EFFECT OF SPATIAL DISPERSION ON THE SPECTRUM OF MAGNETOPLASMA WAVES IN BISMUTH

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The dependence of the speed of magnetoplasma waves in bismuth has been studied as a function of the magnetic field strength at a frequency of 9 MHz and for strong spatial dispersion due to electron drift along the wave vector. The wave spectrum is calculated on the basis of an ellipsoidal model of the Fermi surface of bismuth, with account taken of spatial dispersion, and the results of the calculations are compared with experiments. The speed of the magnetoplasma wave is found to depend on the distance from the sample surface in the case of strong Landau damping.

INTRODUCTION

THE appearance of Landau damping of magnetoplasma waves in bismuth was discovered in[1]. It arises in a strong spatial dispersion when the projection of the drift velocity of the current carriers on the direction of the wave vector is equal to the phase velocity of the wave, i.e., $\mathbf{k}\cdot\overline{\mathbf{v}}/\omega = 1$. This phenomenon was used in[1,2] for the measurement of the Fermi velocity v_F of electrons of the limiting point.

The spatial dispersion appears not only in the damping of the wave but also in the dependence of its speed on the magnetic field H. This is precisely the cause of the deviation from periodicity (as a function of the reciprocal field H^{-1}) of the oscillations of the surface resistance of bismuth, which are produced by the excitation of standing magnetoplasma waves, which discovered in[3] (see Fig. 4 in[3]). By developing this theory, we can calculate the spectrum of the waves in the presence of dispersion if the shape of the Fermi surface is known. Excellent agreement of the calculated results, which use the ellipsoidal model of the Fermi surface, with experiment in the asymptotic region of strong fields (i.e., for $(\omega/\Omega)^2 \ll 1$, $(\mathbf{k}\cdot\mathbf{v}_F/\omega)^2 \ll 1$)[4] allows us to hope that this model will describe the behavior of waves in the case considered by us with sufficient accuracy.

Starting out from what has been said, we calculate in the present work the velocity and damping of magnetoplasma waves in bismuth for the cases $\mathbf{H} \parallel \mathbf{k} \parallel C_3$[1)] and $\mathbf{k} \parallel C_2$, $\mathbf{H} \perp C_3$, $\sphericalangle(\mathbf{H}, C_3) \ll 1$ with the use of the parameters of the ellipsoidal model of the Fermi surface of bismuth, given in[4]. The results of the calculations are compared with experiment. According to the analysis given above, the theory suggested in[3] is applicable only so long as the Landau damping does not surpass the value $(\omega\tau)^{-1}$ (τ is the relaxation time of the current carriers). For very strong Landau damping, the wave field cannot be described by the law $\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$, and the velocity of the wave depends on the coordinates. An experiment was undertaken which confirmed this phenomenon.

EXPERIMENT

The experiments were carried out at a frequency ~9 GHz in fields up to ~10 kG at a specimen temperature of ~1.5°K. The experimental technique was similar to that described in[1,4]. Bismuth samples were used in the form of disks of 18 mm diameter and 1 mm thickness. The normal **N** to the plane of the specimen surface coincided, with an accuracy ~1–2°, with the direction of the trigonal axis C_3 or the binary axis C_2 of the bismuth crystal.

The direction of the magnetic field $\mathbf{H} \parallel C_3$ was determined with an accuracy ~10′ on a sample with $\mathbf{N} \parallel C_3$ from the equality of the velocities of magnetoplasma waves of two types.[4] The direction $\mathbf{H} \parallel C_2$ for a sample with $\mathbf{N} \parallel C_2$ was established with an accuracy ~10′ using the cyclotron resonance for electrons shifted by the Doppler effect,[1] and the direction $\mathbf{H} \parallel C_1 \perp \mathbf{N}$ with an accuracy ~30′ using the cyclotron resonance.

1. WAVE SPECTRUM FOR $\mathbf{H} \parallel \mathbf{K} \parallel C_3$

The theoretical consideration of the behavior of the waves for $\mathbf{k}\cdot\overline{\mathbf{v}}/\omega \approx 1$ we begin with the simplest case $\mathbf{H} \parallel \mathbf{k} \parallel C_3$. For such an orientation of the field and wave vector, all the nondiagonal components of the conductivity tensor vanish, because of the high symmetry in the approximation $(\omega/\Omega)^2 \ll 1$, and the dispersion equation breaks up into two identical equations

$$i\sigma_\perp = \frac{4\pi}{c}k^2,$$

where $\sigma_\perp = \sigma_{11}(3) = \sigma_{22}(3)$[2)]. In the consideration of the problem at hand, it is necessary to take it into account that, in accord with the estimates made from the recording of the experiments (Fig. 1), $(\omega/\Omega)^2 \lesssim 0.1$ when $\mathbf{k}\cdot\mathbf{v}_F/\omega \approx 1$ (v_F and Ω in this case are taken for electrons), and therefore, for the calculation

[1)]The following notation is used for the axes of the bismuth crystal: C_1 –bisector, C_2–binary, C_3 –trigonal.

[2)]Here and below, we use the same notation as in [3].

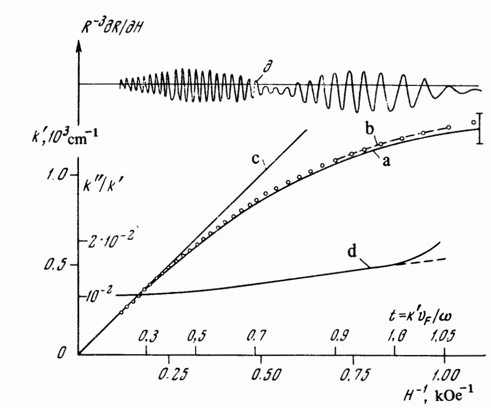

FIG. 1. Spectrum of a wave for the case of strong spatial dispersion. Points—experimental for $H\|k\|C_3$ and frequency 9.35 GHz. Curve a—calculation for $(\omega/\Omega)^2 \ll 1$, dashed b—calculation with account of correction for time dispersion; straight line c—asymptote for $H^{-1} \to 0$; curve d—calculation of damping for $\omega\tau^{-1} \approx 2 \times 10^{-2}$. Vertical bar on curve a denotes the accuracy of the calculation. The decrease in the amplitude of the oscillations in the middle of the recording of the experiment with decrease in H^{-1} is connected with the increase in R^3, and the subsequent increase, with the more rapid growth of the amplitude $\partial R/\partial H$ at the expense of a decrease in the period of ΔH^{-1}; as $H^{-1} \to 0$, the amplitude of the oscillations again falls off because of the increase in R^3.

of $\sigma_\perp$ one must use Eq. (6) of[3].3) The difference from the asymptotic case $H^{-1} \to 0$ lies in the fact that the first term in this formula is of the order of the second. All the integrals entering in the conductivity tensor can be computed within the framework of the ellipsoidal model. Since the field enters in $\sigma_\perp$ in the form of the factor H^{-2}, it follows that H^{-1} is expressed in explicit form as a function of the parameter $t = \mathbf{k}\cdot\mathbf{v}_F/\omega$. The results of the calculation, carried out for t varying in the range t = 0 – 1.05, is shown in Fig. 1. There the dashed curve is the theoretical curve, which takes into account the correction due to the successive terms of the expansion in $(\omega/\Omega)^2$. The agreement of the calculation with experiment is seen to be satisfactory; one can expect, a priori, significant divergences, associated with the difference in the shape of the electron Fermi surface from ellipsoidal,[5] which is particularly appreciable close to the limiting points.

In addition to the real part k', one can also compute the imaginary part k'' in this case. By introducing the complex $k = k' + ik''$ in the conductivity tensor, as well as $\omega + i/\tau$ (in place of ω), and assuming $k''/k' \ll 1$ and $1/\omega\tau \ll 1$ (which is seen to be true in the region of fields under study), we can obtain the $k''(t)$ dependence in explicit form from the dispersion equation (here $t = k'v_F/\omega$); it is represented in Fig. 1. The relative damping grows smoothly as t increases, from the value $k''/k' = \frac{1}{2}\omega\tau$ for t = 0 to $\approx 0.8/\omega\tau$ for t = 1. Upon further increase in t, an additional term appears, connected with the Landau damping, and is proportional to $\chi^2(t-1)^2$, where χ is the angle of inclination of the electron ellipsoid with the basal plane. Setting $\omega\tau = 50$ (which is comparable, for example, with the value obtained in[1]), it is not difficult to find that when t changes from 1 to 1.05 (at this value we have $k''/k' \approx 1/\omega\tau$) the amplitude of the oscillations for a sample of thickness of 1 mm falls by a factor of about 2, which is in qualitative agreement with experiment (see Fig. 1). The relative value of the impedance oscillations is $\Delta R/R \approx 10^{-2}$ under these conditions.

A solution in the form of an exponentially damped wave (which was assumed in the calculation of the conductivity tensor in[3]) exists only so long as $k''/k' < 1/\omega\tau$; as soon as these values are comparable, the solution of the problem in this form fails. This is natural, since the field in the depth of the plasma, for strong Landau damping, changes according to a more complicated law than the exponential,[6] and allowance for the effect of the boundary of the sample becomes very important. This was not made in the calculation carried out for an infinite metal. It is possible that allowance for the boundary also brings about some corrections in the solution for $k''/k' < 1/\omega\tau$ obtained by us.

In passing, we note that Kaner and Skobov,[7] in consideration of the Landau damping of magnetoplasma waves, obtained the result formulated in the following way: for **H** parallel to **k** and directed along the axis of symmetry of higher order (third and above), the Landau damping is absent.4) This statement is in contradiction with the calculation given above and with experiment. The fact is that the results of[7] refer only to the case in which the portions of the Fermi surface that are capable of making a contribution to the Landau damping possess high symmetry; in our case, each separate ellipsoid does not have such symmetry and only three ellipsoids in the set possess symmetries of third order. Besides, for the absence of Landau damping, it suffices that the field be directed along the axis of symmetry of second order of the part of the Fermi surface which makes a contribution to the damping. For example, for bismuth, for $\mathbf{H} \parallel \mathbf{k} \parallel C_2$, the electrons of the ellipsoid whose major axis is perpendicular to the binary axis under study have the maximum velocity. From the symmetry requirements for this case, it follows that

$$v_x(\varphi) = -v_x(\pi+\varphi), \quad v_y(\varphi) = -v_y(\pi+\varphi), \quad v_z(\varphi) = v_z(\pi+\varphi).$$

Expanding v_x, v_y, and v_z in Fourier series in φ, making use of the definition of $\psi(\varphi)$ and the expression for the conductivity tensor from[3], it is easy to see that the nondiagonal components $\sigma_{zx} = \sigma_{xz} = \sigma_{zy} = \sigma_{yz} = 0$ in any order of expansion in ψ/Ω, while the coefficients in σ_{xx}, σ_{yy} and σ_{xy} are equal to zero for terms containing $\omega - \mathbf{k}\cdot\mathbf{v}_z$ in the denominator. This leads to the absence of Landau damping for both waves, in agreement with experiment.[1]

The absence of Landau damping in the case under study is easily explained. Actually, the electrons that contribute to the damping, which move along with the wave in the coordinate frame connected with the wave,

3)We note that the sign in front of the first term of Eq. (6) in [3] is incorrect.

4)In the paper of Kaner and Skobov [8] it was reported that the Landau damping is absent only in the particular case when the wave vector **k** and the field **H** are parallel to axes of third or sixth order and the centers of the orbits of each group of carriers are located on a line parallel to this symmetry axis.

are situated in a constant electric field perpendicular to the magnetic (this is guaranteed by the vanishing of the components $\sigma_{z\alpha}$) and the action of the electric field, averaged over the cyclotron period, on an electron moving along a symmetric orbit does not give dissipation of the energy.

Similar considerations allow us to understand the results given above for $\mathbf{H} \parallel \mathbf{k} \parallel C_3$. The electron orbits in this case are inclined to the magnetic field. It is evident that the action of a spatially inhomogeneous electric field on an electron moving along such orbits leads to the appearance of finite Landau damping due to the inclination of the orbit to the magnetic field, because of the slope of the latter relative to the axis of the ellipsoid. Since the diameter of the orbit for electrons moving in phase with the wave increase with increase in t, beginning with a zero value at t = 1, it follows that for t = 1 the damping is equal to zero and successively increases, as was obtained in the calculation.

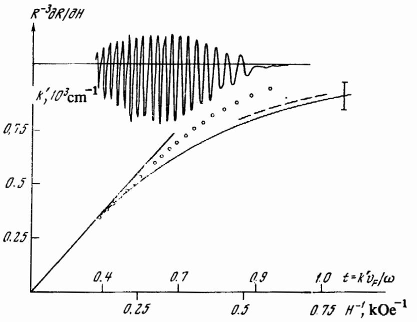

FIG. 2. Spectrum of the wave for strong spatial dispersion. Points—data of experiment for 9.62 GHz, $\mathbf{k} \| C_2$, $\mathbf{H} \perp C_3 \| \mathbf{J}$, $\vartheta = 4°\,40'$. Continuous curve—calculation for $\omega/\Omega \ll 1$, dashed curve—with account of the correction for time dispersion. Straight line—asymptote as $H^{-1} \to 0$. The vertical bar indicates the accuracy of the calcualtion.

2. SPECTRUM OF THE WAVES FOR $\mathbf{k} \parallel C_2$, $\mathbf{H} \perp C_3$, $\measuredangle(\mathbf{H}, C_3) \ll 1$

It is possible, by the method outlined in Sec. 1, to consider the speed of magnetoplasma waves, within the framework of the ellipsoidal model, for H directed not parallel to any symmetry axis. However, in this case, the calculation can be very complicated, because of the necessity of accounting for a large number of components σ_{ik}. Thus, for $\mathbf{k} \parallel C_2$, $\mathbf{H} \perp C_3$, $\measuredangle(\mathbf{H}, \mathbf{k}) = \vartheta \neq 0$, $\mathbf{J} \parallel C_3$ (J is the microwave current) it is necessary to take into account σ_{xx}, σ_{zx}, and σ_{zz}. The dispersion equation here takes the form

$$\frac{4\pi}{c}k^2 = i\left[\sigma_{xx} + \frac{\sigma_{zx}^2}{\sigma_{zz}}\right].$$

The result of the calculation of the wave velocity for $\vartheta = 4°40'$ is represented in Fig. 2. Qualitatively, the behaviors of the experimental and computed curves are the same, although, in contrast with the case of Fig. 1, a notable (~10%) quantitative difference is observed. This divergence can evidently be due to the fact that in the calculation, under the condition $\mathbf{k}\cdot\mathbf{v}_F/\omega \approx 1$, the electrons of the limiting points, near which the Fermi surface differs from an ellipsoid, make the chief contribution.[6]

Allowance for the terms σ_{zx} and σ_{zz} in the dispersion equation leads to great difficulties in the calculation of the damping as a function of the parameter t for values of the latter that are close to unity, i.e., in the most interesting region. This is connected with the fact that as $t \to 1$ the value of $|\sigma_{zz}|$, neglecting the dissipation processes, approaches infinity, while $\mathrm{Re}\,\sigma_{zz}$ jumps from zero at $t < 1$ to π for $t > 1$ (see Eq. (5) in[3]). Allowance for the collisions and for the finite value of k''/k' leads to the result that $|\sigma_{zz}|$ remains finite, but is a rapidly changing function of t, and hence, in a narrow range of t ($\Delta t \approx 1/\omega\tau$), the damping of the wave increases jumpwise; its value is proportional to ϑ^2 and is comparable with the relaxation damping at $\vartheta \approx 3-4°$.

An exact calculation of the current has not yet been carried out, not only because of the necessity of practically guessing the solution, but also because of fundamental difficulties. The fact is that all the formulas in[3] were obtained under the assumption T = 0°; in the experiment, $T \approx 1.5°$K and $T/T_F \approx 10^{-2} \approx 1/\omega\tau$, which obviously leads to the necessity of calculation of the temperature smearing of the Fermi level; on the other hand, allowance for the Landau quantization, which is especially significant near the limiting point, can also contribute a significant correction to the calculation. If we turn to a comparison with experiment, then the results of an exact calculation at the present stage are not required, since measurement of the absolute damping has not been made, and the picture of the sharp decrease in the amplitude of oscillations with increase of H^{-1} in the region $t \approx 1$ has been observed only qualitatively. For "oblique" directions of the field H (i.e., for $\vartheta \neq 0$) the value of the region Δt in which damping increases strongly is evidently small, as can be judged from a comparison of the electron velocity, obtained under different conditions of observation, and calculation according to the model of the Fermi surface of bismuth (Fig. 6 from[1]).

3. PROPAGATION OF ELECTROMAGNETIC EXCITATIONS FOR $\mathbf{k}\cdot\mathbf{v}_F/\omega > 1$ UNDER CONDITIONS OF STRONG LANDAU DAMPING

In the consideration of this question, we limit ourselves to a qualitative discussion. As has already been shown in Sec. 1, for $k''/k' \gtrsim 1/\omega\tau$, the solution in the form of exponentially damping waves ceases to satisfy the system of Maxwell's equations and the kinetic equation. This can be explained in the following way: let a wave be propagated from the surface of the sample with a phase velocity $v_{ph} = \omega/k < v_F$. The presence of an electric field disturbs the equilibrium distribution of current carriers. Since there are electrons moving with a speed $v_F > v_{ph}$, they "carry" information on the electromagnetic field to the depth of the free path l in a time $\tau \approx l/v_F < l/v_{ph}$. If the damping length is less than l (which is determined by the condition $k''/k' > 1/\omega\tau$), then, at distances of the order of l from the surface, the field will be determined by "fast" electrons. In other words, the phase velocity increases as one moves into the body of the metal, and tends to the value v_F.

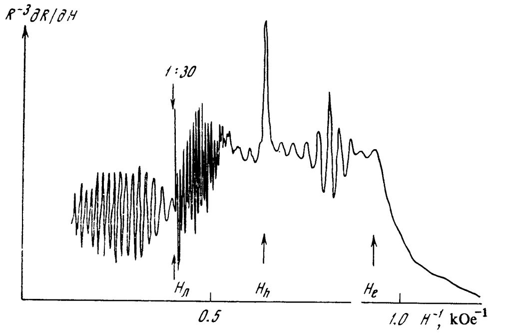

FIG. 3. Recording of the oscillations of the surface resistance (9.62 GHz, $\mathbf{k} \| C_2$, $\mathbf{H} \perp C_3 \| \mathbf{J}$, $\vartheta = 3°\ 10'$). H_e–field of the cyclotron resonance on electrons, shifted by the Doppler effect. H_L–limiting field of Landau damping. Partial oscillations are connected with the wave whose ultrahigh frequency currents in strong fields are close to the direction of the C_1 axis. H_h is the cyclotron resonance (shifted by the Doppler effect) on holes associated with this same wave. 1:30 is the position of change of the magnification of the picture by a factor of 30.

The described distribution of the field in the metal is similar to that which takes place in the anomalous skin effect. This similarity is not accidental, since the distribution of the high-frequency field corresponding to the anomalous skin effect (in a metal without a magnetic field) is the limiting case as $H \to 0$.

These simple qualitative considerations were confirmed by experiment (Fig. 3). Upon appearance of Landau damping, the amplitude of the oscillations falls off by two order of magnitude; however, the oscillations are traced by means of the cyclotron resonance of the electrons, shifted by the Doppler effect. The value of the wave vector, averaged over the thickness of the sample, can be found by a calculation of the number of oscillations: for the experiment considered in Fig. 3, $\bar{k} = (1.38 \pm 0.06) \times 10^3\ \mathrm{cm}^{-1}$. In addition, knowing the effective mass[5] and the velocity v_F (which is determined from the same recording from the limit of the Landau damping) we can calculate the value of the wave vector at the surface from the resonance condition $\Omega = \omega + \mathbf{k} \cdot \mathbf{v}_F$: $k_s = (1.76 \pm 0.1) \times 10^3\ \mathrm{cm}^{-1}$. The difference between k and k_s, which exceeds the error of the measurement, is agreement with the qualitative considerations given above: as the distance from the surface is increased, k decreases, i.e., the phase velocity of the wave increases.

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