the eigenfunctions and eigenvalues of which, when the vector potential is chosen in the form A = (0, Hx, 0), are given by the following expressions:[5]

$$\psi_\alpha=\varphi_n(x-x_\alpha)\exp[iky+ikz],$$
$$E_\alpha=\hbar\Omega\left(n+\frac{1}{2}\right)+\frac{\hbar^2k_z^2}{2m}\equiv E_n+\frac{\hbar^2k_z^2}{2m}, \tag{9}$$

where φ_n is the normalized wave function of the harmonic oscillator in state n, $x_\alpha=-a_H^2k_y$ plays the role of the center of the oscillator, and $a_H=(c\hbar/eH)^{1/2}$ is the so-called Landau oscillator length. The set of Landau quantum numbers (n, k_y, k_z) below will be denoted by the Greek letters (α, β, etc.).

To calculate (8), we must know the Green's function averaged over the positions of the scatterers:

$$G_{\beta\alpha}(u)=\left\langle\left(\beta\left|\frac{1}{u-\mathcal{H}}\right|\alpha\right)\right\rangle. \tag{10}$$

It has been shown by Skobov[6,7] that G(u) can be described in the form of a series

$$G(u)=\frac{1}{u-\mathcal{H}_0}+\sum_j\frac{1}{u-\mathcal{H}_0}\langle T^j(u)\rangle\frac{1}{u-\mathcal{H}_0}$$
$$+\sum_{j\neq h}\frac{1}{u-\mathcal{H}_0}\langle T^j(u)\frac{1}{u-\mathcal{H}_0}T^h(u)\rangle\frac{1}{u-\mathcal{H}_0}+\ldots \tag{11}$$

$$T_{\beta\alpha}{}^j(E+i\varepsilon)=\psi_\beta^*(\mathbf{r}_j)\psi_\alpha(\mathbf{r}_j)\frac{2\pi f\hbar^2}{m}\frac{1}{1+iK(E)f} \tag{12}$$

Here f is the scattering amplitude of the free electron of zero energy by a potential V_j,

$$K(E)=\sum_{n=0}^{N+1}\frac{\hbar}{(2m)^{1/2}a_H{}^2}(E-E_n)^{-1/2}, \tag{13}$$

where N is an integer such that one imaginary term remains in the sum (13). The quantity $T^j_{\beta\alpha}(E+i\epsilon)$ is the amplitude of transition from state α to β in scattering from the j-th center.

Summation of the form (11) under the conditions $\hbar\Omega\gg T$, $f\ll\lambda$, $f\ll\bar{a}$ (λ is the wavelength of the electron, $\bar{a}$ the mean distance between the scatterers) gives

$$G_{\beta\alpha}(E+i\varepsilon)=\delta_{\alpha\beta}[E-E_\alpha+i\hbar\nu(E)/2]^{-1}, \tag{14}$$

where $\nu(E)$ is satisfied by the equation

$$\nu(E)=\nu_0(E)\,(2mE)^{-1/2}\mathrm{Re}\,K(E+i\hbar\nu(E)/2). \tag{15}$$

The quantity $\nu_0(E)=4\pi f^2n_s(2E/m)^{1/2}$ is the scattering probability in the absence of a magnetic field.

If the condition

$$\hbar\Omega\ll(\zeta\hbar\nu_0)^{1/2}, \tag{16}$$

is satisfied, then $\nu(E)\sim\nu_0(E)$ for all energies. In the opposite case, as $E\to\hbar\Omega(n+\frac{1}{2})$, an increase of $\nu(E)$ takes place.[8] In the case $\hbar\nu_0\ll T$, the condition (16) can be replaced by the less stringent one

$$\hbar\Omega\ll(\zeta T)^{1/2}, \tag{17}$$

which is easily satisfied in semimetals. This is connected with the fact that the increase of $\nu(E)$ over energy ranges that are small in comparison with T leads to finite results because of averaging over the Fermi distribution.

For what follows, it is convenient to introduce a diagram technique, analogous to that employed in[8,9]. We associate the matrix element of the operator $(u-\mathcal{H}_0)^{-1}_{\gamma\delta}$ with a thin solid line with indices $\gamma\delta$ at the ends, and the transition amplitude $T^j_{\beta\alpha}$ with a cross with index j. The crosses that correspond to scattering by a given impurity are connected by dashed lines. In diagrams of any order, there should not be two or more crosses in a row corresponding to scattering from the same center, since we have already associated the exact transition amplitude with each cross. Summation is assumed over all indices in the intermediate states, and then averaging over the coordinates of the scatterers. We associate the exact Green's function $G_{\beta\alpha}=\langle(\beta|(u-\mathcal{H})^{-1}|\alpha)\rangle$

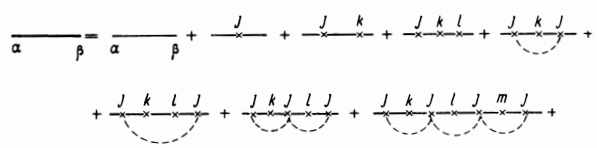

FIG. 1.

with a boldface line. Thus, Eq. (10) can be represented in the form of a sum of diagrams as shown in Fig. 1.

Diagrams in which the dashed lines intersect give a small contribution for $f \ll \lambda$, $f \ll \bar{a}$, $\hbar\Omega \ll \zeta$ and can be neglected.[3]

For the calculation of (8), we introduce two axes. We associate $(E - E_\alpha \pm i\epsilon)^{-1}$ with the thin line on the upper axis and $(E + \hbar\omega - E_\alpha \pm i\epsilon)^{-1}$ with the line on the lower axis. We shall draw the matrix element of the interaction operator U in the form of a wavy line joining the upper and lower axes. The simplest diagram is shown in Fig. 2a. It corresponds to the expression

$$\sum_{\alpha\alpha'} \frac{|U_{\alpha\alpha'}|^2}{(E - E_\alpha \pm i\varepsilon)(E + \hbar\omega - E_{\alpha'} \pm i\varepsilon)}. \tag{18}$$

Summation of all diagrams in which there are no crosses corresponding to the same scatterer on the upper and lower axes reduces to the replacement of the thin lines by the boldface ones, which amounts analytically to replacement of the quantities $(E - E_\alpha \pm i\epsilon)^{-1}$ in (18) by the matrix elements of the exact Green's function, i.e., by $[E - E_\alpha \pm i\hbar\nu(E)/2]^{-1}$.

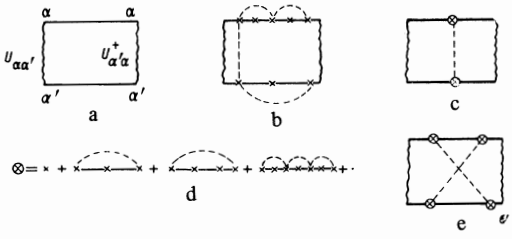

FIG. 2

Let us consider the diagrams which contain multiple scattering by a single impurity both on the upper and the lower axes (Fig. 2b). Summation of diagrams of such a type leads to the diagrams shown in Fig. 2c, where the cross in a circle corresponds to a sum of diagrams (Fig. 2d), calculation of which gives the following analytic expression:[3]

$$\tau_{\beta\alpha}{}^j = \psi_\beta^*(\mathbf{r}_j)\psi_\alpha(\mathbf{r}_j)\frac{2\pi\hbar^2}{m}\left\{1 + ifK\left(E + \frac{i\hbar}{2}\nu(E)\right)\right\}^{-1}. \tag{19}$$

It can be shown that diagrams of the type shown in Fig. 2e make a small contribution in comparison with the diagrams without intersection of the dashed lines.

Thus, for calculation of (8) it is necessary to sum the ladder of diagrams (Fig. 3), which leads to the integral equation

$$|U_0|^2\sum_{\alpha\alpha'}\Pi_{\alpha\alpha'}J_{\alpha'\alpha} = \sum_{\alpha\alpha'}\frac{|U_0|^2|J_{\alpha'\alpha}|^2}{(E - E_\alpha + i\hbar\nu/2)(E + \hbar\omega - E_{\alpha'} + i\hbar\nu/2)}$$
$$+ |U_0|^2\sum_j\sum_{\alpha\alpha'\beta\beta'}\frac{1}{E - E_\alpha + i\hbar\nu/2}\langle\tau_{\alpha\beta}{}^j\Pi_{\beta\beta'}\tau_{\beta'\alpha'}{}^j\rangle\frac{J_{\alpha'\alpha}}{E - E_\alpha + \hbar\omega + i\hbar\nu/2} \tag{20}$$

Here $U_0^*\Pi_{\alpha\alpha'}$ denotes an expression corresponding to

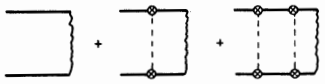

FIG. 3

the sum of the diagrams shown in Fig. 3:

$$J_{\alpha\alpha'} = \int \psi_{\alpha'}^*(\mathbf{r})e^{i\varkappa\mathbf{r}}\psi_\alpha(\mathbf{r})\,d\mathbf{r} = \delta(k_z' - k_z - \varkappa_z)\delta(k_y' - k_y - \varkappa_y)I_{n'n} \tag{21}$$

where

$$I_{n'n} = \int \varphi_{n'}(x - x_{\alpha'})\exp(i\varkappa_x x)\varphi_n(x - x_\alpha)\,dx.$$

We shall set $\kappa_x = 0$ below.

We average Eq. (20) over the coordinates of the centers of the oscillators $x_\alpha = -a_H{}^2k_y$. The first term on the right side of (20) does not depend on x_α, and averaging of the second gives

$$\sum_j\langle\langle\tau_{\alpha\beta}{}^j\Pi_{\beta\beta'}\tau_{\beta'\alpha'}{}^j\rangle\rangle = \left(\frac{2\pi f\hbar^2}{m}\right)^2\left\{\left[1 + ifK\left(E + \frac{i\hbar\nu}{2}\right)\right]\right.$$
$$\left.\times\left[1 + ifK\left(E + \hbar\omega + \frac{i\hbar\nu}{2}\right)\right]\right\}^{-1}\sum_j\langle\langle\psi_\alpha^*(\mathbf{r}_j)\psi_\beta(\mathbf{r}_j)\Pi_{\beta\beta'}\psi_{\beta'}^*\psi_{\alpha'}\rangle\rangle. \tag{22}$$

Since Eq. (22) is multiplied by $J_{\alpha'\alpha} \propto \delta(\mathbf{k}_{\alpha'} - \mathbf{k}_\alpha - \boldsymbol{\kappa})$, we can set $\mathbf{k}_{\alpha'} = \mathbf{k}_\alpha + \boldsymbol{\kappa}$ (here $\mathbf{k}_\alpha$ and $\boldsymbol{\kappa}$ are two-dimensional vectors (k_y, k_z) and (κ_y, κ_z)). After this it is not difficult to set

$$\sum_j\langle\langle\psi_\alpha^*\psi_\beta\Pi_{\beta\beta'}\psi_{\beta'}^*\psi_{\alpha'}\rangle\rangle = n_s J_{\alpha'\alpha}^* J_{\beta'\beta}\Pi_{\beta\beta'}, \tag{23}$$

where n_s is the scatterer concentration. Thus the equation is uncoupled in the indices α and β.

By denoting

$$|U_0|^2\left\langle\sum_{\alpha\alpha'}\Pi_{\alpha\alpha'}J_{\alpha'\alpha}\right\rangle_{x_\alpha} \equiv |U_0|^2\Pi(E),$$

we have

$$\Pi(E) = \sum_{\alpha\alpha'}|J_{\alpha'\alpha}|^2\left(E - E_\alpha + \frac{i\hbar\nu}{2}\right)^{-1}\left(E - E_\alpha + \hbar\omega + \frac{i\hbar\nu}{2}\right)^{-1}$$
$$\times\left\{1 - \left(\frac{2\pi f\hbar^2}{m}\right)^2 n_s\frac{1}{1 + iKf}\frac{1}{1 + iK'f}\right.$$
$$\left.\times\sum_{\alpha\alpha'}|J_{\alpha'\alpha}|^2\frac{1}{E - E_\alpha + i\hbar\nu/2}\frac{1}{E + \hbar\omega - E_{\alpha'} + i\hbar\nu/2}\right\}^{-1}. \tag{24}$$

Let us consider the expression in the curly brackets in (24) in more detail. For $\kappa_z \neq 0$, it is not difficult to show that the addition to unity is a quantity of the order of $(\kappa_z l)^{-1} \ll 1$ and can be neglected. In the case $\kappa_z = 0$, all the terms in the sum entering into the curly bracket in (24) for which $E - E_n \sim \hbar\Omega$, $E - E_{n'} \sim \hbar\Omega$ are quantities of the order of $\hbar\nu_0/\sqrt{\hbar\Omega E} \ll 1$. Consequently, only the term for which $|E - E_n| \sim \hbar\nu \ll T$ can be significant. It can be shown that when the energy approaches the value E_n, the value of this term increases, while in the narrow range $|E - E_n| \sim \hbar\nu \ll T$, it has the order of $(\kappa R)^{-1}(\hbar\Omega/\zeta)^{1/2}$. Outside this region, it is of the order of $\hbar\nu/(\hbar\Omega E)^{1/2} \ll 1$. Since the condition $(\kappa R)^{-1}(\hbar\Omega/\zeta)^{1/2} \ll 1$ is usually satisfied in experiment, the contribution to the curly bracket in (24) is small and can be neglected in the calculation of the smooth part of the absorption. The oscillating part of the absorption coefficient is of the order of $(\hbar\Omega/\zeta)^{1/2}$ relative to the smooth part; there-

fore, this contribution must be taken into account in the calculation of the value of the term at resonance. However, aside from the narrow region ($\Delta E \sim \hbar\nu \ll T$) near resonance, it can be neglected even in the calculation of the oscillating part.

Under these assumptions, we obtain

$$I(E)=\frac{|U_0|^2}{(2\pi)^2}\sum_{\alpha\alpha'}|J_{\alpha'\alpha}|^2\frac{\hbar\nu}{(E-E_\alpha)^2+(\hbar\nu)^2/4}\frac{\hbar\nu}{(E+\hbar\omega-E_{\alpha'})^2+(\hbar\nu)^2/4}. \tag{25}$$

We investigate this expression for small κ_z. Setting $\hbar^2\kappa_z^2/2m \ll \hbar\nu$ and $E = E_n + \Delta$, and carrying out integration over k_z and summation over the coordinates of the centers of the oscillators, we obtain the result that the principal contributions are made only by the terms of the sum (25) with identical oscillator quantum numbers. With account of this, we have

$$I(E)\propto\sum_n\frac{1}{\sqrt{E-E_n}}\left|I_{nn}\left(\frac{\kappa_\perp^2 a_H^2}{2}\right)\right|^2\nu(E)$$
$$\times\left\{\left[\left(\frac{2\kappa_z}{\sqrt{2m}}\sqrt{E-E_n}+\omega\right)^2+\frac{\nu^2}{4}\right]^{-1}+\left[\left(\frac{2\kappa_z}{\sqrt{2m}}\sqrt{E-E_n}-\omega\right)^2+\frac{\nu^2}{4}\right]^{-1}\right. \tag{26}$$

Using the explicit form of the Fermi distribution function, it is easy to establish the fact that the absorption coefficient is proportional to the integral of the product of two functions

$$\Gamma\propto\int_0^\infty\frac{dy}{1+(By-\omega/v_0)^2}\sum_n\mathrm{ch}^{-2}\left(\frac{y^2-A_n}{2}\right), \tag{27}$$

where

$$B=\left(\frac{2T}{m}\right)^{1/2}\frac{\kappa\cos\theta}{v_0},\qquad A_n=\frac{\zeta-\hbar\Omega(n+1/2)}{T}.$$

The integrand in (27) is itself the product of two rapidly varying functions:

$$\left[1+\left(By-\frac{\omega}{v_0}\right)^2\right]^{-1},\qquad \sum_n\mathrm{ch}^{-2}\frac{y^2-A_n}{2},$$

which give a set of peaks of unit height. If the condition

$$\cos\theta\gg(\zeta/\hbar\Omega)^{1/2}(\kappa L)^{-1}, \tag{28}$$

is satisfied,[2)] the distance between the maxima of the second function is seen to be much larger than the width of the maximum of the first. In this case, there is either only one group of electrons (with definite k_z) on the Fermi surface which satisfies the laws of conservation of energy and momentum, or there are no such electrons. Thus giant oscillations are possible and one "resonant" group of electrons is responsible for the absorption in this case.

In the case when the angle θ is close to $\pi/2$, one can obtain the following interpolation formula for the contribution of the "resonant" group of electrons at the maximum:[3)]

$$\Delta\Gamma_{res}^{max}=\Gamma_0\left|I_{NN}\left(\frac{a_H^2\kappa_\perp^2}{2}\right)\right|^2\frac{\cos\theta}{\cos^2\theta+mwv_0/\kappa T}\frac{\hbar\Omega}{4T}. \tag{29}$$

Here Γ_0 is the value of the sound absorption coefficient obtained from classical theory for $\kappa \parallel H$, w is the speed of sound. The contribution of the "nonresonant" groups of electrons is a smooth function of the magnetic field plus small oscillations (of the de Haas–van Alphen type), brought about by oscillations in the density of state in the magnetic field.

From the expression (29), it is easy to see that the amplitude of the giant oscillations is a rapidly varying function of the angle θ and for

$$\cos\theta=(mwv_0/\kappa T)^{1/2} \tag{30}$$

it reaches a maximum whose value is

$$\Delta\Gamma_{max}^{max}=\Gamma_0\left|I_{NN}\left(\frac{a_H^2\kappa_\perp^2}{2}\right)\right|^2\frac{\Omega}{8v_0}\frac{\hbar\kappa}{mw}, \tag{31}$$

and the effective width is of the order

$$\Delta(\cos\theta)\sim(mwv_0/\kappa T)^{1/2}. \tag{32}$$

In the subsequent approach of the angle θ to $\pi/2$ the contribution of the "resonant" group of electrons, and with it the amplitude of giant oscillations, decreases and as $\theta \to \pi/2$ becomes less than the nonresonant part.

We note that the condition (30) is convenient to use for estimate of the relaxation time of the electrons.

We present the results for the absorption coefficient in the case in which the condition

$$\cos\theta \ll w/v_F \tag{33}$$

holds and the absorption is basically determined by the non-resonant groups of electrons. Using the asymptotic expression for the matrix element

$$|I_{nn}(x)|^2=\frac{1}{\pi}\frac{1}{\sqrt{nx}}\cos^2\left(\sqrt{nx}-\frac{\pi}{4}\right), \tag{34}$$

and also the formula for Poisson summation, it is not difficult to obtain the following expression:

A. For $\kappa R \gtrsim 1$ (R is the Larmor radius)

$$\Gamma=\Gamma_1+\Gamma_2, \tag{35}$$

where

$$\Gamma_1=\frac{m^2|\Lambda_{ik}\kappa_i h_k|^2\Omega}{2\pi^2\rho h^3 w\kappa v_0}\frac{1}{1+(\omega/v_0)^2}[1+N_0(\kappa R)], \tag{36}$$

coincides with the value calculated in classical strong magnetic fields if $\kappa \perp H$.[11] This part of the absorption coefficient describes the oscillating behavior of the matrix elements in a magnetic field (the so-called oscillations of geometric resonance).[4)]

For Γ_2 it is not difficult to obtain the following result, which describes the quantum oscillations of the absorption coefficient:

$$\Gamma_2=\frac{m^2|\Lambda_{ik}\kappa_i h_k|^2\Omega v_0}{\pi^2\rho h^3 w\kappa(\omega^2+v_0^2)}\left(\frac{\hbar\Omega}{\zeta}\right)^{1/2}\sum_{k=1}^{\infty}\left\{J_0\left(\pi k\frac{\zeta}{\hbar\Omega}\right)\cos\frac{\pi k\zeta}{\hbar\Omega}\right.$$
$$\left.+\frac{1}{\sqrt{2k}}\sin(\kappa R)\cos\left(\frac{2\pi k\zeta}{\hbar\Omega}-\frac{\pi}{4}\right)+\frac{(-1)^{k+t}}{k}\cos\left(\frac{2\pi k\zeta}{\hbar\Omega}-\frac{\pi}{4}\right)\right\}$$
$$\times\frac{2\pi^2 kT}{\hbar\Omega}\mathrm{sh}^{-1}\left(\frac{2\pi^2 kT}{\hbar\Omega}\right). \tag{37}$$

Here t = 0 if $\omega \ll \nu_0$, and t = 1 if $\omega \gg \nu_0$. The third term in the curly brackets is caused by the quantum os-

[2)]Condition (28) is a sufficient condition for the existence of giant oscillations. As Kaner and Skobov have shown, [10] the necessary and sufficient condition has the form $\cos\theta \gg (\zeta T)^{1/2}(\hbar\Omega\mathcal{H}L)^{-1}$. It is obtained from a comparison of the contribution of the "resonance" group of electrons with the contribution of the nonresonant groups.

[3)]Formula (29) is introduced in a fashion similar to formula (4.15) of [10].

[4)]In formulas (36)-(38), ρ is the density of the crystal, h_k the components of the polarization vector of the sound wave, w the speed of sound, $N_0(x)$ the Neumann function, $J_0(x)$ the Bessel function.

cillations of the relaxation time in the magnetic field.

B. For $\kappa R \ll 1$, the oscillations of geometric resonance disappear. The expression

$$\Gamma = \frac{|\Lambda_{ik} \varkappa_i h_k|^2}{2\rho w} \int_0^\infty \frac{\nu(\varepsilon)}{\omega^2 + \nu^2} \bar{\rho}(\varepsilon) \frac{\partial f}{\partial \varepsilon} d\varepsilon, \tag{38}$$

is valid for the absorption coefficient. Here $\bar{\rho}(\epsilon)$ is the density of states in the magnetic field, which has been well studied in a number of works.

In conclusion, I take the occasion to thank V. L. Gurevich for direction of the research and V. G. Skobov for useful discussions.

[1] V. L. Gurevich, V. G. Skobov and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. 40, 3 (1961) [Sov. Phys.-JETP 13, 1 (1961)].

[2] A. M. Toxen and S. Tansal, Phys. Rev. 137, A211 (1965).

[3] V. G. Skobov, Zh. Eksp. Teor. Fiz. 40, 1446 (1961) [Sov. Phys.-JETP 13, 1014 (1961)].

[4] L. D. Landau and E. M. Lifshitz, Elektrodinamika sploshnykh sred (Electrodynamics of Continuous Media) (Gostekhizdat, 1957, p. 451).

[5] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics) (Gostekhizdat, 1948, p. 534).

[6] V. G. Skobov, Zh. Eksp. Teor. Fiz. 37, 1467 (1959) [Sov. Phys.-JETP 10, 1039 (1960)].

[7] V. G. Skobov, Zh. Eksp. Teor. Fiz. 38, 1304 (1960) [Sov. Phys.-JETP 11, 941 (1960)].

[8] Yu. A. Bychkov, Zh. Eksp. Teor. Fiz. 39, 1401 (1960) [Sov. Phys.-JETP 12, 971 (1961)].

[9] A. A. Abrikosov, L. P. Gor'kov and I. E. Dzyaloshinskii (Metody kvantovoi teorii polya v statisticheskoi fizike (Methods of Quantum Field Theory in Statistical Physics) (Fizmatgiz, 1962, p. 425).

[10] E. A. Kaner and V. G. Skobov, Zh. Eksp. Teor. Fiz. 53, 375 (1967) [Sov. Phys.-JETP 26, 251 (1968)].

[11] V. L. Gurevich, Zh. Eksp. Teor. Fiz. 37, 71 (1959) [Sov. Phys.-JETP 10, 51 (1960)].

Translated by R. T. Beyer
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