PRIMARY SPECIFIC IONIZATION OF RELATIVISTIC PARTICLES IN GASES

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Formulas are proposed and calculations are performed for the primary specific ionization in noble gases and also in H_2 , N_2 and O_2 by taking into account polarization of the medium. It is shown that at relativistic energies the dependence of the primary specific ionization on the particle Lorentz factor γ is similar to that for specific energy losses; however, the primary ionization plateau begins at smaller values of γ . The minimum absolute values of the primary specific ionization in He, Ne, Ar, Xe, H_2 , N_2 and O_2 at normal temperatures and pressures are calculated to be respectively 3.5, 11.4, 25.8, 49.6, 5.1, 27.1, 28.9 ion pairs per cm; the plateau exceeds the minimum correspondingly by 48.7, 58.7, 41.6, 40.6, 36.1, 43.8 and 52.2%. Some simplified expressions for the calculation of the primary specific ionization at various gas pressures are given. The mean excitation potentials of the gases are calculated. The results are compared with the available experimental data.

 ${f M}$ EASUREMENTS of the specific ionization are used in high-energy physics to determine the velocity of fast charge particles or to identify these particles. One usually measures the total specific ionization nt or the specific energy loss $(-dE/dx) = wn_t$ (w is the energy consumed in the production of one pair of ions, and is a constant for any given substance). Another possibility is to measure not the total but the primary specific ionization n₁, i.e., the number of primary ion pairs produced by the charged particle per unit path in the substance. One of the advantages of measuring the primary ionization is that its statistical fluctuations obey a Poisson distribution which is narrower and more symmetrical than the distribution of the energy loss (-dE/dx) (Landau distribution). The measurements of n_1 are best carried out with the aid of a multilayer spark chamber with spark-gap efficiency $\eta < 1^{\lfloor 1-4 \rfloor}$ or with a streamer chamber^[5,6].

1. FORMULAS FOR THE CALCULATION OF THE PRIMARY SPECIFIC IONIZATION

The dependence of n_1 on the particle Lorentz factor γ for most gases is unknown. The Bethe formula^[7] makes it possible to calculate n_1 in atomic hydrogen without taking into account the effect of the density of the medium. The calculations of Budini et al.^[8,9], in which this effect is taken into account, were performed only for H and He.

 $\ln^{[3]}$ we proposed formulas which make it possible to calculate the specific energy losses of a charged particle $(-dE/dx)_{W_{\rm m}} < w < _{T}$ with energy transfer in each individual collision act from $W_{\rm m}$ to T, and also the specific number of collisions $(dN/dx)_{W_{\rm m}} < w < _{T}$ with allowance for the polarization of the medium. The medium was represented in the form of oscillators with a continuous frequency spectrum. In particular

$$\left(\frac{dN}{dx}\right)_{W_m < W < T} = \frac{2\pi e^4 z^2 n}{mc^2 \beta^2} \left\{ \int_{W_m/\hbar\nu_p}^{T/\hbar\nu_p} \frac{f(\mathbf{v})}{\hbar\nu_p \mathbf{v}} d\mathbf{v} \left[\ln \frac{2mc^2 \beta^2}{\hbar\nu_p \mathbf{v} \left(1 - \beta^2\right)} - \beta^2 + \left(1 - \frac{\hbar\nu_p \mathbf{v}}{T}\right) \right] + \left(\frac{1}{W_m} - \frac{1}{T}\right) \int_{0}^{W_m/\hbar\nu_p} f(\mathbf{v}) d\mathbf{v} - \Delta\left(\varepsilon\right) \right\} \left[\mathbf{cm}^{-1} \right] (1)$$

where z is the charge of the particle, n is the number of electrons per cm³ of matter, ν is the frequency expressed in units of the plasma frequency ν_p = $(4\pi e^2 n/m)^{1/2}$, $f(\nu)$ is the density of the oscillator strengths¹⁾, and $\Delta(\epsilon)$ is the correction for the density effect, which takes into account the polarization of the medium and the Cerenkov radiation:

$$\Delta(\varepsilon) = \frac{1}{\hbar v_p} \left\{ \int_{W_m/h_p}^{T/h_p} \frac{f(\mathbf{v}) \, d\mathbf{v}}{\mathbf{v}} \left[\ln \frac{1 - \beta^2}{\sqrt{(1 - \beta^2 \operatorname{Re} \varepsilon)^2 + \beta^4 \operatorname{Im}^2 \varepsilon}} \right. \\ \left. + \beta^2 (1 - \operatorname{Re} \varepsilon) \right] + 2 \int_{W_m/h_p}^{T/h_p} \left(\beta^2 - \frac{\operatorname{Re} \varepsilon}{|\varepsilon|^2} \right) d\mathbf{v} \right\}$$
(2)

which depends on the complex dielectric constant of the medium $\epsilon(\nu)$ (see, for example, ^[8]).

To calculate the primary ionization n_1 , the threshold energy W_m in formulas (1) and (2) should be set equal to I_0 , the ionization potential of the outer atomic electron²⁾, and the contribution of the Cerenkov radiation (the last integral in (2)) must be eliminated. A calculation of the complex dielectric constant $\epsilon(\nu)$ shows that the condition for the emission of Cerenkov radiation $\beta^2 \operatorname{Re}(\epsilon) > 1$ can be satisfied only for frequencies up to the first ionization potential I_0 , and also in narrow regions below the absorption edges. Thus, for example, according to our estimates, the contribution of the Cerenkov photons corresponding to the K-absorption edge of Ne and the L and K absorption edges of Ar amounts to a fraction of one per cent. Therefore we can assume with a great degree of accuracy that

$$(dN/dx)_{I_0 < W < T} = n_1.$$
 (3)

When so defined, n_1 includes also the excitation of the internal electron shells, which, however, has low probability and is accompanied in most cases by emission of Auger electrons or autoionization of the atom.

¹⁾The oscillator strength density f(v) takes into account also discrete transitions corresponding to excitation of the atom.

²⁾When $W_m = I_0$, formulas (1) and (2) differ from the corresponding formulas for the primary ionization as given by Budini et al. [^{8,9}].

In practice, the use of formulas (1) and (2) is made complicated by the need for calculating $\epsilon(\nu)$, and additional computer time is required. The calculations can be greatly simplified by replacing the continuous distribution of the oscillator strength by an equivalent discrete distribution (the model of the medium in the form of a set of undamped oscillators). As a result, expressions (1) and (2), with (3) taken into account, are transformed into formulas obtained by us earlier^[10]:

$$n_{1} = \frac{2\pi e^{4} z^{2} n}{m c^{2} \beta^{2}} \left\{ \sum_{i=s+1}^{n} \frac{f_{i}}{I_{i}} \left[\ln \frac{2m c^{2} \beta^{2}}{I_{i} (1-\beta^{2})} - \beta^{2} \right] + \sum_{i=s+1}^{h} f_{i} \left(\frac{1}{I_{i}} - \frac{1}{T} \right) + \sum_{i=1}^{s} f_{i} \left(\frac{1}{I_{0}} - \frac{1}{T} \right) - \Delta \right\} \ [cm^{-1}]$$
(4)

where Δ is the correction to the effect of the density of the medium³:

$$\Delta = \sum_{j=s+1}^{h} \frac{1}{I_j} \left[2v_j (1-\beta^2) (v_j - v_j') + \sum_{i=1}^{h} f_i \ln \left| \frac{(l_i - v_j) (l_i + v_j')}{(l_i - v_j') (l_i + v_j)} \right| - f_j \ln \frac{(1-\beta^2)}{1-\beta^2 \varepsilon(l_j)} - f_j \beta^2 \right].$$
(5)

Here I_i and f_i are the energy and strength of the i-th discrete oscillator with frequency $\nu_i = I_i/\hbar\nu_p$ (ν_i increases with increasing i), k is the total number of oscillators, s is the number of oscillators with frequency $\nu_i < I_0/h\nu_p$, $l_i^2 = \nu_i^2 + f_i$, T is the maximum energy of the δ electron, and

$$\varepsilon(l_j) = 1 + \sum_{i=1}^{k} \frac{f_i}{(v_i^2 - l_j^2)}.$$
 (6)

The values of ν'_i are roots of the equation

$$(1-\beta^2)/\beta^2 = \sum_{j=1}^{k} f_j/(v_j^2 - v'^2), \qquad (7)$$

which are real for all j > 1. For j = 1, the root ν'_1 is real when $\beta < \beta_0$ and imaginary when $\beta > \beta_0$, where

$$\beta_{0} = \left(\sum_{j=1}^{n} f_{j} / v_{j}^{2} + 1\right)^{-2}.$$
 (8)

In the latter case, ν'_1 in (5) should be replaced by 0. Since $1/I_0 \gg 1/T$ and $1/I_1 \gg 1/T$, the terms 1/T in expression (4) should be omitted.

At particle velocities $\beta < \beta_0$ the correction Δ for the effect of the density of the medium can be neglected. In all the gases, at normal temperature and pressure $(t = 0^{\circ}C, P = 1 \text{ atm}), \beta_0$ is in the region of the logarithmic rise. Thus, in a wide range of particle energies, the primary specific ionization can be calculated from the simple formula

$$n_1(\beta < \beta_0) = \frac{A_1}{\beta^2} P \frac{273}{273 + t} \left[A_2 + \ln \frac{\beta^2}{1 - \beta^2} - \beta^2 \right] \text{ [cm}^{-1} \text{], (9)}$$

where A_1 and A_2 are constants for the given medium:

$$A_{1} = 0,1536 \cdot 10^{6} \rho \frac{Z}{A} \sum_{i=i+1}^{k} \frac{f_{i}}{I_{i}} \left[\text{cm}^{-1} \cdot \text{atm}^{-1} \right]$$

$$A_{2} = 1 + \left(\frac{1}{I_{0}} \sum_{i=1}^{s} f_{i} + \sum_{i=i+1}^{k} \frac{f_{i}}{I_{i}} \ln \frac{2mc^{2}}{I_{i}} \right) \bigg| \sum_{i=i+1}^{k} \frac{f_{i}}{I_{i}}; \quad (10)$$

Table I. Primary specific ionization in gases under normal conditions P = 1 atmosphere, $t = 0^{\circ}C$

Substance	Gas density $\rho \cdot 10^3$, g/cm ³	Primary ionization potential I ₀ , eV	Primary spe tion,	cific ioniza- cm ⁻¹	Relativistic growth R, %	Start of plateau γ _{pl}	Parameters of for- mulas (9) and (10)			
			n ₁ min	ⁿ i pl			Yo		A ₂	
He Ne Ar Xe H ₂ N ₂ O ₂	$\begin{array}{c} 0.17847\\ 0.90035\\ 4.78370\\ 5.8510\\ 0.08988\\ 1.25055\\ 1.42904 \end{array}$	24.56 21.66 15.75 12.08 15.4 15.5 12.2	3.5 11.4 25.8 49.6 5.1 27.1 28.9	5.2 18.1 36.5 69.6 6.9 39.0 44.0	48.7 58.7 41.6 40.6 36.1 43.8 52.2	200 209 155 200 123 102 110	121 90 46 35 61 52 48	0.244 0.844 1.828 3.554 0.329 1.941 2.079	11.64 10.89 11.45 11.31 12.73 11.43 11.28	

P is the gas pressure in atmospheres, ρ is the gas density at P = 1 atmosphere and t = 0°C, Z is the atomic number, A is the atomic weight of the substance, and I₁ are expressed in eV. The values of A₁, A₂, and γ_0 = $\gamma(\beta_0)$ are listed in Table I. For atomic hydrogen, expression (9) coincides within 3% with the Bethe formula^[7]. The latter quantity can be regarded as an estimate of the accuracy of the formulas (4) and (9) obtained by us.

2. OSCILLATOR STRENGTHS AND AVERAGE IONIZA-TION POTENTIALS

In the calculations of (-dE/dx), the oscillator strengths are frequently assumed equal to the relative populations of the corresponding atomic shells, i.e., to the number of electrons in these shells, divided by the total number of electrons $Z^{[11]}$. In such an approach, however, no account is taken of the fact that the sum rule⁴

$$\int_{\bullet}^{\infty} f(\mathbf{v}) d\mathbf{v} = \sum_{i=1}^{k} f_i = 1,$$
(11)

should be satisfied for all the electrons of the atom as a whole, and not for each shell separately. In particular, f_i of the internal shells of the heavy atoms are smaller than their relative populations in connection with the forbiddenness of transitions to occupied levels, and the role of the outer shells is enhanced by the collective interaction of the electrons.

In this paper we calculate f_i and I_i by starting from the oscillator strength density distribution $f(\nu)$, which is determined by the photo-absorption cross section $\sigma(E)$:

$$f(\mathbf{v})d\mathbf{v} = \frac{\sigma(E)dE}{2\pi^2 r_0 \hbar cZ},$$
(12)

where E is the photon energy and $r_0 = e^2/mc^2$. The $\sigma(E)$ dependence for noble gases, H_2 , N_2 , and O_2 was measured in a wide energy range (see, for example^[12]). The distribution $f(\nu)$ was broken up into k intervals, each corresponding to the strength of the equivalent discrete oscillator f_i :

$$f_{i} = \int_{v_{i}, l}^{v_{i}, \mathbf{u}} f(v) dv, \qquad (13)$$

where $\nu_{i,l}$ and $\nu_{i,u}$ are respectively the lower and upper limits of the i-th interval.

The calculations of f_i have shown that $\sum_{i=1}^{n} f_i$, with allowance for the transitions to the discrete spectrum

³⁾The value of Δ has been refined in comparison with [¹⁰].

⁴⁾The summary oscillator strength is normalized not to the total number of electrons in the atom Z, but to unity.

(for which the values of f_i and I_i were taken from ^[12, 13]), are always somewhat smaller than unity. The differences amounted respectively to approximately 0.3% for He, 4% for Ne, 6% for Ar, 7% for O₂, and 9% for N₂ and Xe; they are apparently connected with the errors in the measurements of the photo-absorption cross section, which amount to 5–10%. The values of f_i were normalized in the calculations in such a way as to satisfy the sum rule (11) rigorously.

A distinction must be made between the values of I_i , depending on whether we are interested in the specific energy losses or in the primary specific ionization. In the calculation of (-dE/dx), the values of I_i are found as the mean logarithmic values in the interval $(\nu_{i,l}, \nu_{i,u})^{5}$:

$$I_i^{(E)} = \hbar v_p \exp\left[\frac{1}{f_i} \int_{v_{i_i}}^{v_{i_i}} f(v) \ln v dv\right].$$
 (14)

In the calculation of n_1 , the best approximation for I_i turns out to be the average of the reciprocal frequencies (see formulas (1) and (4))

$$I_{i}^{(n_{i})} = \hbar v_{p} \left[\frac{1}{f_{i}} \int_{v_{i}}^{v_{i}} v^{-1} f(v) \, dv \right]^{-1}.$$
 (15)

The calculations of the primary specific ionization were made for k = 10 intervals, but the accuracy turned out to be sufficient even when k = 6. Thus, for example, the values of n_1 for He at k = 6 and k = 10 differ by only 1%.

Knowing the density distribution of the oscillator strengths, and the oscillator strengths of the discrete transitions, or else the values of f_i and I_i obtained from them, we can calculate the average logarithmic ionization potential of the medium I, which enters in the Bethe-Bloch formula:

$$I = \hbar \mathbf{v}_p \exp \left(\int_{\mathbf{a}}^{\infty} \ln \mathbf{v} f(\mathbf{v}) d\mathbf{v} = \exp \left(\sum_{i=1}^{k} f_i \ln I_i^{(E)} \right).$$
(16)

The values of I calculated from this formula are in fair agreement with the values obtained by comparing the measured slowing-down ability of the medium with the Bethe-Bloch formula (Table II).

3. RELATIVISTIC GROWTH OF PRIMARY SPECIFIC IONIZATION

Results of the calculation of the dependence of the primary specific ionization of relativistic charged particles $n_1(\gamma)$ in He, Ne, Ar, Xe, H₂, N₂, and O₂ at normal temperature and pressure are shown in Figs. 1a and 2, and also in Table I. Figure 1b shows for comparison the plots of (-dE/dx) against γ in noble gases, calculated in accordance with the Bethe-Bloch formula with allowance for the effect of the density of the medium^[11], but with values of f and I(E) obtained in accordance with i i expressions (13) and (14).

The dependence of the primary specific ionization of the relativistic particles on γ , as well as the depen-

 Table II. Average ionization potentials of gases



dence of (-dE/dx) on γ , has a minimum at $\gamma = 3-4$, as well as a region of logarithmic rise that goes over into a plateau. For all noble gases at normal temperature and pressure, the plateau of the primary ionization begins in the same region, $\gamma_{pl} \approx 2 \times 10^2$, whereas the start of the plateau of (-dE/dx) is shifted towards higher energies and corresponds to $\gamma_{pl} \approx (2-5) \times 10^2$ in the case of light gases (He, Ne) and $\gamma_{pl} \approx (1-3) \times 10^3$ in the case of heavy gases (Ar, Xe).

The fact that $n_1(\gamma)$ reaches the plateau faster than (-dE/dx) is due to the fact that remote collisions with small energy transfers, which are suppressed to the greatest degree by the effect of the density of the medium, enter in n_1 with a relatively larger weight than in (-dE/dx). Indeed, unlike the Bethe-Bloch formula, the oscillator strengths f_i in (4) are divided by the values of I_i corresponding to them, thereby emphasizing the role of the outer electrons with low ionization potential.

Whereas the relativistic increase of (-dE/dx) increases with increasing Z (Fig. 1b), the largest growth of



FIG. 1. Relativistic growth of specific primary ionization (a) and of specific energy loss (b) in noble gases.

⁵⁾The values of I_i were calculated in [¹¹] from the average ionization potential I and from the values of f_i, assuming that I_i is proportional to the energies of the absorption edges of the corresponding electron shells. When Z > 2, however, only the absorption edges corresponding to deep shells are clearly pronounced. In addition, σ (E) behaves entirely differently at small and large values of E. Therefore the proportionality coefficient should depend on I_i.



FIG. 2. Relativistic growth of primary specific ionization in $\rm H_2$, $\rm N_2$, and $\rm O_2$.

 n_1 for noble gases takes place in Ne and He, and the smallest in Xe (Fig. 1a, Table I). This is also due to the fact that the values of I_0 for Ne and He are approximately twice as large as for Xe. Under conditions when the primary specific ionization is the same for all the noble gases and is equal to $n_1 = 3.5$ cm⁻¹, corresponding to He, Ne, Ar, and Xe pressures of respectively 1, 0.31, 0.14, and 0.07 atmospheres, the largest relativistic increase is observed as before in Ne, followed by Xe, Ar, and He. The same sequence is retained if other identical values of n_1 are chosen. Thus, the most convenient gas from the point of view of using the relativistic growth of the primary specific ionization for the measurement of energy or for the identification of fast charged particles, is Ne.

In order to trace the variation of the primary specific ionization of relativistic particles with changing gas pressure P, we shall characterize the relativistic growth by means of the quantity

$$R = n_{1 \text{ pi}} / n_{1 \text{ min}} - 1, \tag{17}$$

where n_{1pl} and n_{1min} are the values of the primary ionization on the plateau and at the minimum, respectively, and the start of the plateau is characterized by the Lorentz factor of the particle γ_{pl} , which is determined from the condition that the relativistic increase of n_1 from the minimum value to $n_1(\gamma_{pl})$ amounts to 0.9 R:

$$n_1(\gamma_{\rm pl}) / n_1 \min - 1 = 0.9R.$$
 (18)

Calculations show (Table III) that R and γ_{pl} depend on the temperature and pressure of the gas approximately in the following manner:

$$R(P_{2}t) = R(1,0) - a \lg \frac{273P}{273+t},$$
(19)

$$\gamma_{\rm pl}(P,t) = \gamma_{\rm pl}(1,0) \left(\frac{273P}{273+t}\right)^{-b}$$
 (20)

where P is in atmospheres and t is in degrees C; R(1, 0) and $\gamma_{pl}(1, 0)$ are the values of R and γ_{pl} at P = 1 atmosphere and t = 0°C, while a and b are the constants listed in Table III. The accuracy of these approximations at gas pressures 0.03-30 atmospheres amounts to $\delta R \approx 0.01$, and $\delta \gamma_{pl} / \gamma_{pl} \approx 5\%$ ($\approx 15\%$ for Ar).

Thus, with the aid of the simple formula (9), which is valid up to $\gamma < \gamma_0$, where γ_0 likewise depends on the

Table III. Dependence of relativistic growth R of the primary specific ionization, and the points γ_{pl} where the plateau is reached on the gas pressure

Element	P, atm	R, %	۲pl	a	b	Element	P, atm	R, %	[∨] pl	a	ь
Не	$\left \begin{array}{c} 0.1 \\ 0.56 \\ 1.0 \\ 10.0 \end{array} \right $	65.6 52.7 48.7 33.4	484 238 200 76	0.162	0.40	Ar	$\begin{cases} 0.1 \\ 1.0 \\ 10.0 \\ (0.03) \end{cases}$	57.7 41.6 25.3 65.4	355 155 89 530	0.162	0,30
Ne	$\left\{ \begin{matrix} 0.2 \\ 1.0 \\ 5.0 \end{matrix} \right.$	70.6 58.7 46.4	407 209 105	0.172	0,42	Xe	$\begin{cases} 1.00 \\ 32.80 \end{cases}$	40.6 13.8	200 71	0,170	0,29

pressure and temperature of the gas (see (8)),

$$\gamma_0(P,t) = \left[\frac{\gamma_0^2(1,0) - 1}{273P/(273 + t)} + 1\right]^{1/2}$$
(21)

and for values of R(P, t) and $\gamma_{pl}(P, t)$ calculated in accordance with (19) and (20), we can plot $n_1(\gamma)$ for different pressures and temperatures of the gas, without resorting to cumbersome calculations of the correction for the density effect (5).

Let us trace the variation of the relativistic growth of R as a function of the sensitivity threshold of the detector W_m (Fig. 3). Under real conditions, W_m can be smaller than I₀ because of the ionization of the impurities and of the production of molecular ions in collisions, with a part taken by excited gas atoms, by photoionization of impurities, and also by the photoeffect on the walls of the apparatus^[6]. On the other hand, when n_1 is measured with a gas-discharge instrument, the presence in the gas of electronegative impurities hinders the development of the discharge at a low ionization density, which is equivalent to increasing W_{m} compared with I₀. The increase of R in the region $W_m \leq I_0$ (Fig. 3) is due to the raising of the level of the plateau, owing to the contribution of the Cerenkov-emission photons. The increase of R when $W_{m} > I_{0}$ is connected with the decreasing role of the collisions with the external atomic electrons. The subsequent decrease of R is explained by the fact that at very large W_m the atomic electrons behave like free electrons, and the collisions have a Rutherford character.

It is interesting to note that by specially introducing electronegative additives to the primary-ionization de-



FIG. 3. Dependence of relativistic growth R of the primary specific ionization in noble gases and in H_2 on the threshold detector energy W_m . The arrows designate the values of R corresponding to the primary specific ionization.

tector, which works with Ar or Xe, it is possible to increase greatly the relativistic growth of the ionization (Fig. 3).

4. COMPARISON OF CALCULATIONS WITH EXPERI-MENTS

The experimental data on primary ionization were obtained in experiments with low-efficiency counters $^{[19-21]}$, cloud chambers $^{[22-25]}$, and a streamer chamber^[5,6]. A comparison of these data with calculations entails definite difficulties, since the results of the measurements of the primary ionization are distorted by secondary processes in the detector gas^[6]. All the processes (with the exception of electron capture by the electronegative impurities) overestimate slightly the measured values of n₁. However, owing to the small difference between the energies of the excitation levels of the outer electrons of the atom and I_0 , the processes occurring in the detector gas do not change the relativistic growth of the primary specific ionization, making it possible to compare the calculated and the measured values of R. Figure 4 shows that a calculation of the relativistic growth of the primary specific ionization in He (P = 0.6 atmospheres, t = 20° C) agrees with the experimental data obtained in a streamer chamber^[5].

The calculated and measured values of the absolute primary ionization in noble gases, and also in H_2 , N_2 , O_2 , and air, are compared in Table IV. The calculations agree best with measurements of n_1 in a streamer chamber filled with helium, where special measures were used to rid the gas of impurities^[6], than with the measurements of McClure, who used low-efficiency counters with H₂ added^[19]. Nonetheless, the experimental data for He, Ne, Ar^[19,25] can be reconciled with the calculations by taking into account the additional ionization in the processes in which excited atoms take part (the results of the corresponding calculations are given in the parentheses).

The calculated value of the primary specific ionization in Xe is close to the measured one, but a detailed comparison is impossible, since the experimental conditions and the measurement errors are unknown. Calculations of the primary specific ionization in H_2 agree with the experimental data^[19,21,22]. At the same time, the calculated values of n_1 in N_2 , O_2 , and air exceed somewhat the results of measurements in cloud chambers, this apparently being due to failure to take into account^[22,25] or to taking incorrectly into account^[23,24] the overlap of the images of the clusters of drops, and



FIG. 4. Relativistic growth of primary specific ionization n_1 in He (P = 0.6 atmospheres, $t = 20^{\circ}C$). Solid curve-calculation by formula (4), dashed-calculations of Budini et al. [9]. The experimental data were obtained in a streamer chamber [5].

Table IV. Comparison of calculated and experimental data on the primary specific ionization in gases at P = 1 atm, $t = 0^{\circ}C$ (the values of n_1 were taken near the minimum of the ionization)

ance		n ₁ ,	c m -1	_			
Subst	Ÿ	Calculation* Experiment		Experimental method, year			
He	$\left\{ \substack{4.03\\3.4} \right\}$	3.49 (5.26) 3.5 (5.29)	5.02 ± 0.06 3.83 ± 0.11	Low-efficiency counter [¹⁹] (1953) Streamer chamber [⁶] (1963)			
Ne	4.03	11.4 (12.4)	12.4 ± 0.13	Low-efficiency counter [¹⁹] (1953)			
Ar	$\big\{{}^{4,03}_{3.5}\\ {}^{3.5}_{3.5}$	25.8 (28.5) 25.8 (28.5) 25.8 (23.5)	27.8 ± 0.31 28.6 ± 0.5 26.4 ± 1.8	Low-efficiency counter [¹⁹] (1953) Cloud chamber [²⁵] (1957) Diffusion chamber [²⁵] (1957)			
Xe	4.0	49.6	44	Low-efficiency counter [20] (1946)			
H_2	$\big\{ \begin{matrix} 4.03 \\ 3.2 \\ 3.16 \end{matrix} \big\}$	$5.40 \\ 5.12 \\ 5.41$	$\begin{array}{c} 5,32 \pm 0.06 \\ 4.55 \pm 0.35 \\ 5.1 \pm 0.8 \end{array}$	Low-efficiency counter [19] (1953) Low-efficiency counter [21] (1948) Cloud chamber [22] (1930)			
N_2	4.92	27.3	19,3	Cloud chamber [23] (1936)			
O_2	4.3	28.9	22.2 ± 2.3	Cloud chamber [22] (1930)			
Air	${9.4 \\ 3.5}$	29.5 27.5	$25.4 \\ 18.5 \pm 1.3$	Cloud chamber $[24]$ (1954) Diffusion chamber $[25]$ (1957)			

"The values of n₁ contained in the parentheses were calculated with allowance for the contribution of the excitation of the atoms.

also the contribution made to the calculation results by the dissociation of the molecules of these gases.

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