QUANTUM OSCILLATIONS OF THE POTENTIAL DIFFERENCE IN A NONUNIFORM MAGNETIC FIELD

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An electron gas in a weakly inhomogeneous magnetic field is considered. The potential difference that appears in this case is determined. It is shown that for a semimetal the potential difference is a measurable quantity.

1. ELECTRON MOTION IN AN INHOMOGENEOUS MAGNETIC FIELD

We consider the motion of conduction electrons in a weakly inhomogeneous magnetic field ($R \ll \delta$, where R is the radius of the electron orbit and δ is the characteristic dimension of the magnetic field). Assume that in this case the intersections of the Fermi surface with the plane $(\mathbf{p}\cdot\mathbf{H})/H = p_H = \text{const}$ are finite during the course of the motion. Then, as is well known[1], the electron motion can be represented as a finite motion around the center of the orbits, motion of the center of the orbit along the magnetic field, and a drift of this orbit perpendicular to the magnetic field. The drift velocity is equal to the average velocity (over the period of revolution around the center of the orbit) in a plane perpendicular to the magnetic field, and differs from zero only in an inhomogeneous magnetic field; its order of magnitude is $V_{dr} \sim V_F R/\delta$. The quantity $S(\epsilon, p_H)/H$, where $S(\epsilon, p_H)$ is the area of the intersection of the surface $\epsilon = \text{const}$ and the plane $p_H = \text{const}$, is an adiabatic invariant when the electron moves in a homogeneous magnetic field. If $R/\delta \ll 1$, then the drift shift of the center of the orbit during the revolution period is $\Delta_{dr} \sim R^2/\delta \ll \delta$, and the shift of the center of the orbit along the magnetic field is $\Delta H \sim R \ll \delta$. Therefore the magnetic field and p_H can be regarded as slowly varying parameters of the motion. Thus, S/H is an adiabatic invariant in a weakly inhomogeneous magnetic field[1].

The quantization of the electron energy levels is analogous to the quantization in a homogeneous magnetic field, and it depends on the coordinates as a parameter.

2. THERMODYNAMICS OF AN ELECTRON GAS IN AN INHOMOGENEOUS MAGNETIC FIELD

The chemical potential of the electron gas depends on the total number of particles N, and is a functional of the inhomogeneous magnetic field; in thermodynamic equilibrium, it is constant along the sample. The chemical potential can be represented in the form $\zeta = \epsilon_0 + \delta\zeta$, where $\delta\zeta = \delta\zeta\{H\}$ is the part of the chemical potential which depends on the distribution of the magnetic field, and ϵ_0 is the Fermi energy. Assume, for simplicity, that $l \ll \delta$, where l is the electron mean free path. We subdivide the sample into regions with dimension d, where $l \ll d \ll \delta$. By virtue of the foregoing assumptions, we can introduce a chemical potential ζ_{loc} in each of these regions, and the magnetic field can be regarded as constant. The quantity ζ_{loc} introduced in this manner is a function of the number of particles and of the magnetic field. In analogy with the foregoing, ζ_{loc} can be represented in the form $\zeta_{loc} = \zeta_0 + \delta\zeta_{loc}$, where $\delta\zeta_{loc}(\kappa, H)$ is a function of the number of particles and of the magnetic field, and consequently is a function of the coordinates.

In thermodynamic equilibrium, the chemical potential is constant along the sample, leading to the equality $\delta\zeta_{loc}(\mathbf{r}) = \delta\zeta\{H\} = \text{const}$. The quantity $\delta\zeta_{loc}(n, H)$ may remain constant along the sample only if the number of particles in a unit volume depends on the coordinates. It is clear here that

$$n - n(\varepsilon_0) = \delta n \approx -\frac{\partial n}{\partial \varepsilon}[\delta\zeta_{loc}(n, H) - \delta\zeta].$$

This circumstance leads to the appearance of an uncompensated charge density

$$\rho' \approx e\frac{\partial n}{\partial \varepsilon}[\delta\zeta_{loc}(n_0, H) - \delta\zeta].$$

The uncompensated charge density produces an electric field $\varphi(\mathbf{r})$, which in turn changes the particle-number distribution by an amount

$$\delta n \approx -\frac{\partial n}{\partial \varepsilon}e\varphi, \qquad \rho' = \frac{\partial n}{\partial \varepsilon}e^2\varphi$$

etc.

In order to obtain a self-consistent solution of the problem, it is necessary to include simultaneously in consideration an inhomogeneous magnetic field and an electric field $\varphi(\mathbf{r})$, to obtain from the thermodynamic calculation the density of the uncompensated charge $\rho' = \rho'(\varphi, H)$, and to solve the Poisson equation for φ, namely $\Delta\varphi = -4\pi\rho'(\varphi, H)$.

Let us stop to discuss the solution of the last equation in greater detail. From the preceding estimates we find that $4\pi\rho' = \lambda^2\{\varphi + e^{-1}\delta\zeta_{loc}\}$, where φ is renormalized to the constant quantity $\delta\zeta(\varphi \to \varphi e^{-1}\delta\zeta)$, and $\lambda^2 = 4\pi e^2 \partial n/\partial\epsilon \sim 1/a^2$ ($a \sim 1/n_0^{1/3}$). Then

$$\Delta\varphi + \lambda^2\varphi = -\lambda^2 e^{-1}\delta\zeta_{loc}.$$

In the left side of the equation, the term with $\Delta\varphi \sim \varphi/\delta^2$ is much smaller than the term $\lambda^2\varphi \sim \varphi/a^2$, since $(a/\delta)^2 \ll 1$. Therefore the equation assumes the form $\lambda^2\varphi = -\lambda^2 e^{-1}\delta\zeta_{loc}$, from which we obtain, accu-

rate to $(a/\delta)^2$,

$$e\varphi \approx -\delta\zeta_{\text{loc}} \sim \hbar\Omega\,(\hbar\Omega/\varepsilon_0)^{1/2},$$

where Ω is the cyclotron frequency of revolution in the magnetic field. This shows that $\rho' = 0$ with accuracy $(a/\delta)^2$ (see[2]), and the equation $\rho' = 0$ is equivalent, with the same accuracy, to the equation[1)] $\Delta\varphi = -4\pi\rho'$. Thus, to find out how we can use the equation $\rho' = 0$. We now proceed to solve the problem.

3. DETERMINATION OF THE POTENTIAL DIFFERENCE IN AN INHOMOGENEOUS MAGNETIC FIELD

According to the foregoing, the electric potential $\varphi(\mathbf{r})$ arising in an homogeneous magnetic field should be determined from the electroneutrality condition $\rho' = 0$ or $n(\mathbf{r}) = n(\epsilon_0)$ (we neglect magnetostriction), where

$$n(r) = \frac{\delta N}{\delta V} = \frac{\delta}{\delta V}\frac{\partial\Omega}{\partial\zeta}$$

However, in the case of a weakly inhomogeneous magnetic field, the calculation of any thermodynamic quantities is much simpler. We can use the corresponding result obtained for a homogeneous field. It is necessary here to replace the chemical potential ζ_{hom} by $\zeta - e\varphi$. The chemical potential can be represented in the form $\zeta = \epsilon_0 + \delta\zeta\{H\}$. Then the combination $\epsilon_0 - e\varphi + \delta\zeta\{H\}$ enters in all the formulas. The electric-field potential is defined apart from an arbitrary constant, in which the quantity $\delta\zeta\{H\}$ can also be included. Thus, to obtain a thermodynamic quantity in a weakly inhomogeneous field it is necessary to replace in the corresponding quantity for the homogeneous field the chemical potential by $\epsilon_0 - e\varphi$, and to replace the volume V by the integral over the volume.

Using this, we get

$$n(\varepsilon_0) = n(\varepsilon_0 - e\varphi, H), \quad n(\zeta, H) = n(\zeta) + \widetilde{\Delta n}(\zeta, H),$$

where[3]

$$\widetilde{\Delta n}(\zeta, H) = -\sum_m \frac{dS_m}{d\zeta}\frac{1}{S_m}(H\tilde{M}_m),$$

$$\tilde{M}_m = -\frac{1}{\sqrt{2\pi}\,\pi^2\hbar^3}\left(\frac{e\hbar H}{c}\right)^{3/2}\left(\frac{\partial^2 S_m}{\partial p_z^2}\right)^{-1/2}\frac{S_m}{H}\left(\frac{dS_m}{d\zeta}\right)^{-1}$$

$$\times\sum_{m=1}^{\infty}\frac{\psi(k\lambda)}{k^{3/2}}\sin\left\{\frac{kcS_m}{e\hbar H}\pm\frac{\pi}{4}+2\pi k\gamma\right\}\cos\left(\frac{k}{2m_0}\frac{dS_m}{d\zeta}\right),$$

S_m is the extremal section of the Fermi surface and $\tilde{M}_m$ is the magnetic moment of the m-th group of electrons. We note that in the expression for $n(\zeta, H)$ we neglected the non-oscillating diamagnetic and paramagnetic terms, since they are small at low temperatures[3].

Since $e\varphi/\hbar\Omega \ll 1$ (see the preceding section), we can set in the argument of the sine function $S_m(\epsilon_0 - e\varphi)$ equal to $S_m(\epsilon_0)$. We then get

$$n(\varepsilon_0 - e\varphi) + \widetilde{\Delta n}(\varepsilon_0, H) = n(\varepsilon_0),$$

$$\varphi = -\frac{1}{e\nu(\varepsilon_0)}\sum_m \frac{dS_m}{d\zeta}\frac{1}{S_m}(H\tilde{M}_m),$$

where $\nu(\epsilon)$ is the density of states.

Since only the potential difference has a physical meaning, we obtain ultimately

$$e\Delta\varphi = -\frac{1}{\nu(\varepsilon_0)}\sum_m \frac{dS_m}{d\zeta}\frac{1}{S_m}\{(\tilde{M}_m H)|_{r_2} - (\tilde{M}_m H)|_{r_1}\}.$$

Let us now estimate the order of magnitude of the result. The quantity $\nu(\epsilon_0)$ is determined mainly by the electrons with the largest mass $m^*_{\max}$

$$\nu(\varepsilon_0) \sim (m^*_{max})^{3/2}\varepsilon_0^{1/2}\hbar^{-3};$$

$(\tilde{M}_m H)$ is determined by the electrons with the smallest mass (owing to the factor $\exp[-\pi^2 T/\hbar\Omega]$) and

$$(\tilde{M}_m H) \sim \frac{\varepsilon_0}{\hbar^3}\left(\frac{e\hbar H}{c}\right)^{3/2}$$

(see[3]). We have $S_m^{-1}\,dS_m/d\zeta \sim 1/\epsilon_0$. Thus

$$e\Delta\varphi \sim \hbar\Omega_{min}\,(\hbar\Omega_{min}/\varepsilon_0)^{1/2},$$

$$\Omega_{min} = \frac{eH}{m^*_{max}c}, \quad \frac{e\Delta\varphi}{\hbar\Omega_{min}} \sim \left(\frac{\hbar\Omega_{min}}{\varepsilon_0}\right)^{1/2} \ll 1.$$

It is seen from the estimates that the period of the oscillations is determined by the electrons having the smallest mass, and the oscillation amplitude is determined by the electrons with the largest mass.

If the field is $H \sim 10^4$ Oe, then for semimetals such as Bi we get $\Delta\varphi \sim 10^{-4}$–$10^{-5}$ V, which is readily measurable. For normal metals $\Delta\varphi \sim 10^{-6}$–$10^{-7}$ V at $H \sim 10^4$ Oe.

From the obtained value of the potential $\varphi(\mathbf{r})$ we can determine the uncompensated charge density ρ' with the aid of the equation $\Delta\varphi = -4\pi\rho'$. It is easily found that $\rho' \sim (a/\delta)^2(\epsilon_0/\hbar\Omega)^{1/2}\rho_0$ in order of magnitude.

1)$A \sim 10^{-8}$ cm for good metals and $a \sim 10^{-7}$ – 10^{-6} cm for semimetals; in fields $H \sim 10^4$ Oe we have $R \sim 10^{-3}$ cm, and consequently $\delta \geqslant 10^{-2}$ cm. Thus, $(a/\delta)^2 \leqslant 10^{-12}$ – 10^{-8}, which greatly exceeds the accuracy of further calculations.

[1] I. M. Lifshitz, A. A. Slutskin, and V. M. Nabutovskiĭ, Zh. Eksp. Teor. Fiz. 41, 939 (1961) [Sov. Phys.-JETP 14, 669 (1962)].

[2] M. Ya. Azbel' and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. 49, 572 (1965) [Sov. Phys.-JETP 22, 399 (1966)].

[3] I. M. Lifshitz and A. M. Kosevich, Zh. Eksp. Teor. Fiz. 29, 730 (1955) [Sov. Phys.-JETP 2, 636 (1956)].

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