TWO-TEMPERATURE HYDRODYNAMICS FOR GAS MIXTURES WITH A LARGE DIFFERENCE OF COMPONENT MASSES

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A two-temperature hydrodynamics is developed for binary gas mixtures with components greatly differing in molecular weight. Propagation of ultrasound and of shock waves in such a system is considered. When the sound frequency exceeds the inverse relaxation time with respect to energy, the lighter component ceases to participate in the oscillations and the sound velocity drops. The damping is smaller than that obtained from ordinary hydrodynamics. If the shock wave force exceeds the critical value, a "partial-isothermal" discontinuity arises in which the density and temperature of the heavy component sharply change. In an extremely strong shock wave there are two discontinuities, a density jump of the light component located at the beginning of the wave and a density and temperature jump of the heavy component.

IT follows from papers devoted to the propagation of sound[1] and shock waves[2-4] in binary gas mixtures that the most interesting effect should be expected in the case of a large difference between the molecular weights of the components. In this case, however, the exchange of energy between the molecules of different types is difficult, so that it is necessary to make use of two-temperature hydrodynamics, similar to that developed consistently by Braginskiĭ for a fully-ionized plasma. Hamel[6] obtained two-fluid transport equations for such gas mixtures from Boltzmann's equations with a model collision integral. We derive below similar equations for the case of a Boltzmann collision integral and analyze the propagation of sound and of a shock wave in such a system.

If the sound frequency exceeds the reciprocal time of energy exchange between the components, then, as shown qualitatively in[7], the light component does not take part in the oscillations, so that the velocity and damping of sound are determined by the heavy component. This leads, in particular, to an appreciable negative dispersion of the speed of sound.

Owing to the presence of the light component, the thermal conductivity and the diffusion in the mixture are anomalously large, so that the characteristic scale of variation of the quantities in the shock wave exceeds the free path length. This makes it possible to analyze the structure of the wave with the aid of transport equations, as was done in[8,9] for the case of a plasma. When the mach number is sufficiently large, a "partially-isothermal" jump is produced, in which the density and the temperature of the heavy component become discontinuous. The region in front of this discontinuity is richer in light molecules, and in the case of a shock wave of maximum intensity there is produced one more discontinuity—a discontinuity of the concentration of the light component, separating the wave from the unperturbed gas; this phenomenon has no plasma analog. Both temperatures and transport coefficients vanish in this case.

1. TRANSPORT EQUATIONS

To obtain the transport equations we use Grad's method of 13 moments[10]. When the distribution function is expanded in Sonine polynomials, this method yields the transport coefficient in the first non-vanishing approximation. If the cross section does not depend very strongly on the velocity, then the error arising thereby is small; for the elastic-sphere model and for a Lennard-Jones potential[6-12], the error, for example, is of the order of 1–3% (the error is relatively large, on the order of 25%, only for the thermal diffusion coefficient[11,12]).

We assume that

$$m \ll M, \quad n_1\Sigma_{12} / n_2\Sigma_{12} \gg 2m / M, \qquad (1)$$
$$n_2\Sigma_{22} / n_1\Sigma_{12} \gg 2\sqrt{m / M}, \quad T_1 / T_2 \gg m / M.$$

Here m is the mass of the light molecule, M the mass of the heavy molecule, n the concentration, T the temperature, Σ the effective cross section. The index 1 denotes quantities pertaining to the light component and the index 2 pertains to the heavy component. The conditions (1) denote that upon relaxation, the Maxwellian distributions with $T_1 \neq T_2$ are established much earlier than the temperature common to the mixture; the relative velocity in the collisions is the velocity of the light molecules.

We write the distribution function in the form

$$f_\alpha = \left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{3/2} \exp\left(-\frac{m_\alpha c_\alpha^2}{2T_\alpha}\right)\left\{ n_\alpha + \frac{m_\alpha c_{\alpha i} j_{\alpha i}}{T_\alpha} + \frac{m_\alpha}{2T_\alpha^2} \times c_{\alpha i} c_{\alpha h} p_{\alpha ih} + \frac{m_\alpha}{5T_\alpha^2} h_{\alpha i} c_{\alpha i}\left(\frac{m_\alpha c_\alpha^2}{T_\alpha} - 5\right)\right\}, \qquad (2)$$

where $\mathbf{e}_\alpha$ is the velocity of the molecules relative to the average mixture velocity $\mathbf{u}$;

$$\mathbf{j}_\alpha = n_\alpha(\mathbf{u}_\alpha - \mathbf{u}) = \int \mathbf{c}_\alpha f_\alpha d\mathbf{c}_\alpha$$

is the diffusion flux of the component α;

$$p_\alpha = n_\alpha T_\alpha = \frac{1}{3}\int m_\alpha c_\alpha^2 f_\alpha d\mathbf{c}_\alpha$$

is the partial pressure;

$$p_{\alpha ik} = m_\alpha \int \left(c_{\alpha i} c_{\alpha k} - \frac{1}{3} c_\alpha^2 \delta_{ik} \right) f_\alpha d\mathbf{c}_\alpha$$

is the tensor of the viscous stresses;

$$\mathbf{q}_\alpha = \mathbf{h}_\alpha + \frac{5T_\alpha \mathbf{j}_\alpha}{2} = \frac{m_\alpha}{2} \int \mathbf{c}_\alpha c_\alpha^2 f_\alpha d\mathbf{c}_\alpha$$

is the partial heat flux.

We multiply the kinetic equations for the functions f_α by the Sonine polynomials:

$$H_\alpha = 1, \quad H_{\alpha i} = c_{\alpha i}, \quad H_{\alpha ik} = c_{\alpha i} c_{\alpha k} - \frac{T_\alpha}{m_\alpha} \delta_{ik},$$
$$H_{\alpha ikk} = c_{\alpha i} \left(c_\alpha^2 - \frac{5T_\alpha}{m_\alpha} \right) \tag{3}$$

and integrate over the velocities. Using the orthogonality of the polynomials (3), we obtain a system of equations for the moments of the distribution function. Taking (1) into account and assuming that all the quantities change little over the mean free path[1] and during the time between collisions, we get

$$\partial n_\alpha / \partial t + \nabla n_\alpha \mathbf{u}_\alpha = 0 \tag{4a}$$

or[14]

$$\partial \rho / \partial t + \nabla \mathbf{u} \rho = 0, \tag{4b}$$

$$\rho \partial C_1 / \partial t + \mathbf{u} \nabla C_1 = -m \nabla \mathbf{j}_1 = m \nabla (\mathbf{u}_1 - \mathbf{u}) n_1, \tag{4c}$$

where $\rho = mn_1 + Mn_2$; $C_1 = mn_1/\rho$ is the mass concentration of the light molecules). In our case, in view of the smallness of the ratio m/M, it is more convenient, generally speaking, to use the molar concentration $\alpha = n_1/n = n_1/(n_1 + n_2)$; the equations for it can be readily obtained from (4)).

The diffusion flux (with allowance for (1)) is

$$m\mathbf{j}_1 = -\rho D \left(\nabla C_1 + \frac{K_T}{T_1} \nabla T_1 + \frac{k_p}{p} \nabla p - \frac{\alpha m}{(1-\alpha)^2 M} \frac{\mathbf{X}_1}{T_1} \right)$$
$$= -\frac{1}{\nu} (\nabla p_1 + k_T n \nabla T_1 - n_1 \mathbf{X}_1); \tag{5a}$$

Here $\mathbf{X}$ is the force acting on the particle; $p = p_1 + p_2$ is the total pressure; ν is the effective frequency of collision between light and heavy molecules, which is connected with the customarily employed diffusion coefficient D:

$$\nu = \frac{16}{3} n_2 \Omega_{12}^{11} (1-\Delta) = \frac{T_1 (1-\alpha)}{mD},$$
$$\Delta = \frac{(5\Omega_{12}^{11} - 2\Omega_{12}^{12})^2 (1-\alpha)}{2\Omega_{12}^{11} [\alpha \Omega_{11}^{12} + (1-\alpha)(^{25}/_2 \Omega_{12}^{11} - 10\Omega_{12}^{12} + 2\Omega_{12}^{13})]}$$
$$\Omega_{\alpha\beta}^{ls} = \left[\frac{T(m_\alpha + m_\beta)}{2\pi m_\alpha m_\beta} \right]^{1/2} \int_0^\infty e^{-x^2} x^{2s+3} (1 - \cos^l \vartheta_{\alpha\beta}) d\sigma_{\alpha\beta} \tag{5b}$$

are known integrals, introduced in[11] (if $\alpha = \beta = 2$, then $T = T_2$; in the opposite case, $T = T_1$);

$$k_T = \frac{M}{m} (1-\alpha)^2 K_T = a\alpha(1-\alpha)$$
$$= \frac{5\alpha(1-\alpha)(5\Omega_{12}^{11} - 2\Omega_{12}^{12})}{2[\alpha\Omega_{11}^{12} + (1-\alpha)(^{25}/_2 \Omega_{12}^{11} - 10\Omega_{12}^{12} + 2\Omega_{12}^{13})]} \tag{5c}$$

is the coefficient of thermal diffusion of the light gas in accordance with[11,12], K_T is in accordance with[14]; a is the thermodiffusion ratio[11]. The diffusion velocity $\mathbf{u}_1 - \mathbf{u}$, which is small compared with the thermal velocities of the light molecules, may exceed the thermal velocities of the heavy molecules. The barodiffusion coefficient is

$$k_p = (M/m) C_1^2 (1 - C_1).$$

The Navier-Stokes equation takes the form

$$\rho (\partial / \partial t + \mathbf{u}\nabla)\mathbf{u} + \nabla p - n_1 \mathbf{X}_1 - n_2 \mathbf{X}_2 - 2\nabla(\eta \varepsilon_{ik}) = 0. \tag{6a}$$

Neglecting the viscosity of the light molecules, we obtain

$$\eta = \frac{5T_2}{8\Omega_{22}^{22}}, \quad \varepsilon_{ik} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) - \frac{1}{3} \delta_{ik} \nabla \mathbf{u}. \tag{6b}$$

The partial temperatures are determined by the relations

$$\frac{3}{2} n_1 \left(\frac{\partial}{\partial t} + \mathbf{u}_1 \nabla \right) T_1 + p_1 \nabla \mathbf{u}_1 - \nabla (\varkappa_1 \nabla T_1) + \nabla \left(\frac{5}{2\alpha} k_T T_1 \mathbf{j}_1 \right)$$
$$- \frac{m\nu}{n_1} j_1^2 + \frac{k_T}{\alpha} \mathbf{j}_1 \nabla T_1 = \frac{3m}{M} \frac{\nu n_1}{1-\Delta} (T_2 - T_1), \tag{7a}$$

$$\frac{3}{2} n_2 \left(\frac{\partial}{\partial t} + \mathbf{u}_2 \nabla \right) T_2 + p_2 \nabla \mathbf{u}_2 - \nabla (\varkappa_2 \nabla T_2) - 2\eta \varepsilon_{ik} \frac{\partial u_i}{\partial x_k}$$
$$= \frac{3m}{M} \frac{\nu n_1}{1-\Delta} (T_1 - T_2). \tag{7b}$$

The thermal conductivity coefficients are of the form

$$\varkappa_1 = \frac{75T_1}{32m} \left[\Omega_{11}^{22} + \frac{1-\alpha}{\alpha} \left(\frac{25}{2} \Omega_{12}^{11} - 10\Omega_{12}^{12} + 2\Omega_{12}^{13} \right) \right]^{-1},$$
$$\varkappa_2 = \frac{75T_2}{32M\Omega_{22}^{22}}. \tag{7c}$$

The heat due to the diffusion is released in the light gas, and that due to viscosity in the heavy gas[5].

A generalization of the system (4)–(7) to polyatomic molecules is generally difficult; we note, however, that the relaxation times of the rotational degrees of freedom are usually of the order of the time between the collisions, whereas for the oscillations these times as a rule are quite appreciable. Thus, in many cases we can confine ourselves to a replacement of the factor $^3/_2$ in the left side of (7a) and (7b) by $(\gamma_\alpha - 1)^{-1}$, where $\gamma_\alpha = (c_p/c_v)$, and introduce, in accordance with Mandel'shtam and Leontovich, a second viscosity, which takes into account the relaxation of the oscillation[14]. The values of η, D_{12}, and k_T depend little on the structure of the molecules, whereas the dependence of the thermal conductivity can be taken into account with satisfactory accuracy with the aid of the Euken correction[12].

2. PROPAGATION OF SOUND

The influence of the large difference between the masses of the mixture molecules on the propagation of the sound was considered in[7] in two limiting cases, $\omega \gg (m/M)\nu$, and $\omega \ll (m/M)\nu$ (ω–cyclic frequency). In this case the velocity of sound changes, for example, from a value c_0 to c_∞, where

$$c_0 = \{\gamma T / [\alpha m + (1-\alpha)M]\}^{1/2}, \quad c_\infty = (\gamma_2 T / M)^{1/2}, \tag{8}$$

and

$$\gamma = \frac{\alpha(\gamma_1 - \gamma_2) + \gamma_2(\gamma_1 - 1)}{\alpha(\gamma_2 - \gamma_1) + \gamma_1 - 1}.$$

An analysis of the general dispersion equation (of sixth order in ω) is quite difficult. Its solution, applicable for all frequencies, can be obtained only when

[1] We exclude from consideration here a number of interesting special cases, such as the barodiffusion in a viscous stream, which was investigated in [13].

$\alpha \ll 1$. The influence of the light gas on the sound is small in this case, so that in the zeroth approximation in α we can neglect the terms containing u_1 and T_1 in Eqs. (6a) and (7b), find from them u_2 and T_2, and substitute in (5a) and (7a). Expression then u_1 and T_1 in terms of u_2 and T_2, and again using (6a) and (7b), we obtain equations containing only u_2 and T_2. The solution of the corresponding dispersion equation for the particular case $\gamma_1 = \gamma_2 = \gamma$ is of the form (k—wave vector, $\Omega = M\omega/m\nu$, and the perturbation is in the form $\exp(-i\omega t + ikx)$:

$$\omega = k\sqrt{\frac{T\gamma}{M}} - \frac{i}{2}\left\{\frac{k^2}{Mn_2}\left(\frac{4}{3}\eta + \zeta + \frac{M\varkappa_2(\gamma-1)^2}{\gamma}\right) + i\alpha\omega\frac{A}{B}\right\}, \quad (9)$$

where

$$A = 1 - i\Omega\left[\frac{3\gamma^2-5\gamma+3}{3\gamma(\gamma-1)} + \frac{m\nu\varkappa_1}{Tn_1\gamma^2}(\gamma-1)^2\right] + \Omega^2\frac{m\nu\varkappa_1}{Tn_1}\frac{3\gamma^2-5\gamma+3}{3\gamma^3},$$

$$B = 1 - i\Omega\frac{3-2\gamma}{3\gamma(\gamma-1)} + \Omega^2\left[\frac{1}{3(\gamma-1)} + \frac{\varkappa_1 m\nu}{3Tn_1\gamma}\left(1 - \frac{i\Omega}{\gamma}\right)\right].$$

In the calculations we have neglected the inertia and the viscosity of the light component, whose contributions are in higher order of $\sqrt{m/M}$, and we have left out the rather cumbersome terms due to thermal diffusion and the Dufour effect (heat flux connected with relative velocity). Usually these terms are small (on the order of several per cent of those taken into account) and do not change the result appreciably.

Figure 1 shows the velocity of sound (curve 1) and the absorption (curve 2), calculated in accordance with (9) for a mixture 10% He—90% Xe. The atoms were assumed to be hard spheres with diameters $\sigma_1 = 217$ Å and $\sigma_2 = 4.92$ Å, in accordance with[11]. Under normal conditions $\omega = 1.66 \times 10^{-8}\,\Omega\,[\sec^{-1}]$. For comparison, we show the values of the sound absorption in the same model, obtained in accordance with[1] from ordinary hydrodynamics (straight line a); the straight line b represents the part of the absorption due to the heavy component (first term in the curly bracket (9)). The collision frequency of the Xe atoms corresponds to a value $\Omega \sim 7$; at such frequencies, our analysis no longer holds.

3. STRUCTURE OF SHOCK WAVE

An analysis of a small-amplitude shock wave yields results that agree, in the main, with those obtained by D'yakov[3] (see also[13]; namely, the width of the shock wave L is proportional to the attenuation coefficient of the (low-frequency) sound, divided by ω^2. However, when $\gamma_1 \neq \gamma_2$ there is produced in the mixture an additional attenuation of the sound, due to the slowness of the heat exchange between the component[7]:

$$(-\Delta\omega)_{\text{heat exch.}} = \frac{i\alpha\omega^2}{6mM\nu c_p c_v}\left(\frac{1}{\gamma_1-1} - \frac{1}{\gamma_2-1}\right)^2$$

(c_p and c_v are the specific heat per unit mass of the mixture). This circumstance can lead to a broadening of the front of the shock wave by 10–20%.

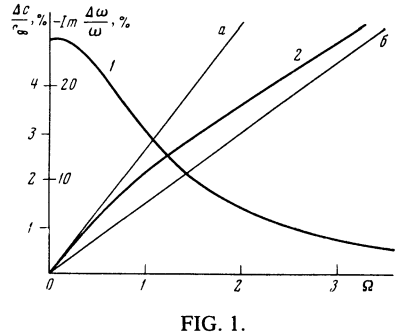

FIG. 1.

In the case of a strong shock wave, the transport coefficients can no longer be regarded as constant, so that the problem reduces to a numerical integration of the system (5)—(7) for a definite molecular model (we note that an estimate in accordance with[14] of the width of the strong wave yields $L \sim l\sqrt{M/m}$, where l is the mean free path; thus, the equations (6) themselves are applicable in this case, too). Let us write out the equations for the shock wave using the same simplifications as in the derivation (9). We disregard likewise the viscosity and the thermal conductivity of the heavy components. We put $\partial/\partial t = 0$ and introduce the dimensionless variables

$$v = \frac{u_1}{u^{(0)}}, \quad w = \frac{u_2}{u^{(0)}}, \quad \theta = \frac{2T_1}{M_0^2\gamma T^{(0)}},$$

$$\tau = \frac{2T_2}{M_0^2\gamma T^{(0)}}, \quad z = \sqrt{\frac{2\gamma}{\gamma+1}}\frac{2m\nu^{(1)}}{Mc_0^{(1)}(1-\alpha)}x,$$

$$\varkappa' = \frac{2m\nu^{(1)}\varkappa_1}{Mn_2u^{(0)}c_0^{(1)}(\gamma-1)}\sqrt{2\gamma(\gamma+1)}. \quad (10)$$

The superscripts 0 and 1 correspond to the gas of preceding and following the wave front; the Mach number is $M_0 = u^{(0)}/c_0^{(0)}$; α is the concentration of the light component in the unperturbed gas. We have

$$\frac{d}{dz}\left(\frac{\theta}{v}\right) = -\frac{\nu}{\nu^{(1)}}\frac{v-w}{v},$$

$$\tau = \frac{2w}{1-\alpha}\left(1 - w - \frac{\alpha\theta}{2v} + \frac{1}{\gamma M_0^2}\right),$$

$$\frac{d}{dz}\left[\tau(1-\alpha) + \frac{\gamma-1}{\gamma}w^2\right] =$$

$$= -\frac{\nu}{\nu^{(1)}}\frac{\alpha(1-\alpha)(\gamma-1)}{2\gamma v}\left[\frac{3}{1-\Delta}(\theta-\tau) + \frac{2w}{1-\alpha}(v-w)\right],$$

$$\varkappa'\frac{d\vartheta}{dz} = \frac{\gamma}{\gamma+1}\left[\alpha\theta + (1-\alpha)\tau - \frac{2}{\gamma M_0^2}\right] + w^2 - 1. \quad (11)$$

Eliminating τ and writing out the expression for dw/dz, we can verify, by using the relations between the initial and final values of[14,15], that is $M_0 > (M_0)_{\text{crit}}$, then the denominator passes through zero; when $M_0 = (M_0)_{\text{crit}}$, the zero of the denominator coincides with the final point. The critical Mach number is

$$(M_0^2)_{\text{crit}} = \frac{2\gamma + \gamma^*(\gamma-1)}{\gamma(2\gamma^* + 1 - \gamma)}, \quad \gamma^* = \gamma_2\left(1 - \alpha + \frac{m}{M}\alpha\right). \quad (12)$$

This means that a continuous variation of w (and τ) becomes impossible, and a jump is produced; the discontinuous quantities are w, τ, $d\theta/dz$, and dv/dz. When M_0 increases from $(M_0)_{\text{crit}}$, this jump moves from the final point towards the beginning of the wave. The value of $(M_0)_{\text{crit}}$ can be obtained by considering the stability of the jump against acoustic perturbations[9,14], which yields the condition for the formation of the discontinuity in the form $u^{(1)} = c_\infty^{(1)}$, which agrees with (12). The necessary dissipation is produced in the jump by the viscosity and by the thermal conductivity of the heavy component, but allowance for their contri-

butions is beyond the scope of the present approximation. It is easy to see that for such a mixture, in which

$$2\gamma^* < \gamma - 1, \tag{13}$$

the jump is not produced at all (see, however, Sec. 4).

It is of interest to compare (12) and (13) with the results of Cowling[2], who calculated, starting from the usual hydrodynamic equations, the upper limit of the amplitude of the shock wave with allowance for barodiffusion only. We present data for the He-Xe mixture ($(M_0^2)_C$—Cowling's data):

α:	0.1	0.3	0.5	0.7	0.8	0.9
$(M_0^2)_{\text{crit}}$:	1.11	1.46	2.24	5.52	27.5	∞
$(M_0^2)_C$:	1.18	1.78	3.03	16.35	∞	∞

We see that although the thermal conductivity of the light component was taken into account in the derivation of (12), this condition is more stringent than that given by Cowling.

Figure 2 shows the results of a numerical solution of Eqs. (11) for the 50% He—50% Xe mixture (the parameters for the molecules are the same as in Sec. 2) at $M_0 = 2$. The solution near the initial and final points can be readily obtained by linearizing the system (11). The condition of monotonic variation of the quantities behind the wave front makes it possible to choose one of the three roots of the resultant characteristic equation; the initial point, on the other hand, is a generalized saddle, so that it is necessary to construct a family of integral curves and to choose a monotonic curve from among them. The solutions constructed in this manner are joined together at the point $\theta_I = \theta_{II}$, $v_I = v_{II}$. The velocity w of the heavy component changes at the jump from 0.69 to 0.83; the temperature θ decreases ahead of the wave much more slowly than v. The mean free path of the heavy molecule in the region of the jump amounts to 0.5—0.6 in our units.

Let us consider now the case of an extremely strong shock wave $M_0 \to \infty$. The coefficients of the thermal conductivity and diffusion vanish in this case at a certain point ahead of the wave; the velocity and concentration of the light component experience a discontinuity at this point. Let us write out Eqs. (11) for the case of molecules interacting in accordance with a power law $F_{\alpha\beta} = \chi_{\alpha\beta}/r^{\mu}$:

$$\frac{d\theta}{dz} = \theta^{-k}\left(\frac{a}{1-\alpha}+\frac{bv}{\alpha w}\right)\left[\frac{\alpha\gamma}{\gamma+1}\theta\left(1-\frac{w}{v}\right)+(1-w)(w-w^{(1)})\right] \tag{14a}$$

$$\frac{dv}{dz} = \frac{v}{\theta}\frac{d\theta}{dz} + \theta^{-k}\frac{\gamma-1}{\gamma+1}\frac{v}{w}(v-w), \tag{14b}$$

$$\frac{dw}{dz} = \theta^{1-k}\frac{(\gamma-1)^2}{\gamma(\gamma+1)}\frac{a}{2v}\left[3\theta\left(\frac{1-\alpha}{w}-\frac{\alpha}{v}\right)-\right.$$
$$\left.-6(1-w)-\frac{v}{\gamma-1}(v-w)\right]\left[1-\frac{\gamma+1}{\gamma}w-\frac{\alpha\theta}{2v}\right]^{-1}, \tag{14c}$$

where $k = (\mu + 3)/2(\mu - 1)$; a and b are constants on the order of unity. Dividing (14b) by (14c) we can easily see that v remains finite when $\theta \to 0$ only if

$$\lim_{\theta\to 0}\frac{1-w}{\theta(1-v)} \neq \infty.$$

From (14b) and (14c) it follows that $(1 - w) \sim \theta$, whereas when $\theta \to 0$ we have $\lim v = v_0 \neq 1$. Thus, the solution of (14) near the initial point is given by

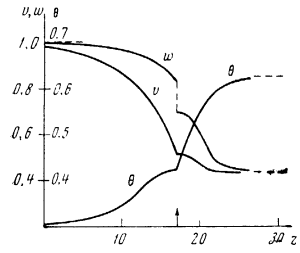

FIG. 2.

The value of v_0 can be found from the requirement that dv/dz be finite when $\theta \to +0$.

Figure 3 shows the results of a numerical solution for the same 50% mixture of spheres with masses and radii corresponding to helium and xenon; in this case $v_0 = 0.626$. In order to determine the behavior of the solution near the initial point (the coefficient c_1), it is necessary to retain also the coefficients a_2 and b_2. The numerical calculation simplifies greatly, since there is only one solution near the two singular points (initial and final); this solution is finite when $z \to \pm\infty$. The mean free path of the heavy molecule in the region of the second jump is of the order of 0.7—0.9; the concentration of the light molecules in the hot region behind the jump is practically constant.

4. INFLUENCE OF MIXTURE CONCENTRATION

We shall assume for simplicity that all the collision cross sections are comparable. Figure 4 shows schematically the dependence of the collision frequencies on the concentration; the frequencies corresponding to the vertical lines differ by an approximate factor $\sqrt{m/M}$; the shaded area is the one bounded by the inequalities (1). The reciprocal momentum and energy relaxation times are $\nu_{\alpha\beta}$ and $\nu_{\alpha}^{(e)}$; $\nu_{21} \sim \nu_{21}^{(e)}$. If the concentration of the light component is lower then $\alpha_1 = m\Sigma_{12}/M\Sigma_{11}$, then the energy and momentum relaxation of the light particles takes place on the heavy particles, and the partial temperature of the light particles becomes meaningless; Eqs. (6) remain qualitatively in force in this case, but the expressions for the transport coefficients (7) for the light particles have to be modified. On the other hand, if $\alpha_2 < \alpha < \alpha_3$, where

$$\theta = \sum_{n=1}^{\infty} a_n z^{n/k}, \quad w = 1 + \sum_{n=1}^{\infty} b_n z^{n/k}, \quad v = v_0 + \sum_{n=1}^{\infty} c_n z^{n/k}.$$

then the density of the mixture is determined by the heavy component, but the transport coefficients for the heavy particles are determined by their relaxation on the light particles, and in this case T_2 becomes mean-

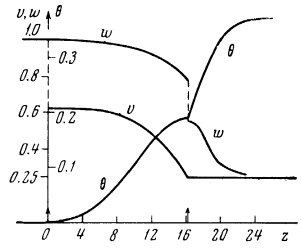

FIG. 3.

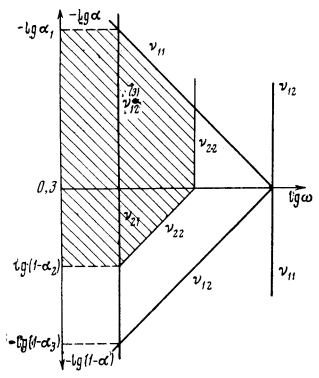

FIG. 4.

ingless. The mean free path l_2 increases with increasing α, and when $\alpha \sim \alpha_2$ it reaches a value

$$1-\alpha_2=\frac{\Sigma_{12}}{\Sigma_{11}}\sqrt{\frac{m}{M}},\quad 1-\alpha_3=\frac{m}{M},$$

This length determines the width of the viscous jump, so that when $\alpha > \alpha_2$ the difference between the continuous and discontinuous solutions for the shock wave becomes meaningless. The heavy particles take part in the acoustic oscillations at such concentrations only when $\omega < \nu_{21} + \nu_{22}$; on the other hand, if the frequency exceeds ν_{12} (but it is smaller than ν_{11}), then sound can propagate through the light component; in this case the speed of sound is $c' = \sqrt{T_{\gamma_1}/m}$. Thus, when sound propagates in a mixture with $1 \gg 1 - \alpha > 1 - \alpha_2$, both positive (when $\omega \sim \nu_{12}$) and negative (when $\omega \sim \nu_{12}$) dispersion of the speed of sound are possible.

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