SOME FEATURES OF THRESHOLD ABSORPTION OF HIGH-FREQUENCY MAGNETIC FIELD BY A UNIAXIAL ANTIFERROMAGNET

L. D. FILATOVA and V. M. TSUKERNIK

Physico-technical Institute, Ukrainian Academy of Sciences

Submitted February 7, 1969

Zh. Eksp. Teor. Fiz. 57, 498–502 (August, 1969)

Absorption of the energy of a high-frequency magnetic field by a uniaxial antiferromagnet is considered for the case when the stationary field and high-frequency magnetic field are perpendicular to the easy axis. It is shown that in the harmonic approximation the absorption consists of a resonant part and a threshold part. If the stationary magnetic field strength is less than a certain critical value, the absorption coefficient tends to infinity on the right-hand boundary of the absorption band, and this anomalous behavior is related to the properties of the spin-wave dispersion law.

IF a constant magnetic field and a high-frequency magnetic field are directed along the easy axis of an antiferromagnet, then the absorption in such a system has a non-resonant character. In particular, when $\omega\tau_S \gg 1$ (τ_S is the smallest relaxation time in the spin-wave system and ω is the frequency of the alternating field), the absorption is connected with the decay of a quantum $\hbar\omega$ into two non-interacting spin waves, and this mechanism is a threshold mechanism, i.e., it contributes only in a definite frequency interval. On the boundaries of this interval, the absorption coefficient vanishes, and its derivative with respect to the frequency has a root singularity (see, for example,[1]). The situation is different if the constant and high-frequency fields are perpendicular to the easy axis.

In the present paper we find the high-frequency magnetic susceptibility for this case, and show that when $\omega\tau_S \gg 1$ the absorption of the high-frequency field consists of a resonant component and a threshold component, the resonant frequency being located to the left of the threshold-absorption band.

If the constant magnetic field H exceeds a certain critical field H_{cr}, then the threshold absorption does not differ qualitatively from the one mentioned above[1]. On the other hand, if $H < H_{cr}$, then it turns out that the absorption coefficient vanishes as before on the left boundary, increases monotonically, and then tends to infinity on the right boundary of the absorption band. Such an anomalous behavior of the absorption is connected, as will be shown below, with the properties of the dispersion law of the spin waves as a function of the constant magnetic field.

1. The Hamiltonian of the system considered by us has the following form:

$$\mathcal{H} = J\sum_{l,m}\mathbf{S}_{l1}\mathbf{S}_{m2} - \alpha\sum_{l}(S_{l1}^{z^2}+S_{l2}^{z^2}) - \mu H\sum_{l}(S_{l1}^{y}+S_{l2}^{y}). \qquad (1)$$

Here J is the constant of exchange interaction between the nearest neighboring atoms in the antiferromagnet, $\alpha > 0$ is the anisotropy constant, μ is the magneton, and H is the external magnetic field, directed along the y axis. The indices 1 and 2 number the magnetic sublattices, the z axis coincides with the easiest mechnetization axis, and $\mathbf{S}_{lj}$ (j = 1, 2) is the spin-vector operator in the l-th site.

Employing, as usual, the quasiclassical analysis, we can easily find the ground state of the antiferromagnet with Hamiltonian (1). It is characterized by the angles θ_1 and θ_2 of the moments of the first and second sublattices respectively with the easiest magnetization axis. As the result of minimization of the energy with respect to these angles, we find that

$$\sin\theta_1 \equiv \sin\theta = H/H_A, \quad H \leqslant H_A, \qquad (2)$$

and $\theta_2 = \pi - \theta$. The field H_A for a simple cubic lattice is determined by the equation $\mu H_A = 12\,JS + 2\alpha S$. If $H \geq H_A$, both angles are equal to $\pi/2$.

Starting from the obtained ground state, we express the Hamiltonian (1) in terms of the creation and annihilation operators, accurate to the quadratic terms in these operators[2]. As the result, the Hamiltonian (1) will contain, besides the energy of the ground state, also terms of second and higher orders in the Bose operators. The quadratic part of the Hamiltonian is diagonalized, as the result of which we obtain

$$\mathcal{H} = \mathcal{H}_0 + \sum_{\mathbf{k}}(\varepsilon_{\mathbf{k}1}a_{\mathbf{k}1}^{+}a_{\mathbf{k}1} + \varepsilon_{\mathbf{k}2}a_{\mathbf{k}2}^{+}a_{\mathbf{k}2}) + \mathcal{H}', \qquad (3)$$

where $\mathcal{H}_0$ is the energy of the ground state and $\mathcal{H}'$ contains terms of third and higher orders in the operators $a_{\mathbf{k}}^{+}$ and $a_{\mathbf{k}}$. The energies of $\epsilon_{\mathbf{k}1}$ and $\epsilon_{\mathbf{k}2}$ of the two branches of the spin-wave spectrum are determined in our case of a primitive cubic lattice by the equation

$$\begin{aligned}\varepsilon_{\mathbf{k}1}^2 &= (A - B_{\mathbf{k}})^2 - (C - D_{\mathbf{k}})^2,\\ \varepsilon_{\mathbf{k}2}^2 &= (A + B_{\mathbf{k}})^2 - (C + D_{\mathbf{k}})^2,\end{aligned} \qquad (4)$$

where

$$A = \frac{\mu H_A}{2} + \alpha S\cos^2\theta, \quad B_{\mathbf{k}} = 2JS\sin^2\theta\sum_{j=1}^{3}\cos k_j a,$$
$$C = \alpha S\sin^2\theta, \quad D_{\mathbf{k}} = 2JS\cos^2\theta\sum_{j=1}^{3}\cos k_j a. \qquad (5)$$

(the angle θ is determined from (2), k_j is the projection of the wave vector of the spin wave on the crystal axes (j = x, y, z), and a is the crystal-chemical lattice constant.

It is seen from (4) and (5) that the second branch of the spectrum is obtained from the first by making the shift $k_j \to k_j + \pi/a$.

2. The interaction with the alternating magnetic field

$h_t = h_0 e^{-i\omega t}$, polarized along the constant field, is described by the Hamiltonian

$$\mathcal{H}_t = -\mu \sum_l (S_{l1} + S_{l2}^y) h_t. \tag{6}$$

The alternating addition to the projection of the magnetic moment of the antiferromagnet on the y axis is given by[3]

$$\langle \mu S^y(t) \rangle = -\frac{i\mu^2}{\hbar} h_0 e^{-i\omega t} \int_0^\infty e^{i\omega\tau} \operatorname{Sp} \rho_0 [S^y, S^y(\tau)]\, d\tau, \tag{7}$$

where $\rho_0 = z^{-1} e^{-\beta\mathcal{H}}$ is the equilibrium Gibbs distribution for the magnetic system with Hamiltonian (1) (β is the reciprocal temperature), and $S^y(\tau)$ is the operator of the total moment of both sublattices in the interaction representation.

In the high-frequency case considered by us ($\omega\tau_s \gg 1$) the main contribution to the magnetization is made by the terms of the Hamiltonian (3) which do not contain anharmonicities, and the interaction between the spin waves can be taken into account with the aid of perturbation theory.

Confining ourselves to the quadratic terms in the Hamiltonian (3), we obtain after rather cumbersome calculations

$$\langle \mu S^y(t) \rangle = -\frac{\mu^2}{\hbar} h_0 e^{-i\omega t} \Big\{ NS \cos\theta \sqrt{\frac{\alpha}{\alpha + 6J}} \Big(\frac{1}{\omega + \varepsilon_{01}/\hbar + i\nu} - \frac{1}{\omega - \varepsilon_{01}/\hbar + i\nu} \Big) + \frac{\sin^2\theta}{8} \sum_{\mathbf{k}} \Big[\frac{(D_{\mathbf{k}} - C)^2}{\varepsilon_{\mathbf{k}1}^2} (2n_{\mathbf{k}1} + 1) \Big(\frac{1}{\omega + 2\varepsilon_{\mathbf{k}1}/\hbar + i\nu} - \frac{1}{\omega - 2\varepsilon_{\mathbf{k}1}/\hbar + i\nu} \Big) + \frac{(D_{\mathbf{k}} + C)^2}{\varepsilon_{\mathbf{k}2}^2} (2n_{\mathbf{k}2} + 1) \Big(\frac{1}{\omega + 2\varepsilon_{\mathbf{k}2}/\hbar + i\nu} - \frac{1}{\omega - 2\varepsilon_{\mathbf{k}2}/\hbar + i\nu} \Big) \Big] \Big\}, \quad \nu \to +0. \tag{8}$$

The first term in the curly brackets corresponds to antiferromagnetic resonance at the frequency $\epsilon_{01}/\hbar$ ($\epsilon_{01} = \epsilon_{\mathbf{k}1|\,\mathbf{k}=0}$). It should be noted that the frequency connected with the second branch of the spectrum is missing from the resonant term. This is the consequence of the fact that the high-frequency field, which is polarized along the y axis, excites a normal oscillation corresponding to only one natural frequency $\epsilon_{01}/\hbar$. This can be readily verified by considering directly the equations of the homogeneous oscillations of the magnetic moments of the sublattices.

The remaining two terms are due to decay of the photon $\hbar\omega$ into two spin waves with opposite quasimomenta (the photon momentum is assumed to be zero).

3. The imaginary part of the magnetic susceptibility χ'' determines the absorption coefficient of the field $\kappa(\omega)$. Confining ourselves only to the nonresonant part of the absorption, we get from (8)[1)]

$$\varkappa(\omega) = \omega\chi''(\omega) = \frac{\pi\mu^2}{2(2\pi)^3\hbar^2} \operatorname{cth}\frac{\beta\hbar\omega}{4} \frac{\sin^2\theta}{\sqrt{\cos 2\theta\,[\omega_0^2(\theta) - \omega^2]}} \times \Big\{ (D_{\mathbf{k}} - C)^2 \Big|_{x=x_1} \int_{x=x_1} \frac{dS_{\mathbf{k}}}{|\partial x/\partial \mathbf{k}|} + (D_{\mathbf{k}} - C)^2 \Big|_{x=x_2} \int_{x=x_2} \frac{dS_{\mathbf{k}}}{|\partial x/\partial \mathbf{k}|} \Big\}. \tag{9}$$

Here $x = \sum_j \cos k_j$ for a simple cubic lattice, x_1 and x_2 are the roots of the equation $\epsilon_{\mathbf{k}_1} \equiv \epsilon_1(x) = \hbar\omega/2$ and lie in the interval $-3 \le x \le 3$, with

$$\varepsilon_1^2(x) = (\mu H_A/2 + \alpha S \cos 2\theta + 2JSx \cos 2\theta) \Big(\frac{\mu H_A}{2} + \alpha S - 2JSx \Big). \tag{10}$$

the frequency ω_0 is determined by

$$\omega_0^2(\theta) = \frac{(\mu H_A \cos^2\theta + 2\alpha S \cos 2\theta)^2}{\hbar^2 \cos 2\theta}, \tag{11}$$

with $\omega_0^2(\theta) > 0$ only when $\cos 2\theta > 0$.

The integrals in (9) are taken over the equal-energy surfaces $x = x_1$ and $x = x_2$, respectively (we note that $\epsilon_{\mathbf{k}}$ depends on $\mathbf{k}$ only via x).

It is clear that the equation $\epsilon_1(x) = \hbar\omega/2$ has at least one real root in the interval $-3 \le x \le 3$, provided only the frequency satisfies the inequalities

$$2\varepsilon_1^{min} \leqslant \hbar\omega \leqslant 2\varepsilon_1^{max}.$$

It follows therefore that the nonresonant absorption has a threshold character both on the low and on the high frequency sides. It is easy to verify that the lower limiting frequency at any value of the constant field $H \le H_A$ is equal to double the energy of the spin wave with $\mathbf{k} = 0$, i.e.,

$$\hbar\omega_{min} = 2\varepsilon_1(3) = 2\sqrt{2\mu H_A \alpha S} \cos\theta. \tag{12}$$

As to the upper limiting frequency, it is different in different field-value intervals. If $H < H_{cr}$, where H_{cr} is determined from the condition

$$\Big(\frac{H_{cr}}{H_A} \Big)^2 = \frac{6J}{18J + \alpha}, \tag{13}$$

then the upper limiting frequency coincides with the frequency (11), which is double the maximum value of the frequency $\epsilon_1(x)/\hbar$ in the interval $-3 \le x \le 3$. On the other hand, if $H > H_{cr}$, then the maximum of $\epsilon_1(x)/\hbar$ is reached outside this interval. In this case the upper limit of the absorption band is reached on the left edge of the interval and coincides with the frequency

$$2\varepsilon_1(-3) = 2[\mu H_A (\mu H_A \sin^2\theta + 2\alpha S \cos 2\theta)]^{1/2}. \tag{14}$$

4. We now investigate the character of the absorption near the limits of the band.

Let $H > H_{cr}$. Then the equation $\epsilon_1(x) = \hbar\omega/2$ has only one root $x = x_1$. Near the lower limit of the absorption band, i.e., at $(\omega - \omega_{min})/\omega_{min} \ll 1$, we have

$$x_1 = 3 - \frac{\sqrt{\alpha(6J + \alpha)}\hbar(\omega - \omega_{min}) \cos\theta}{2JS(6J\cos^2\theta + \alpha \sin^2\theta)}.$$

In this case the surface $x = x_1$ reduces to a sphere of small radius

$$(ka)^2 = \frac{\sqrt{\alpha(6J + \alpha)}\hbar(\omega - \omega_{min}) \cos\theta}{JS(6J\cos^2\theta + \alpha\sin^2\theta)}.$$

After integration over the sphere, we obtain for the absorption coefficient near the lower limit of the band the following expression:

$$\varkappa(\omega) = \frac{\mu^2 S}{8\pi a^3 \hbar} \frac{\sin^2\theta (6J\cos^2\theta - \alpha\sin^2\theta)^2 [\alpha(6J + \alpha)]^{1/4}}{\sqrt{JS}(6J\cos^2\theta + \alpha\sin^2\theta)^{3/2}} \times \operatorname{cth}\frac{\beta\hbar\omega_{min}}{4} [\hbar(\omega - \omega_{min})\cos\theta]^{1/2}. \tag{15}$$

[1)] Owing to the summation over $\mathbf{k}$, the contribution from the terms contained in the first and second branches of the spin-wave spectrum is the same, since $\epsilon_2(k_x, k_y, k_z) = \epsilon_1(k_x + \pi/a, k_y + \pi/a, k_z + \pi/a)$, and $D(k_x, k_y, k_z) = -D(k_x + \pi/a, k_y + \pi/a, k_z + \pi/a)$. This property holds also for lattices having a different symmetry.

We see therefore that, as usual, the absorption coefficient near the edge of the band tends to zero in proportion to the square root, and the derivative $d\kappa/d\omega$ has a root singularity. A similar result is obtained also near the upper limit of the band, except that the coefficient of $(\omega_{max} - \omega)^{1/2}$ will then be different.

We now consider the field region $H < H_{cr}$. In this case, as already indicated, the upper limiting frequency is the frequency ω_0, defined by expression (11). The equation $\epsilon_1(x) = \hbar\omega/2$ has at $H < H_{cr}$, as before, one root in the frequency interval

$$2\varepsilon_1(3)/\hbar \leqslant \omega \leqslant 2\varepsilon_1(-3)/\hbar$$

and two roots if the frequency lies in the interval

$$2\varepsilon_1(-3)/\hbar \leqslant \omega \leqslant \omega_0.$$

Then, near the lower edge, i.e., near the frequency $2\epsilon_1(3)/\hbar$, expression (15) remains in force for the absorption coefficient.

The situation is different near the upper edge, i.e., near the frequency ω_0. As seen from (9), when $\omega \to \omega_0$ the radical in the denominator tends to zero, whereas the surface integrals remain finite. As a result, if $H < H_{cr}$, then the absorption coefficient increases without limit like $(\omega_0 - \omega)^{1/2}$ as $\omega \to \omega_0$. However, if account is taken of the relaxation mechanisms connected with the anharmonicities in the Hamiltonian (3), then the value obtained for $\kappa(\omega)$ is finite, albeit larger.

If the field H is close to H_{cr}, and the frequency ω close to ω_0, then the surface integrals tend to zero like $(A\Delta\omega + B\Delta H)^{1/4}$, where A and B are constants containing only J, α, and S, $\Delta\omega = \omega - \omega_0$, and $\Delta H = H_{cr} - H$. Since the denominator in (9) tends to zero like $(A\Delta\omega + B\Delta H)^{1/2}$ in this case, the absorption coefficient is proportional to $(A\Delta\omega + B\Delta H)^{-1/4}$.

We note that the lower limit of the absorption band in a field $H \sim H_{cr}$ is equal in order of magnitude to $S\hbar^{-1}\sqrt{J\alpha} \sim 10^{12}$ Hz, and the upper one is $JS/\hbar \sim 10^{13}$ Hz.

In addition, as seen from (9), the absorption is proportional to $(H/H_A)^2$, and therefore the effect is noticeable at sufficiently strong fields.

[1] M. I. Kaganov and N. E. Chupis, Zh. Eksp. Teor. Fiz. 44, 1695 (1963) [Sov. Phys.-JETP 17, 1141 (1964)].

[2] T. Holstein and H. Primakoff, Phys. Rev. 58, 1098 (1940).

[3] R. Kubo, J. Phys. Soc. Japan 12, 1203 (1957).

Translated by J. G. Adashko
57