ENERGY SPECTRUM AND ELECTRON MOBILITY IN A THIN FILM WITH NON-IDEAL BOUNDARY

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We propose a model that takes into account the irregularity of the surface of a thin (quantizing) film of a semimetal or a semiconductor. The influence of the surface scattering of electrons on their spectrum and on their conductivity in a weak constant electric field is clarified. The residual resistance of the film and the temperature dependence of the surface resistance are obtained at low and high temperatures.

THE question of the influence of the irregular structure of the surface on the spectrum and kinetics of the carriers in thin films is of considerable interest. This problem is the subject of a number of works;[1-3] Ziman's book[1] also contains a bibliography of the problem. In the approach developed in [1], use is made of the diffraction of electrons by the irregular surface, and the Huyghens principle is actually employed. In that case the purpose is to determine the diffusion coefficient, i.e., the idea of the kinetic equation employed. When used for thin films, such an approach is doubtful because of the quantization of the transverse motion of the electrons when the dimensions of the surface defects are small compared with the electron wavelength.

In the present paper we consider a pure thin film of a semimetal or conductor at a sufficiently low temperature. We assume that the mean free path with respect to any volume scattering mechanism is infinitely large. Under these conditions, the only cause of relaxation of a state with a definite longitudinal momentum is scattering by the boundaries of the film.

We propose the following model, which takes into account the irregularity of the surface. The wave function of the electron satisfies the boundary condition

$$\psi(x,y,a)+u(x,y)\frac{\partial}{\partial z}\psi(x,y,z)\,|_{z=a}=0. \tag{1}$$

The film occupies the region $0 \leq z \leq a$, and $u(x, y)$ is a random function with specified correlation properties. For simplicity we assume that only one boundary of the film, $z = a$, is irregular, and therefore for $z = 0$ we put $\psi(x, y, 0) = 0$. The case when a condition of the type (1) is satisfied on both surfaces can be treated analogously. The considered model is a reasonable approximation to the real situation. Indeed, when the wavelength of the electron is much larger than the height of the surface irregularities, Eq. (1) is an expansion of the exact condition $\psi = 0$ on a surface of random form $z = a + u(x, y)$. On the other hand, (1) specifies the logarithmic derivative of the wave function of the surface, i.e., it takes into account in a certain sense the interaction between the electron and the short-range surface centers. Inside the film we assume that the electron is a free particle with isotropic effective mass.

Our purpose is to find the Green's function, the state density, and the electron mobility.

1. GREEN'S FUNCTION AND DENSITY OF STATES

The problem consists of solving the equation

$$(\Delta+k^2)G_R(\mathbf{r},\mathbf{r}_0;E)=-\delta(\mathbf{r}-\mathbf{r}_0),\qquad \hbar=m=1,\qquad k^2=2E, \tag{2}$$

with the boundary conditions given above. We seek the solution in the form of a potential of a simple layer

$$G_R(\mathbf{r},\mathbf{r}_0;E)=\int\mu(x_1,y_1)\left[\frac{\exp(ik|\mathbf{r}-\mathbf{r}_1|)}{|\mathbf{r}-\mathbf{r}_1|}-\frac{\exp(ik|\mathbf{r}-\mathbf{r}_1'|)}{|\mathbf{r}-\mathbf{r}_1'|}\right]dx_1\,dy_1 + \frac{1}{4\pi}\left(\frac{\exp(ik|\mathbf{r}-\mathbf{r}_0|)}{|\mathbf{r}-\mathbf{r}_0|}-\frac{\exp(ik|\mathbf{r}-\mathbf{r}_0'|)}{|\mathbf{r}-\mathbf{r}_0'|}\right). \tag{3}$$

Here $\mu(x, y)$—density of the "sources," $\mathbf{r} = (x, y, z)$, $\mathbf{r}_0 = (x_0, y_0, z_0)$, $\mathbf{r}_0' = (x_0, y_0, -z_0)$, $\mathbf{r}_1 = (x_1, y_1, a)$, and $\mathbf{r}_1' = (x_1, y_1, -a)$.

We choose a retarded Green's function, i.e., k^2 contains an infinitesimally small imaginary increment $i\delta$ ($\delta > 0$). Taking the Fourier transform with respect to x, y, x_0, y_0 we get

$$G_R(\mathbf{p},\mathbf{p}',z,z_0)=\frac{4\pi\mu(\mathbf{p})}{R(p)}\sin(R(p)z)\exp(iR(p)a) + i\frac{2\pi^2}{R(p)}[\exp(iR(p)|z-z_0|)-\exp(iR(p)|z+z_0|)]\,\delta(\mathbf{p}-\mathbf{p}'); \tag{4}$$

$\mathbf{p}$ and $\mathbf{p}'$ are two-dimensional momenta; $(R(p) = \sqrt{k^2-p^2}$.

The boundary condition (1) leads to the following integral equation

$$\frac{\operatorname{tg}(R(p)a)}{R(p)}\nu(\mathbf{p},\mathbf{p}')+(2\pi)^{-2}\int u(\mathbf{p}-\mathbf{q})\nu(\mathbf{q},\mathbf{p}')\,d\mathbf{q} = -2\pi\delta(\mathbf{p}-\mathbf{p}')\sin(R(p)z_0)/R(p)\cos(R(p)a). \tag{5}$$

In (5) we have introduced the notation

$$\nu(\mathbf{p},\mathbf{p}')=\mu(\mathbf{p})[1+\exp(2iaR(p))] + 2\pi i\delta(\mathbf{p}-\mathbf{p}')\sin(R(p)z_0)\exp(iR(p)a).$$

We note that when $\sqrt{k^2-p^2} = 0$ all the coefficients of (5) are analytic, so that the question of the choice of the branch of the radical does not arise. Relation (5) can be regarded as an equation describing the motion of a particle in a two-dimensional random potential u(**q**). The role of the "bare" Green's function is played by the quantity R(p) cot (R(p)a), the poles of which coincide with the poles of the true Green's function (4) when u(**p**) ≡ 0 and determine the law of dispersion of the electron in the film with ideal boundaries:

$$k_m^2 = p^2 + (m\pi/a)^2, \quad m = 1, 2, 3, \ldots \tag{6}$$

We are interested in the solution of Eq. (5), averaged over the ensemble of random functions u(x, y). We consider the case when this ensemble is Gaussian. Then the mean values of the products of u break up into paired mean values; for the average solution (5) $\langle \nu(\mathbf{p}, \mathbf{p}') \rangle$ we get a diagram technique which is perfectly analogous to that developed by Abrikosov and Gor'kov in the theory of superconducting alloys (see [4]). The figure shows some of the simplest diagrams. The solid lines correspond to the factors R cot (Ra), the dashed lines to the quantity W(p), which is the Fourier transform of the binary correlation function $W(x - x', y - y') = \langle u(x, y)u(x', y') \rangle$. We shall assume that there are no preferred directions on the surface of the film; then W(**p**) depends only on the modulus of **p**. The average value of u(x, y) is assumed equal to zero. Each diagram must be multiplied by the coefficient of the δ function in the right side of (5). The average quantity $\langle \nu(\mathbf{p}, \mathbf{p}') \rangle$ obviously has the form $\nu(\mathbf{p})\delta(\mathbf{p} - \mathbf{p}')$, where $\nu(\mathbf{p})$ is expressed in known fashion in terms of the irreducible self-energy part Σ(**p**):

$$\nu(\mathbf{p}) = -\frac{2\pi \sin(R(p)z_0)}{R(p)\cos(R(p)a)}\left[\frac{\operatorname{tg}(R(p)a)}{R(p)} - \Sigma(\mathbf{p})\right]^{-1}. \tag{7}$$

In our approximation Σ(**p**) is equal to

$$\Sigma(\mathbf{p}) = \frac{1}{(2\pi)^2}\int W(\mathbf{p} - \mathbf{q})R(q)\operatorname{ctg}(R(q)a)\,d\mathbf{q}. \tag{8}$$

From the estimate of the diagrams follows the criterion for the applicability of (8), namely RΣ ≪ 1. It is easy to verify that the poles of the averaged Green's function (4) are due only to the vanishing of the square bracket in (7). From this we determine the renormalized dispersion of the electrons and the damping of the state with specified m and p. The roots of the equation tan (R(p)a) = R(p)Σ(p) are close to the zeroes of tan (Ra) by virtue of the condition RΣ ≪ 1. Solving this equation approximately, we get

$$k_m^2 = \operatorname{Re} k_m^2 + i\Gamma(m, p) = p^2 + \left(\frac{m\pi}{a}\right)^2\left(1 + \frac{2\Sigma(p)}{a}\right). \tag{9}$$

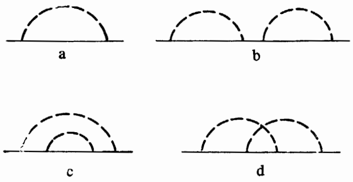
The imaginary part of Σ(**p**), which determines the damping, is due to the circuiting of the poles of cot (R(q)a) in (8). These poles are located on the segment 0 < q < k and their number N(k) equals the integer part of ka/π. As a result we obtain

$$\operatorname{Im}\Sigma(\mathbf{p}) = -\sum_{m=1}^{N(k)} \frac{(\pi m)^2}{2a^3}\overline{W}\left(p, \sqrt{k^2 - (\pi m/a)^2}\right), \tag{10}$$

where $\overline{W}$ is the result of averaging over the angles:

$$\overline{W}(p, q) = (2\pi)^{-1}\int_0^{2\pi} W\left(\sqrt{p^2 + q^2 - 2pq\cos\varphi}\right)d\varphi.$$

As follows from (9) and (10), the imaginary part of the energy Γ is a stepwise function of k. If $\overline{W}$(p) changes little over the interval p ~ k, then Γ(m, p) increases on the average like $(m\pi/a)^2(\overline{W}/a)[p^2 + (m\pi/a)^2]^{5/2}$.



Formula (10) is valid so long as

$$\overline{W}\frac{m\pi}{a}[p^2 + (m\pi/a)^2]^{3/2} \ll 1.$$

In this region, Γ is much smaller than $m\pi/a^2$—the distance between the levels of the transverse part of the energy. The indicated limitation on W has a simple physical meaning. Let the longitudinal and transverse energies be of the same order of magnitude; in this case the de Broglie wavelength of the electron $\underline{\lambda}$ is of the order of a/mπ. An estimate for $\overline{W}$ yields $\overline{W} \sim H^2L^2$, where H and L are the characteristic height and the diameter of the surface inhomogeneities. Then the criterion presented above for the applicability of formula (10) assumes a particularly simple form: $H^2L^2 \ll \lambda^4$.

In the vicinity of the points where ka/π is an integer, it is necessary to take into account the higher approximations for Σ(**p**). It is still possible to neglect here the diagrams with intersections (see Fig. d) with respect to the parameter RΣ ≪ 1. As is well known, in this approximation Σ is determined from the equation

$$\Sigma(\mathbf{p}) = \frac{1}{(2\pi)^2}\int W(\mathbf{p} - \mathbf{q})\left[\frac{\operatorname{tg}(R(q)a)}{R(q)} - \Sigma(q)\right]^{-1}d\mathbf{q}. \tag{11}$$

From (11) it follows that the sharp steps in Γ(E) become smeared out, retaining the smoothed-out relation $\Gamma \sim (2E - p^2)E^{5/2}$. The width of the smearing is of the order of the quantity Γ itself, and is much smaller than the width of the steps. Outside the region RΣ ≪ 1, it is necessary to sum all the self-energy diagrams in order to find the dispersion and the damping, which of course is not realizable. It is interesting, however, that it is possible to ascertain the essential features of the opposite limiting case RΣ ≫ 1. (This is apparently a distinguishing feature of the model under consideration.) Indeed, if RΣ ≫ 1, then the poles of the Green's function are close not to the zeroes but to the poles of tan (Ra). Thus, the change of the dispersion consists of the fact that the quantization of the transverse momentum becomes half-integer (cf. (6)):

$$\operatorname{Re} k_m^2 = p^2 + \left(m + \frac{1}{2}\right)^2\frac{\pi^2}{a^2}\left[1 + O\left(\frac{1}{mR\Sigma}\right)\right].$$

The level width is again small compared with the distance between levels:

$$\Gamma_m = \frac{\operatorname{Im}\Sigma}{a|\Sigma|^2}, \qquad \frac{\Gamma_m}{\Delta k_m^2} \sim (R\Sigma)^{-1} \ll 1.$$

It follows therefore that p and m are good quantum numbers for both small and large W. The reason can be readily seen in the boundary condition (1). With increasing average amplitude u(x, y) we go over from the condition ψ(x, y, a) = 0 (integer quantization), to the concondition ∂ψ(x, y, a)/∂z = 0 (half-integer quantization).

To calculate the density of the states, we use the well known formula

$$\rho(E) = -\frac{1}{\pi}\int \operatorname{Im} G(\mathbf{r},\mathbf{r};E)\,d\mathbf{r}.$$

After simple manipulations, we get

$$\rho(E) = \frac{Sa}{(2\pi)^2}\int_0^\infty \left[1-\frac{\sin(2Ra)}{2Ra}\right] R^{-2}\cos^{-2}(Ra)\operatorname{Im}\left[\frac{\operatorname{tg}(Ra)}{R}-\Sigma\right]^{-1} p\,dp, \tag{12}$$

S is the area of the film. In a film with ideal boundaries, we have $\Sigma(\mathbf{p}) \equiv 0$ and (12) yields the well known discontinuous dependence of the density of states on the energy: $\rho(E)$ is proportional to the integer part of $(a/\pi)\sqrt{2E}$. Scattering by the surface leads to a smoothing of the jumps of $\rho(E)$. The width of the region δE in which the smoothing of the m-th step takes place is determined by the quantity $\Gamma(m, p)$ from (9) at $p = 0$:

$$\delta E \sim \frac{W}{a^4}\frac{(m\pi)^5}{a^2}.$$

We note also that the points of the maxima $d\rho/dE$ shift from the positions $E_m = \frac{1}{2}(m\pi/a)^2$ corresponding to an ideal film, in the direction of higher energies. At these points $\tan(ka) = k \operatorname{Re}\Sigma$. The order of magnitude of the shift is $\Delta E_m/E_m \sim W/aL^3 \sim H^2/aL$.

2. ELECTRIC CONDUCTIVITY

Let us calculate the conductivity of the film in a weak constant electric field parallel to the surface. It is well known (see, for example, [15]), that the conductivity tensor is expressed in terms of the average value of the product of two Green's functions:

$$\sigma_{\alpha\beta} = -8\pi e^2 \int \langle \nabla_\alpha G_E(\mathbf{r},\mathbf{r}')\nabla_\beta G_E(\mathbf{r}',\mathbf{r})\rangle f'(E)\,dE\,d\mathbf{r}\,d\mathbf{r}', \tag{13}$$

where $G_E = G_R - G_A$ is the difference between the retarded and advanced Green's functions, f(E) is the electron distribution function per unit volume, and $f'(E) = df/dE$. Going over to the momentum representation, we obtain

$$\sigma_{xy} = \sigma_{yx} = 0, \qquad \sigma_{xx} = \sigma_{yy} = \sigma;$$

$$\sigma = -4\pi e^2 \int \langle G_E(\mathbf{p},\mathbf{p}',z,z')G_E(\mathbf{p}',\mathbf{p},z',z)\rangle(\mathbf{pp}')f'(E)\frac{d\mathbf{p}\,d\mathbf{p}'\,dz\,dz'}{(2\pi)^4}. \tag{14}$$

As will be shown later, only the vicinity of the poles of the G-functions is significant in the integral (14), so that the Green's function can be replaced by its "pole" part G. Using (4), (5), and the relation $G_A(\mathbf{p}, \mathbf{p}', z, z') = G_R^*(\mathbf{p}', \mathbf{p}, z', z)$, we get

$$G(\mathbf{p},\mathbf{p}',z,z') = \frac{4\pi^2\sin(R(p)z)\sin(R(p')z')}{R(p)R(p')\cos(R(p)a)\cos(R(p')a)}[\tilde{\nu}_A(\mathbf{p},\mathbf{p}')-\tilde{\nu}_R(\mathbf{p},\mathbf{p}')],$$

where $\tilde{\nu}_A$ and $\tilde{\nu}_R$ satisfy the equations (15)

$$\frac{\operatorname{tg}[R\pm i\delta]a}{R\pm i\delta}\tilde{\nu}_{R,A}(\mathbf{p},\mathbf{p}') + \frac{1}{(2\pi)^2}\int u(\mathbf{p}-\mathbf{q})\tilde{\nu}_{R,A}(\mathbf{q},\mathbf{p}')\,d\mathbf{q} = -\delta(\mathbf{p}-\mathbf{p}'). \tag{16}$$

The upper sign pertains to ν_R and the lower to $\tilde{\nu}_A$. The following formula is obtained for σ:

$$\sigma = -4\pi e^2 \int \langle[\tilde{\nu}_A(\mathbf{p},\mathbf{p}')-\tilde{\nu}_R(\mathbf{p},\mathbf{p}')][\tilde{\nu}_A(\mathbf{p}',\mathbf{p})-\tilde{\nu}_R(\mathbf{p}',\mathbf{p})]\rangle\cdot \\ \cdot(\mathbf{pp}')\varphi(p)\varphi(p')f'(E)\,dE\,d\mathbf{p}\,d\mathbf{p}'. \tag{17}$$

We have introduced here the notation

$$\varphi(p) = \tfrac{1}{2}aR^{-2}\cos^{-2}(Ra)[1-\sin(2Ra)/2Ra].$$

In the approximation when $R\Sigma \ll 1$, the averaging reduces to a summation of a "ladder" sequence of skeleton diagrams.[4,5] The summation leads to the integral equation

$$\Delta(\mathbf{p},\mathbf{p}') = g^2(p)\delta(\mathbf{p}-\mathbf{p}') + g^2(p)\int W(\mathbf{p}-\mathbf{q})\Delta(\mathbf{q},\mathbf{p}')\frac{d\mathbf{q}}{(2\pi)^2}, \tag{18}$$

where

$$\Delta(\mathbf{p},\mathbf{p}') = \frac{4\pi^2}{S}\langle[\tilde{\nu}_A(\mathbf{p},\mathbf{p}')-\tilde{\nu}_R(\mathbf{p},\mathbf{p}')][\tilde{\nu}_A(\mathbf{p}',\mathbf{p})-\tilde{\nu}_R(\mathbf{p}',\mathbf{p})]\rangle,$$

$$g(p) = \operatorname{Im}\left[\operatorname{tg}(Ra)/R - \sum\right]^{-1}.$$

We further multiply (18) by $(\mathbf{pp}')\varphi(\mathbf{p}')$ and integrate with respect to $\mathbf{p}'$. Putting

$$\int \Delta(\mathbf{p},\mathbf{p}')(\mathbf{pp}')\varphi(p')\,d\mathbf{p}' = g^2(p)M(p),$$

we obtain the following equation with respect to M(p):

$$M(p) = p^2\varphi(p) + \frac{1}{(2\pi)^2}\int W(\mathbf{p}-\mathbf{q})g^2(q)\frac{\mathbf{pq}}{q^2}M(q)\,dq. \tag{19}$$

The conductivity is expressed in terms of M(p) as follows:

$$\sigma = -\frac{Se^2}{\pi}\int \varphi(p)g^2(p)M(p)f'(E)\,dE\,dp. \tag{20}$$

In the integral (20), the function $g^2(p)$ has sharp maxima at the points where $p = p_m = [k^2 - (m\pi/a)^2]^{1/2}$; in our approximation we can assume that

$$g^2(p) \approx \frac{\pi}{|\operatorname{Im}\Sigma|}\delta\left(\frac{\operatorname{tg}(Ra)}{R}\right).$$

It is therefore important to know only M_m—the values of M(p) at these points. Inasmuch as the integration in (19) also contains $g^2(q)$, we arrive at a system of linear inhomogeneous equations with respect to M_m:

$$M_m = \frac{a^3}{2(m\pi)^2}M_m + \frac{\pi p_m}{2a^3}\sum_{j=1}^{N(k)}\frac{j^2\tilde{W}_{jm}M_j}{p_j|\operatorname{Im}\Sigma(p_j)|}, \tag{21}$$

where

$$\tilde{W}_{jm} = \frac{1}{2\pi}\int_0^{2\pi} W\left(\sqrt{p_j^2+p_m^2-2p_jp_m\cos\varphi}\right)\cos\varphi\,d\varphi.$$

Finally, we obtain σ in terms of M_m:

$$\sigma = -\pi e^2 S\int f'(E)\sum_{m=1}^{N(k)} M_m(k)/|\operatorname{Im}\Sigma(p_m,k)|\,dE. \tag{22}$$

We put

$$t(E) = \sum_{m=1}^{N} M_m(k)\,|\operatorname{Im}\Sigma(p_m;k)|^{-1}.$$

For the energy E between the levels m and m + 1 $(\pi^2m^2/2a^2 < E < \pi^2(m+1)^2/2a^2)$, the determination of t(E) is connected with the solution of a system of m equations (21).

The simplest case corresponds to an energy within the limits of the first subband: $\pi^2/2a^2 < E < 3\pi^2/2a^2$. From (10) and (21) we get

$$t(E) = (a^6/2\pi^2)(E-E_1)(\overline{W}_{11}-\tilde{W}_{11})^{-1},$$

where $\overline{W}_{jm}$ differs from $\tilde{W}_{jm}$ in the presence of $\cos\varphi$ under the integral sign. In the higher subbands, the formulas for t(E) are quite complicated, and we shall not consider these cases in detail.

Let us calculate first the conductivity of the film at absolute zero. As is well known,[6] in the case of scattering by static obstacles, the conductivity of a system of interacting electrons can be obtained by means of the single-electron formula (13), by regarding f(E) as a Fermi distribution function. From (22) we get $\sigma(0) = (\pi e^2/a)t(\mu_0)$, where μ_0 is the chemical potential at T = 0. Inasmuch as scattering by the boundary is assumed to be weak, it is sufficient to find μ_0 for a film with an ideal surface. In this case, as can be easily verified, μ_0 is determined by the following equation:

$$z^2[z] - {}^1/_3[z]([z] + {}^1/_2)([z] + 1) = na^3/\pi. \tag{23}$$

Here n—electron density, $z = \sqrt{2\mu_0}\,a/\pi$, and the square brackets denote the integer part of the number. To be able to confine oneself to the first subband, the density of the electrons should be smaller than $3\pi/2a^3$. Assuming this condition to be satisfied, we get

$$\begin{gathered}\mu_0 = \pi^2/2a^2 + \pi na,\\ \sigma(0) = \frac{ne^2a^6}{2[\overline{W}_{11}(\mu_0) - \widetilde{W}_{11}(\mu_0)]} = \frac{ne^2a^6}{2W_{tr}(\mu_0)},\end{gathered} \tag{24}$$

where W_{tr} is the analog of the transport cross section:

$$W_{tr} = \frac{1}{2\pi}\int_0^{2\pi} W\left(\sqrt{4\pi na(1-\cos\varphi)}\right)(1-\cos\varphi)\,d\varphi.$$

It can be shown that in the approximation corresponding to formula (10), t(E) experiences jumps at $E = E_m = (m\pi/a)^2/2$, where m is an integer. As a result, $\sigma(0)$ turns out to be a discontinuous (decreasing) function of the electron density. The critical values of the density are determined by Eq. (23): $n_{cr}a^3 = 3\pi/2, 13\pi/2, 17\pi$, etc. Such a behavior of the conductivity should be related with the stepwise energy dependence of the relaxation time of the Green's function. Allowance for the higher approximations leads, of course, to a smearing of all the sharp jumps. Thus, in a quantizing film with non-ideal boundary, there is always a nonvanishing residual resistance even in the absence of any volume relaxation mechanism. It is known[1] that a different result is obtained by using the classical kinetic equation with diffuse boundary condition. The conductivity of the film turns out to be infinite if there is no carrier scattering in the volume. This difference is due to the fact that in the quantum problem the transverse momentum is bounded from below.

Let us consider now finite values of the temperature. We start with the low-temperature limit $T \ll 3\pi^2/2a^2$ (the distance between the lower edges of the first and second subbands). If the electron density is not too close to the first critical value, then only the first subband will be appreciably populated. Namely, the following condition should be satisfied: $3\pi^2/2a^2 - \pi na \gg T$. This criterion can be readily derived from the equation for the chemical potential of an ideal film:

$$\frac{2}{a}\sum_{m=1}^{\infty}\int\left\{\exp\left[\frac{1}{2T}\left(p^2 + \frac{\pi^2m^2}{a^2} - 2\mu\right)\right] + 1\right\}^{-1}\frac{d\mathbf{p}}{(2\pi)^2} = n. \tag{25}$$

We shall assume that $3\pi^2/2a^2$ and πna are quantities of the same order of magnitude, but are not close to each other in the sense indicated above. From (27) we get for finite values of T:

$$\mu(T) = \frac{\pi^2}{2a^2} + \pi na + T\ln\left[1 - \exp\left(-\frac{\pi na}{T}\right)\right]. \tag{26}$$

Substituting (26) in (22), we obtain the conductivity as a function of the temperature. Without dwelling on the calculations, we present the results for $\sigma(T)$ in the vicinity of absolute zero:

$$\sigma(T) = \frac{e^2na^6}{2W_{tr}(\mu_0)} + \gamma T^2;\ \gamma = \frac{\pi e^2a^6}{12}\left[\frac{d^2}{dE^2}\frac{E - E_1}{W_{tr}(E)}\right]_{\mu_0} \tag{27}$$

It is natural to assume that $W_{tr}(E)$ is positive and decreases with increasing E, and therefore the function $(E - E_1)W_{tr}^{-1}$ is convex downward, i.e., $\gamma > 0$. Consequently, $\sigma(T)$ increases with temperature in the region $T \ll \pi na < 3\pi^2/2a^2$. In the opposite limiting case, $T \gg \pi^2/a^2$, it is necessary to take many subbands into account. It is practically impossible to obtain an exact solution, for this would require solution of the system (21) at large values of E. In order to determine roughly the temperature dependence of the conductivity, we can proceed in the following manner. Obviously, the relaxation time of the Green's function $\tau = \Gamma^{-1}(m, p)$ and the transport relaxation time, which determines the electron mobility, are of the same order of magnitude. It is therefore possible to average Γ^{-1} with the Boltzmann distribution function (we assume the electron gas to be degenerate when $T \gg \pi^2/a^2$). It is clear that the averaging operation must be performed on precisely Γ^{-1}, since it is the currents of electrons in different quantum states that are additive:

$$\begin{aligned}\langle\tau(T)\rangle \sim &\left\{\sum_{m=1}^{\infty}\int\Gamma^{-1}(m,p)\exp\left[-\frac{1}{2T}\left(p^2 + \frac{\pi^2m^2}{a^2}\right)\right]d\mathbf{p}\right\}\\ &\times\left\{\sum_{m=1}^{\infty}\int\exp\left[-\frac{1}{2T}\left(p^2 + \frac{m^2\pi^2}{a^2}\right)\right]d\mathbf{p}\right\}^{-1},\end{aligned} \tag{28}$$

where

$$\Gamma(m,p) \approx \left(\frac{\pi m}{a}\right)^2\frac{\overline{W}(p,p)}{a}\left[p^2 + \left(\frac{\pi m}{a}\right)^2\right]^{3/2}.$$

At large T we obtain from (28), in order of magnitude, $\langle\tau(T)\rangle \sim a^3W^{-1}(\sqrt{T})T^{-2}$. It is interesting to note the rather strong dependence of the conductivity on the film thickness (a^6 at T = 0 and a^3 in the high-temperature limit).

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