INFLUENCE OF ELECTRON-PHONON INTERACTION ON HIGH-FREQUENCY PROPERTIES OF SUPERCONDUCTORS

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It is shown that the mechanism proposed by Shin[8] cannot explain the subthreshold absorption of infrared radiation in superconductors

RICHARDS, Tinkham, et al.[1-4] observed anomalous absorption of infrared radiation in Pb, Hg, and Sn. In the case of Pb and Hg, the anomalous absorption was observed at an incident-radiation frequency lower than the frequency of the gas, while for Sn it was higher than the gas frequency. The "precursors" in Pb and Hg were observed in both thin films and bulky samples. The "precursor" is a peak of absorption below the energy of the gap (for Pb, $R_s(\omega)/R_n(\omega) \approx 0.1$–$0.2$, where $R_s(\omega)$ and $R_n(\omega)$ are respectively the resistance of the metal in the superconducting and normal states under conditions of the anomalous skin effect). The experiments were performed at a temperature $T \ll T_c$ (T_c—temperature of transition to the superconducting state). Drew and Sievers[5] recently reported an experiment in which they did not observe a "precursor" in bulky superconducting lead. They assumed that the previously observed absorption peaks was a result of an experimental error.

It is not likely that the "precursors" are due to the anisotropy on the gap on the Fermi surface, since these effects were observed in films and in non-diluted alloys (for example, it is shown in[4] that in the case of Pb with 16% impurity, the "precursor" is located in the same place and has the same magnitude as in pure samples). The appearance of the "precursor" could likewise not be explained as being due to the collective excitations below the energy of the gap[6,7]. Shin[8] proposed that the "precursor" is connected with simultaneous absorption of a Cooper pair of a photon of the incident radiation and a phonon of the lattice; the pair can absorb a photon and decay when $\omega < 2\Delta$, if $\omega + \Omega > 2\Delta$, where ω—frequency of the incident radiation, Ω—phonon frequency, and Δ—parameter of the energy gap[1]. Shin[8] obtained an absorption peak below the energy gap and found the temperature dependence of the position of this peak, but gave no correct calculation of the height of the peak or a comparison of the calculation with experiment.

In the present paper we calculate the contribution made by the mechanism proposed by Shin[8] to the absorption of high-frequency radiation by a superconductor. In the approximation linear in the field we have

$$j_\alpha(\mathbf{k},\omega) = -Q_{\alpha\beta}(\mathbf{k},\omega)A_\beta(\mathbf{k},\omega), \tag{1}$$

the kernel $Q_{\alpha\beta}(\mathbf{k},\omega)$ is expanded in a series in the electron-phonon interaction and the first two nonvanishing terms of this series (in (1), $j_\alpha(\mathbf{k},\omega)$ and $Q_{\alpha\beta}(\mathbf{k},\omega)$ are respectively the projections of the Fourier components of the current density and of the vector potential, and we use the gauge div A = 0). Then, recognizing that the vector $\mathbf{j}(\mathbf{k},\omega)$ is directed along the vector $\mathbf{A}(\mathbf{k},\omega)$, we can rewrite (1) in the form

$$\mathbf{j}(\mathbf{k},\omega) = -[Q^0(\mathbf{k},\omega) + Q^1(\mathbf{k},\omega)]A(\mathbf{k},\omega). \tag{2}$$

The term $Q^0(\mathbf{k},\omega)$ without the electron-phonon interaction was considered in[9-10]. $Q^1(\mathbf{k},\omega)$ is the sum of the contributions of all the diagrams of two types shown in the figure. When $2\Delta - \omega \gg T$, the quantity $\mathrm{Im}\, Q^1(\mathbf{k},\omega)$ contains the exponentially small factor $\exp[-(2\Delta - \omega)/2T)$. The exponential smallness disappears when $2\Delta - \omega \sim T$ and $\mathrm{Im}\, Q^1(\mathbf{k},\omega)$ changes little when ω increases further to $\omega = 2\Delta$. Introducing

$$Q^1(\mathbf{k},\omega) = \frac{3e^2N\Delta}{4mvk}Q^1(\omega), \tag{3}$$

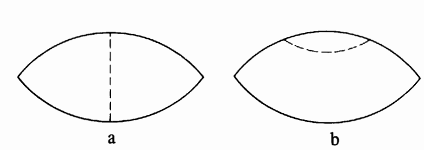
where e and m are the charge and mass of the electron, N is the number of electrons per unit volume, and v is the Fermi velocity, we get at $\omega \sim 2\Delta$ and $T \ll \Delta$

$$\mathrm{Im}\, Q^1(\omega) = 20\zeta\left(\frac{7}{2}\right)\eta\frac{\Delta^2}{\Omega_D^2}\left[\left(\frac{T}{\Delta}\right)^{7/2} + \left(\frac{T}{\Delta}\right)^4 \ln\left(2 - \frac{\omega}{\Delta}\right)\right] \tag{4}$$

(we have used the fact that $N = p_0^3/3\pi^2$ and $g^2 = 2\pi^2\eta/p_0m$, where $\eta \sim 1$). In our notation, p_0 is the Fermi momentum, g the electron-phonon interaction constant, Ω_D the Debye frequency of the phonons, and $\zeta(7/2)$ is the Riemann ζ function. For comparison with experiment, we calculate

$$\frac{\sigma_1}{\sigma_n} = \frac{\Delta}{\pi\omega}\mathrm{Im}\,[Q^0(\omega) + Q^1(\omega)]. \tag{5}$$

It is shown[9] that when $\omega < 2\Delta$ and $T \ll \Delta$ the quantity $\mathrm{Im}\, Q^0(\omega)$ contains the exponentially small factor $\exp(-\Delta/T)$, so that



a b

Diagrams contributing to $Q^1(\mathbf{k},\omega)$ (each solid line represents one of the Gor'kov functions of the superconductor, and the dashed line represents the Green's function of the phonon).

1)We use a system of units in which $\hbar = c = k = 1$.

$$\frac{\sigma_1}{\sigma_n}=\frac{\Delta}{\pi\omega}\operatorname{Im}Q^1(\omega). \tag{6}$$

In (5) and (6), σ_1 is the real part of the surface conductance of the superconductor and σ_n is the surface conductance of the metal in the normal state under the conditions of the anomalous skin effect. Substituting for Pb the values $2\Delta \approx 4T_c$, $T_c \approx 7^\circ K$, $\Omega_D \approx 100^\circ K$, $T = 1^\circ K$, and $\omega = 2\Delta$ we obtain $\sigma_1/\sigma_n \approx 10^{-5}$, whereas experiment yields $\sigma_1/\sigma_n \approx 0.1-0.2$.

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