NONORTHOGONAL QUASISTATIONARY STATES

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The nonorthogonality of unstable states with common decay channels is considered. The mechanism of the formation of nonorthogonal unstable states through the mixing of orthogonal stable levels is analyzed within the framework of the two-level problem. If the quasi-stationary states are nonorthogonal, the change of the total number of systems (particles) with time cannot be described by a sum of exponentials, and the energy spectrum of the decay products cannot be reduced to a sum of Breit-Wigner terms. In the special case of two overlapping nonorthogonal levels the decay law (40) holds, which corresponds to the presence of a second-order pole in the scattering matrix.

1. It is known that the wave functions describing different stationary states of stable quantal systems are mutually orthogonal. This is, in general, not true for quasistationary unstable states. Thus, in the case of the CP violation in the decays of K^0 mesons, the quasistationary wave functions of the particles K_L and K_S, which correspond to definite values of the mass and the lifetime, are in general not orthogonal (cf., e.g.,[1]). This nonorthogonality leads to observable consequences, for example, to the charge asymmetry of the leptonic decays of K_L.[2] The nonorthogonality of the states K_L and K_S is connected with the circumstance that they have common decay channels if CP parity is not conserved. This situation, where two states of a quantal system (two particles) have common decay channels, is of course not a specific feature only of the K^0 mesons. Whenever it occurs, one may expect that the quasistationary wave functions of these states (particles) will be nonorthogonal.

In atomic and nuclear physics one encounters a whole series of phenomena which find a natural physical interpretation in the language of unstable nonorthogonal states. Examples are the "beats" in the total number of unstable systems and decay products, the excitation and decay of atomic and molecular levels under conditions which lead to the appearance of a second-order pole in the scattering matrix,[3] the interference effects in the formation of excited nuclear states,[4] etc.

However, at present the concept of nonorthogonality is fully employed only in the analysis of the problem of the CP violation in the decay of neutral K mesons. In atomic and molecular physics, these phenomena are almost exclusively described within the S matrix formalism. We therefore think it useful to consider in detail the problem of the nonorthogonality of quasistationary states in the general case.

2. Let us consider an unstable two-level system. Let $|\psi_1\rangle$ and $|\psi_2\rangle$ be normalized, in general nonorthogonal quasistationary wave functions of this system; they correspond to the complex energies $E_{1,2} = \mathscr{E}_{1,2} - i\Gamma_{1,2}/2$, where $\Gamma_{1,2}$ are the widths of the two levels. At the initial moment t = 0 we construct a state which is a certain superposition of $|\psi_1\rangle$ and $|\psi_2\rangle$, i.e.,

$$|\psi(0)\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle. \tag{1}$$

Then the wave function $|\psi(t)\rangle$, which describes the behavior of the system at later times t, has the form

$$|\psi(t)\rangle = c_1|\psi_1\rangle e^{-iE_1t/\hbar} + c_2|\psi_2\rangle e^{-iE_2t/\hbar}. \tag{2}$$

It follows from (2) that the number of systems under consideration, N, changes with time in the following way:

$$N(t) \sim \langle\psi(t)|\psi(t)\rangle = |c_1|^2e^{-\Gamma_1t/\hbar} + |c_2|^2e^{-\Gamma_2t/\hbar} + 2\mathrm{Re}\,\{\langle\psi_1|\psi_2\rangle c_1^*c_2e^{i(\mathscr{E}_1-\mathscr{E}_2)t/\hbar}\}e^{-(\Gamma_1+\Gamma_2)t/2\hbar}. \tag{3}$$

If the states $|\psi_1\rangle$ and $|\psi_2\rangle$ are nonorthogonal, i.e., $\langle\psi_1|\psi_2\rangle \neq 0$, then, according to (3), the quantity N experiences "beats" which are damped out in the course of time.

Using (3), we can determine the rate of decay:

$$\frac{\partial}{\partial t}\langle\psi(t)|\psi(t)\rangle = -\frac{\Gamma_1}{\hbar}|c_1|^2e^{-\Gamma_1t/\hbar} - \frac{\Gamma_2}{\hbar}|c_2|^2e^{-\Gamma_2t/\hbar} + 2\,\mathrm{Re}\left\{c_1^*c_2\langle\psi_1|\psi_2\rangle\left[\frac{i}{\hbar}(\mathscr{E}_1-\mathscr{E}_2) - \frac{\Gamma_1+\Gamma_2}{2\hbar}\right]e^{i(\mathscr{E}_1-\mathscr{E}_2)t/\hbar}\right\}e^{-(\Gamma_1+\Gamma_2)t/2\hbar} \tag{4}$$

On the other hand,

$$\frac{\partial}{\partial t}\langle\psi(t)|\psi(t)\rangle = -\sum_m|c_1A_{1m}e^{-iE_1t/\hbar} + c_2A_{2m}e^{-iE_2t/\hbar}|^2, \tag{5}$$

where A_{1m} and A_{2m} are the amplitudes for the decay from the states $|\psi_1\rangle$ and $|\psi_2\rangle$ into certain final states $|m\rangle$; $\sum_m$ includes also the integration over continuous variables, for example, over the angles of emission of the decay products. Comparison of (4) and (5) leads to the following relations:

$$\Gamma_{1(2)} = \hbar\sum_m|A_{1(2)m}|^2, \tag{6}$$

$$\langle\psi_1|\psi_2\rangle = \frac{\hbar R_{12}}{(\Gamma_1+\Gamma_2)/2 - i(\mathscr{E}_1-\mathscr{E}_2)}, \qquad R_{12} = \sum_m A_{1m}^*A_{2m}. \tag{7}$$

Formulas (6) and (7), have been obtained earlier by Bell and Steinberger[5] for application in the analysis of the decay of K^0 mesons.

If we have a system of more than two quasistationary states, then an analogous consideration leads to relations of the type

$$\Gamma_l = \hbar\sum_m|A_{lm}|^2, \tag{8}$$

$$\langle\psi_l|\psi_k\rangle = \frac{\hbar R_{lk}}{(\Gamma_l+\Gamma_k)/2 - i(\mathscr{E}_l-\mathscr{E}_k)}, \qquad R_{lk} = \sum_m A_{lm}^*A_{km}, \tag{9}$$

where the indices l and k refer to the individual states.

According to (7) and (9), the "magnitude" of the nonorthogonality of the states $|\psi_\alpha\rangle$ is completely determined by the difference of the corresponding energies and the amplitudes of the various decays. If the amplitude A and hence, the width Γ, tend to zero for a constant energy difference $\mathscr{E}_1 - \mathscr{E}_2$, then the quantity $\langle\psi_1|\psi_2\rangle$ also tends to zero, and in the limit of stable particles the states $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthogonal. However, the states $|\psi_1\rangle$ and $|\psi_2\rangle$ are also orthogonal when decays occur, as long as the levels 1 and 2 have different quantum numbers which are conserved during the decay.

If the quantum numbers by which the levels 1 and 2 differ are not conserved during the decay, then the quantity $\langle\psi_1|\psi_2\rangle$ can be different from zero, and the wave functions $|\psi_1\rangle$ and $|\psi_2\rangle$ are not orthogonal. In general, the quantity $\langle\psi_1|\psi_2\rangle$ is complex, and its absolute value lies between zero and unity. According to[6] $\langle\psi_1|\psi_2\rangle \neq 0$ means that $|\psi_1\rangle$ and $|\psi_2\rangle$ are not completely different, or "quasi-identical" states, which interfere under any measuring conditions.

3. Orthogonal quasistationary states can go over into nonorthogonal states when an appropriate interaction is switched on. Let $|\varphi_1\rangle$ and $|\varphi_2\rangle$ be the wave functions of two orthogonal quasistationary states (for example, the 2S and 2P states of the hydrogen atom), and let $\epsilon_{1,2}$ and $\gamma_{1,2}$ be their energies and widths. Let us now switch on a mixing interaction (for example, an external electric field). The new quasistationary wave functions satisfy the Schrödinger equation

$$\hat{H}|\psi_{1,2}\rangle = E_{1,2}|\psi_{1,2}\rangle, \tag{10}$$

where $\hat{H}$ is an effective, in general nonhermitian Hamiltonian, and E_1 and E_2 are complex energies. We write $|\psi_{1,2}\rangle$ in the form

$$|\psi_{1,2}\rangle = c_{1(2)1}|\varphi_1\rangle + c_{1(2)2}|\varphi_2\rangle. \tag{11}$$

Then it follows from the normalization condition for $|\psi_{1,2}\rangle$ that

$$|c_{1(2)1}|^2 + |c_{1(2)2}|^2 = 1. \tag{12}$$

Using the standard method for solving (10), we obtain the following expressions for the energies $E_{1,2}$ and the coefficients c:

$$E_{1,2} = \frac{H_{11}+H_{22}}{2} \pm \frac{1}{2}\sqrt{(H_{11}-H_{22})^2 + 4H_{12}H_{21}}; \tag{13}$$

$$c_{11} = \left(1 + \left|\frac{H_{21}}{\alpha}\right|^2\right)^{-1/2}, \qquad c_{12} = \frac{H_{21}}{\alpha}c_{11}; \tag{14}$$

$$c_{22} = \left(1 + \left|\frac{H_{12}}{\alpha}\right|^2\right)^{-1/2}, \qquad c_{21} = -\frac{H_{12}}{\alpha}c_{22}, \tag{15}$$

where

$$\alpha = E_1 - H_{22} = H_{11} - E_2 = \frac{H_{11}-H_{22}}{2} + \frac{1}{2}\sqrt{(H_{11}-H_{22})^2 + 4H_{12}H_{21}}, \tag{16}$$

and $H_{ik} = \langle\varphi_i|\hat{H}|\varphi_k\rangle$ are the matrix elements of the operator $\hat{H}$.

Knowing the coefficients c, we can easily express the "nonorthogonality" of the functions $|\psi_1\rangle$ and $|\psi_2\rangle$ through the matrix elements H_{ik}. We have

$$\langle\psi_1|\psi_2\rangle = c_{11}^* c_{21} + c_{12}^* c_{22}, \tag{17}$$

or

$$\langle\psi_1|\psi_2\rangle = \left(1 + \left|\frac{H_{12}}{\alpha}\right|^2\right)^{-1/2}\left(1 + \left|\frac{H_{21}}{\alpha}\right|^2\right)^{-1/2}\left(\frac{H_{21}^*}{\alpha^*} - \frac{H_{12}}{\alpha}\right) \tag{18}$$

We emphasize that, since the states $|\psi_1\rangle$ and $|\psi_2\rangle$ are only defined up to a phase, we can always choose these phases such that the measure of nonorthogonality $\langle\psi_1|\psi_2\rangle$ be real (positive).

4. We recall that the Schrödinger equation with an effective nonhermitian Hamiltonian can be obtained in the Weisskopf-Wigner approximation, starting from the theory of radiation damping, which deals with a hermitian interaction operator W (cf.[7-9]). Let us write the operator W in the form $W = V' + V$, where V' describes the decays and V is some hermitian interaction which mixes the states $|\varphi_1\rangle$ and $|\varphi_2\rangle$ (for example, the interaction of the system with external electric or magnetic fields). We assume further that the decay interaction does not mix these states. In this case the matrix elements of the effective Hamiltonian have the form

$$H_{11} = \varepsilon_1 + V_{11} - i\frac{\gamma_1}{2}, \quad H_{22} = \varepsilon_2 + V_{22} - i\frac{\gamma_2}{2},$$
$$H_{12} = H_{21}^* = V_{12} = V_{21}^*, \tag{19}$$

where ϵ_1 and ϵ_2, γ_1 and γ_2 are the energies and widths of the levels before the mixing (cf. also[8]). As is seen from (18), $\langle\psi_1|\psi_2\rangle \sim \operatorname{Im}\alpha$ for $H_{12} = H_{21}^*$. It is easy to see [formula (16)] that $\operatorname{Im}\alpha \neq 0$ and hence $\langle\psi_1|\psi_2\rangle \neq 0$, if the widths of the levels before the mixing are not equal to each other ($\gamma_1 \neq \gamma_2$). If $|(\gamma_1-\gamma_2)/(\epsilon_1-\epsilon_2)| \ll 1$, then $|\langle\psi_1|\psi_2\rangle| \ll 1$ independently of the magnitude of the mixing interaction. The "nonorthogonality" is also much smaller than unity if the mixing interaction is small compared with the difference of the energies of the levels $|\varphi_1\rangle$ and $|\varphi_2\rangle$, independently of the difference of their widths. In this limiting case

$$\langle\psi_1|\psi_2\rangle = i\frac{\operatorname{Re}H_{21}}{|H_{11}-H_{22}|^2}(\gamma_2-\gamma_1). \tag{20}$$

We note that, if the quasistationary levels are mixed by an external field, where according to (19), $H_{12} = H_{21}^*$, the widths of the new levels can be expressed in the form

$$\Gamma_i = -2\operatorname{Im}E_i = -2\operatorname{Im}\langle\psi_i|\hat{H}|\psi_i\rangle = |c_{i1}|^2\gamma_1 + |c_{i2}|^2\gamma_2, \tag{21}$$

where c_{i_1} and c_{i_2} are given by (14) and (15). Then

$$\Gamma_1 + \Gamma_2 = \gamma_1 + \gamma_2, \tag{22}$$

i.e., the sum the new widths is equal to the sum of the old widths.

The equality (21) can also be obtained with the help of (6) if one takes account of the linear relation between the new and old amplitudes A_{im} and a_{lm} (i, l = 1, 2):

$$A_{im} = c_{i1}a_{1m} + c_{i2}a_{2m}, \quad \sum_m a_{1m}a_{2m}^* = 0. \tag{23}$$

A simple analysis of the equations (10) also shows that expression (18), with $H_{12} = H_{21}^*$, and expression (7), with account of the linear relation between the new and old amplitudes (23), lead to the same formula

$$\langle\psi_1|\psi_2\rangle = \frac{\langle\psi_2|\hat{H}|\psi_1\rangle^* - \langle\psi_1|\hat{H}|\psi_2\rangle}{E_1^* - E_2} = \frac{c_{11}^*c_{21}\gamma_1 + c_{12}^*c_{22}\gamma_2}{(\gamma_1+\gamma_2)/2 - i(\mathscr{E}_1-\mathscr{E}_2)} \tag{24}$$

We note that in general $H_{12} \neq H_{21}^*$ if the interaction which causes the decays also mixes the states $|\varphi_1\rangle$ and $|\varphi_2\rangle$ ($\sum_m a_{1m}a_{2m}^* \neq 0$); then nonorthogonality occurs for any values of the original widths. Formulas (21) and (24) are invalid in this case, but (22) and (23) remain true.

5. Let us now consider the case where the energies

and widths of the quasistationary levels coincide as a result of the inclusion of the interaction W. It follows from (13) that

$$E_1 = E_2 = \frac{\varepsilon_1 + \varepsilon_2}{2} - i\frac{\gamma_1 + \gamma_2}{4} + \frac{V_{11} + V_{22}}{2} = \frac{H_{11} + H_{22}}{2}, \quad (25)$$

if the expression under the root is zero:

$$(H_{11} - H_{22})^2 + 4H_{12}H_{21} = 0. \quad (26)$$

Using the notation $H_{11} - H_{22} = \delta + i\beta$, (26) can be rewritten in the form of the two conditions

$$\delta^2 - \beta^2 + 4\mathrm{Re}\,(H_{12}H_{21}) = 0, \quad \delta\beta + 2\mathrm{Im}\,(H_{12}H_{21}) = 0. \quad (27)$$

For a hermitian mixing interaction

$$\mathrm{Im}\,(H_{12}H_{21}) = 0, \quad \mathrm{Re}\,(H_{12}H_{21}) > 0,$$

and E_1 can become equal to E_2 if $\delta = 0$ and $\beta^2 = 4|H_{12}|^2$. In this case $\alpha = i(\gamma_2 - \gamma_1)/4$, and the expression for the "nonorthogonality" becomes

$$\langle\psi_1|\psi_2\rangle = i\frac{\gamma_2 - \gamma_1}{|\gamma_2 - \gamma_1|}\frac{H_{12}}{|H_{12}|}. \quad (28)$$

We see that $|\langle\psi_1|\psi_2\rangle| = 1$ in the limit where $E_1 = E_2$. It is easy to verify that also if the mixing interaction is nonhermitian ($H_{12} \neq H_{21}^*$), we have $|\langle\psi_1|\psi_2\rangle| = 1$ for $E_1 = E_2$.

We note here that, irrespective of the "degeneracy" ($E_1 = E_2$), the wave functions $|\psi_1\rangle$ and $|\psi_2\rangle$ entering in (28) are uniquely defined if we regard the above-given relations as limits for $E_1 \to E_2$.

6. Let us now assume that the state $|\psi(0)\rangle$ corresponding to $|\varphi_1\rangle$ has been formed as a result of some process. Let us consider its change with time. To this end we expand $|\varphi_1\rangle$ in the quasistationary states $|\psi_1\rangle$ and $|\psi_2\rangle$. It follows from (11) that this expansion has the form

$$|\psi(0)\rangle = |\varphi_1\rangle = \frac{c_{22}|\psi_1\rangle - c_{12}|\psi_2\rangle}{D}, \quad (29)$$

where $D = c_{11}c_{22} - c_{12}c_{21}$. The development in time of $|\varphi_1\rangle$ is described by

$$|\psi(t)\rangle = \frac{c_{22}}{D}|\psi_1\rangle e^{-iE_1t/\hbar} - \frac{c_{12}}{D}|\psi_2\rangle e^{-iE_2t/\hbar}. \quad (30)$$

The probability for observing the system in the initial state $|\varphi_1\rangle$ at the time t is given by

$$P(t) = |\langle\varphi_1|\psi(t)\rangle|^2. \quad (31)$$

Let us consider (31) for the case where $E_1 \to E_2$. For $E_1 = E_2$ the coefficients in the expansion (29) become singular. The precise prescription for going to the limit leads after some algebraic transformations to the formula

$$P(t) = \left|1 - i\frac{(H_{11} - H_{22})t}{2\hbar}\right|^2 e^{-(\gamma_1+\gamma_2)t/2\hbar}, \quad (32)$$

or

$$P(t) = \left\{1 + \frac{\beta t}{\hbar} + \frac{\beta^2 + \delta^2}{4\hbar^2}t^2\right\} e^{-(\gamma_1+\gamma_2)t/2\hbar}, \quad (33)$$

where β and δ satisfy (27).

It should be noted that, for $t \neq 0$, there exists also a finite probability for observing the system in the state $|\varphi_2\rangle$:

$$Q(t) = \langle\varphi_2|\psi(t)\rangle = \left|\frac{H_{12}t}{\hbar}\right|^2 e^{-(\gamma_1+\gamma_2)t/2\hbar}. \quad (34)$$

Accordingly, the total number of systems (particles) which were originally in the state $|\varphi_1\rangle$ changes with time in the following way:

$$N(t) = P(t) + Q(t) = \left\{1 + \frac{\beta t}{\hbar} + \frac{\delta^2 + \beta^2 + 4|H_{12}|^2}{4\hbar^2}t^2\right\} e^{-(\gamma_1+\gamma_2)t/2\hbar}. \quad (35)$$

If the levels are mixed by an external field, we have, as already noted above, $\delta = \epsilon_1 - \epsilon_2 + V_{11} - V_{22} = 0$. According to (27), the mixing interaction must be severely restricted, viz.,

$$|H_{12}| = |H_{21}| = \tfrac{1}{4}|\gamma_1 - \gamma_2|. \quad (36)$$

Thus the decay law for the state $|\varphi_1\rangle$ is in this case

$$P(t) = \left(1 + \frac{\gamma_2 - \gamma_1}{4\hbar}t\right)^2 e^{-(\gamma_1+\gamma_2)t/2\hbar},$$
$$Q(t) = \frac{(\gamma_2 - \gamma_1)^2}{16\hbar^2}t^2 e^{-(\gamma_1+\gamma_2)t/2\hbar}. \quad (37)$$

If the state $|\varphi_1\rangle$ was stable ($\gamma_1 = 0$, $\gamma_2 = \gamma$) before the mixing, then

$$P(t) = \left(1 + \frac{\gamma t}{4\hbar}\right)^2 e^{-\gamma t/2\hbar}. \quad (38)$$

The expression (37) agrees with the Goldberger-Watson formula for the decay of a pole of second order.[10] If, on the other hand, $\gamma_1 = \gamma$, $\gamma_2 = 0$, we have[1)]

$$P(t) = \left(1 - \frac{\gamma t}{4\hbar}\right)^2 e^{-\gamma t/2\hbar}. \quad (39)$$

We emphasize that our type of non-exponential law for the decay of $|\varphi_1\rangle$ is simply a special limiting case of the general law for the decay of $|\varphi_1\rangle$, which leads to damped beats in the case $E_1 \neq E_2$.

It follows from our analysis that if the two nonorthogonal quasistationary states are degenerate, the decay law is generally of the form

$$|1 + \eta t|^2 e^{-\Gamma t/\hbar}, \quad (40)$$

where η depends on the type of excitation. If $\eta \neq 0$ the amplitude for resonance scattering on the above-considered system of two levels will have a pole of second order at $E_0 = E_1 = E_2$ (cf. also[3,11]).

7. Our theory of the decay of unstable systems is quite general and can be applied to various physical situations. We have pointed out already that a strong nonorthogonality of the quasistationary states requires that the difference of the level widths be comparable with the energy differences. Let us consider, for example, the decay of the excited $2S_{1/2}$ and $2P_{1/2}$ states of the hydrogen atom when they are mixed by an external electric field, and the Lamb shift is compensated by a corresponding magnetic field[12] (the electric and magnetic fields must be perpendicular to each other). In this case $|\varphi_1\rangle$ and $|\varphi_2\rangle$ correspond to the states $2S_{1/2}$ and $2P_{1/2}$ in the absence of the electric field. During the mixing of the levels the nonorthogonal states $|\psi_1\rangle$ and $|\psi_2\rangle$ with the energies E_1 and E_2 appear. This causes the number of excited atoms to change in time according to a non-exponential law [formula (3)]. For $E_1 \to E_2$ we have $\langle\psi_1|\psi_2\rangle \to 1$, and the law for the decay of the system is given by (38) and (39).[3] The amplitude for resonance scattering of photons has in this case a

[1)] See also [3]. The case where one of the widths is zero corresponds to the Lee model. The non-exponential decay in the framework of this model has been considered earlier in [11].

second-order pole in the point $E_0 = E_1 = E_2$, i.e., is proportional to $1/(E - E_0)^2$.

To give a second example, we note that the excited states $^8Be_1^*$ and $^8Be_2^*$, which were considered in[4], are also nonorthogonal. Indeed, both states of beryllium have the same spin and parity and decay into two α particles. Using the experimental data on the widths Γ_1 and Γ_2 and the energy difference $Q = E_1 - E_2$, quoted in[4], we obtain with the help of (7)

$$|\langle {}^8Be_1^* | {}^8Be_2^* \rangle| = \sqrt{\frac{\Gamma_1\Gamma_2}{1/4(\Gamma_1+\Gamma_2)^2+Q^2}} \approx 0.3.$$

It is the nonorthogonality of the states $^8Be_1^*$ and $^8Be_2^*$ which makes it impossible to describe the decay spectrum for the process $^8Be^* \to 2\alpha$ by a sum of two Breit-Wigner terms; it leads to the appearance of an interference term, which has been observed experimentally.[2)]

8. We note, in conclusion, that the state which is formed after the excitation of the system is a superposition

$$|\psi\rangle = f_1|\psi_1\rangle + f_2|\psi_2\rangle, \tag{41}$$

where f_1 and f_2 are certain constants. We choose the normalization such that $\langle\psi|\psi\rangle = \sigma$, where σ is the total cross section for the process. Then

$$\sigma = |f_1|^2 + |f_2|^2 + 2\mathrm{Re}\,(f_1^* f_2 \langle\psi_1|\psi_2\rangle). \tag{42}$$

It should be emphasized in particular, that if $|\psi_1\rangle$ and $|\psi_2\rangle$ are nonorthogonal, the quantities f_1 and f_2 do not coincide with the S matrix elements F_1 and F_2 corresponding to the transition from some initial state $|\Phi\rangle$ to the states $|\psi_1\rangle$ and $|\psi_2\rangle$. Indeed,

$$\begin{aligned} F_1 &= \langle\psi_1|\hat{S}|\Phi\rangle = \langle\psi_1|\psi\rangle = f_1 + f_2\langle\psi_1|\psi_2\rangle, \\ F_2 &= \langle\psi_2|\hat{S}|\Phi\rangle = \langle\psi_2|\psi\rangle = f_2 + f_1\langle\psi_2|\psi_1\rangle. \end{aligned} \tag{43}$$

Solving this system of equations, we obtain the relations

$$f_1 = \frac{F_1 - \langle\psi_2|\psi_1\rangle F_2}{1-|\langle\psi_2|\psi_1\rangle|^2} \qquad f_2 = \frac{F_2 - \langle\psi_1|\psi_2\rangle F_1}{1-|\langle\psi_1|\psi_2\rangle|^2}. \tag{44}$$

For $\langle\psi_1|\psi_2\rangle = 0$ we have $f_1 = F_1$ and $f_2 = F_2$, as was to be expected.[3)]

Let us now consider the distribution over the energies (more precisely, the effective masses) of the decay products of the decay $|\psi_1\rangle, |\psi_2\rangle \to |m\rangle$. It has the following form:

$$d\sigma_m = \frac{\hbar}{2\pi}\left| f_1 \frac{A_{1m}}{\varepsilon - \mathscr{E}_1 + i\Gamma_1/2} + f_2 \frac{A_{2m}}{\varepsilon - \mathscr{E}_2 + i\Gamma_2/2}\right|^2 d\varepsilon, \tag{45}$$

where A_{1m}, A_{2m} are the amplitudes for the decays $|\psi_1\rangle \to |m\rangle$, $|\psi_2\rangle \to |m\rangle$, respectively; E_1 and E_2, Γ_1 and Γ_2 are the energies and the widths of the states $|\psi_1\rangle$ and $|\psi_2\rangle$, and the quantities f_1 and f_2 satisfies the relations (41) to (44).

In the summation over the final states (for example, the integration over the angles) the interference term in (45) vanishes if $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthogonal; if the states $|\psi_1\rangle$ and $|\psi_2\rangle$ are not orthogonal, the interference term in the expression $\sum_m d\sigma_m$ is always different from zero.

The total cross section for the formation of the states $|\psi_1\rangle$ and $|\psi_2\rangle$ is $\sigma = \sum_m \int d\sigma_m$. Using (6) and (7), we arrive at (42), as was to be expected.

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[2)]The states $^8Be_1^*$ and $^8Be_2^*$ are superpositions of states with isospin $T = 0$ and 1, which can be written in the form

$$^8Be_1^* = a|\varphi_{T=0}\rangle + \frac{b}{\alpha}|\varphi_{T=1}\rangle, \qquad ^8Be_2^* = -\frac{b^*}{\alpha}|\varphi_{T=0}\rangle + a|\varphi_{T=1}\rangle,$$

where a is a real number, and α is in the present case necessarily a complex number [cf. formulas (14 to 16, 19)]. This was apparently overlooked by the authors of [4]. For real values of the coefficients the above-given superpositions would evidently be orthogonal.

[3)]L. I. Lapidus and R. M. Ryndin have independently arrived at analogous results.

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