THE SPECTRUM AND DAMPING OF SURFACE ELECTRON STATES IN A MAGNETIC FIELD

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The spectrum and damping of surface electron states in a parallel magnetic field are found. In contrast to the Landau levels, the surface energy levels depend on the coordinate of the rotation center. Damping is due to scattering by random irregularities on the interface. It is shown that in the case of irregularities of atomic dimensions most of the electrons experience diffuse scattering and surface quantization is absent. For "glancing" electrons in weak magnetic fields the reflection is close to specular reflection since the effective "wavelength" of the electrons along the normal to the surface is much greater than the mean height of the irregularities.

1. INTRODUCTION

IT is well known that the Landau energy levels

$$\varepsilon_n(p_z)=\frac{p_z^2}{2m}+\left(n+\frac{1}{2}\right)\hbar\Omega, \qquad n=0,1,2\ldots \tag{1.1}$$

do not depend on the coordinate of the center of rotation $X=-cp_y/eH$, i.e., they are degenerate in p_y. Here ε—energy, p—momentum, m—mass, Ω—cyclotron frequency, $-e$—electron charge ($e>0$), and $2\pi\hbar$—Planck's constant. The z axis is parallel to the magnetic field **H**.

If the metal is bounded, then the electron trajectories near the surface of the metal can cross the interface. A reflection of the electrons then takes place. Let the magnetic field be parallel to the surface of the metal (Fig. 1). Then the electron is reflected many times and drifts along the surface in the y direction (0x—normal to the surface). The character of the reflection is determined by the properties of the interface. If we assume it to be an ideal plane and the electron scattering is assumed to be elastic, then the motion in the 0x direction will be finite and periodic, just as in an unbounded metal. However, the period of this motion will now depend on X—the coordinate of the center of rotation (on p_y). Consequently, the energy levels will also depend on X. In other words, the collisions of the electrons with the surface lift the degeneracy in X. These electron states will be called surface states. The first to study surface levels of electrons in a magnetic field were I. Lifshitz and Kosevich[1] in connection with the de Haas–van Alphen effect in thin films.

In the quasi-classical approximation, the energy levels can be determined from the Born–Sommerfeld quantization condition, if account is taken of the fact that in the magnetic field the quantities p_x and X are canonically-conjugate variables. The quantization condition is of the form

$$S(\varepsilon,p_z,X)\equiv\int p_x\,dp_y=2\pi n\frac{\hbar eH}{c}. \tag{1.2}$$

The integral is taken over the period of the classical motion of the particle. $S(\varepsilon, p_z, X)$ is an area bounded by the curve $\varepsilon(\mathbf{p})=\varepsilon$; $p_z=\text{const}$; $X=\text{const}$ in momentum space between two neighboring turning points. In an unbounded metal, S does not depend on X and equals the area bounded by the entire curve $\varepsilon=\text{const}$; $p_z=\text{const}$. For colliding electrons, S depends on X. In the case of an isotropic quadratic electron dispersion law ($\varepsilon=\mathbf{p}^2/2m$), the quantization condition (1.2) can be represented in the form

$$p\left\{1+\frac{2}{\pi}\operatorname{sign}X\left[\frac{a}{2p^{1/2}}\left(1-\frac{a^2}{4p}\right)^{1/2}+\arcsin\frac{a}{2p^{1/2}}\right]\right\}=2n, \tag{1.3}$$

where we introduce the following notation:

$$p=\frac{\varepsilon-p_z^2/2m}{\hbar\Omega}-\frac{1}{2}, \quad a=\frac{|X|}{l}, \quad l=\left(\frac{\hbar c}{2eH}\right)^{1/2}. \tag{1.4}$$

The quantity $p+\frac{1}{2}$ is the energy of the transverse motion of the electron in $\hbar\Omega$ units, a is the absolute value of the X coordinate of the center of rotation in units of the magnetic length l, and the parameter $a^2/4$ is the energy of the drift motion along the y axis in units of $\hbar\Omega$.

If the X coordinate is negative (the center of the orbit is outside the metal) and $|X|\approx R_\perp=cp_\perp/eH$ ($p_\perp=\sqrt{2m\varepsilon-p_z^2}$), then the electron "glances" along the surface. In this case $a\approx 2p^{1/2}$ and the energy levels are determined by the formula

$$\varepsilon_n(p_y,p_z)=\frac{p_y^2}{2m}+\frac{p_z^2}{2m}+\left(\frac{3\pi}{2}\right)^{2/3}\left[\frac{p_y^2}{2m}(n\hbar\Omega)^2\right]^{1/3}. \tag{1.5}$$

A similar expression was obtained for the case of a cylindrical equal-energy surface (when ε does not depend on p_z) by Nee and Prange.[2] They made use of the surface levels to explain the oscillations of the impedance in weak fields (~1–10 Oe), discovered by Khaĭkin.[3] These impedance oscillations can be treated as a cyclotron resonance due to the transitions between different surface levels. When an electromagnetic-field quantum $\hbar\omega$ is absorbed, the projections of the momentum p_y and p_z are conserved, and the resonance takes place at the frequencies

$$\omega=\omega_{nn_1}\approx\left(\frac{3\pi}{2}\right)^{2/3}(\Omega^2\varepsilon_F/\hbar)^{1/3}(n^{2/3}-n_1^{2/3}). \tag{1.6}$$

FIG. 1. Trajectory of "glancing" electrons near a smooth metal surface.

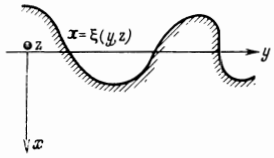

FIG. 2.

Consequently, the resonant values of the magnetic field H_{res} are proportional to $\omega^{3/2}$. This is precisely the connection between H_{res} and ω which was established experimentally.[3,4] The distance between the neighboring surface levels is approximately $(50\varepsilon_F/\hbar\Omega)^{1/3}$ times larger than the cyclotron frequency Ω. Therefore the characteristic resonant frequencies ω_{res} turn out to be appreciably larger than Ω even in a field H $\sim$1–10 Oe ($\omega_{res} \sim 10^{10}$ sec^{-1}).

In [1,2] they considered a smooth interface, i.e., it was actually assumed that the electron reflection is specular. Random, even microscopic irregularities on the surface of the metal lead to diffuse scattering and to the attenuation of the surface states. The purpose of the present paper is to determine the damping and the change of the spectrum of such states as a result of the scattering of electrons by random roughnesses on the metal surface.

2. FORMULATION OF PROBLEM

Let us consider a metal bounded by an uneven surface $x = \xi(y, z)$ (Fig. 2). We assume the roughnesses to be random. A magnetic field **H** parallel to the averaged surface $x = 0$ (zy plane, $\mathbf{H} \| z$). The electron reflection is assumed to be locally elastic (specular). In other words, the metal boundary is modeled by a potential U which vanishes inside the metal and is infinite on the boundary and in the vacuum. The presence of such a potential corresponds to specular reflection of the electrons from the plane boundary and to a partially diffuse scattering in the case of an uneven surface. The function $\xi(y, z)$ is a random function of the coordinates, with zero mean value, i.e.,

$$\overline{\xi(y, z)} = 0. \tag{2.1}$$

We assume that the roughnesses are statistically homogeneous and introduce the correlation function

$$\overline{\xi(y, z)\xi(y+\eta, z+\zeta)} = \sigma^2 W(\eta, \zeta), \tag{2.2}$$

where $\sigma^2 = \overline{\xi^2}$ is the mean square height of the roughnesses, and $W(\eta, \zeta)$ is the correlation coefficient. It is seen from (2.2) that $W(0, 0) = 1$ and $W(\eta, \zeta)$ is an even function of its variables. The characteristic correlation radius L (horizontal dimension of the inhomogeneities) is defined as the distance over which the function $W(\eta, \zeta)$ decreases appreciably.

To find the electron spectrum it is necessary to solve the Schrödinger equation with a potential U. The stationary wave function of the electron $\psi(\mathbf{r})$ should vanish on the surface $x = \xi(y, z)$. The function $\psi(\mathbf{r})$ will be a functional of the random function $\xi(y, z)$ and can therefore be represented in the form

$$\psi(\mathbf{r}) = \overline{\psi(\mathbf{r})} + \psi'(\mathbf{r}), \quad \overline{\psi'(\mathbf{r})} = 0. \tag{2.3}$$

It is obvious that both functions $\overline{\psi}$ and ψ' are solutions of the Schrödinger equation. The exact energy levels should be determined from the boundary conditions

$$\psi = 0 \tag{2.4}$$

on the surface and at $x = \infty$. However, we are interested not in the exact states of the electrons but in the average ones. The averaging leads not only to a change of the spectrum of the surface states, but also to attenuation of these states. This corresponds to the obvious fact that a rough boundary reflects "in the mean" the electrons not strictly specularly, but partly in a diffuse manner.

We confine ourselves henceforth to the quasi-classical approximation. In this approximation, the wave function $\overline{\psi}$ near the surface of the metal can be represented in the form of a sum of two plane waves—incident and reflected. The reflection of the plane wave from a random rough surface was considered by Isakovich and Bass in connection with the scattering of radio waves from the surface of the sea.[5,6] They calculated the effective reflection coefficient V of a plane wave in different cases (for a plane $V = -1$). Knowing the reflection coefficient V, we can write the function $\overline{\psi}$, following collisions between the electron and the surface, in the form

$$\overline{\psi(\mathbf{r})} = A(y, z)[\exp(-ik_x x) + V(k_x)\exp(ik_x x)], \tag{2.5}$$

where V is the reflection coefficient and k_x is the projection of the wave vector of the electron on the x axis ($k_x = p_x/\hbar$). The difference between $|V|$ and unity leads to the attenuation of the surface state.

If we assume the random roughnesses to be sufficiently smooth or relatively small, then we can calculate V by using the methods of geometrical optics or perturbation theory. This is precisely how this problem was solved in the previously cited papers.[5,6] Consequently, the spectrum and damping of the surface states can be represented in the form

$$\varepsilon = \varepsilon^{(0)} + \delta\varepsilon - i\gamma, \tag{2.6}$$

where $\varepsilon^{(0)}$ are the unperturbed levels (for a plane boundary), $\delta\varepsilon$ is the energy shift, and γ is the attenuation of the state. The quantities $\delta\varepsilon$ and γ are functions of the reflection coefficient V.

We shall first consider the problem of finding the unperturbed energy levels. We shall then determine with the aid of (2.5) the level shift $\delta\varepsilon$ and the damping γ as a result of diffuse scattering of the electrons from the uneven surface.

3. UNPERTURBED ENERGY LEVELS

The Schrödinger equation inside the metal has the usual form

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \left[\varepsilon - \frac{p_z^2}{2m} - \frac{m\Omega^2}{2}(x - X)^2\right]\psi = 0. \tag{3.1}$$

(We are considering a metal with isotropic quadratic

electron dispersion law). We have used the Landau gauge for the vector potential **A**:

$$A_x = A_z = 0, \qquad A_y = Hx.$$

In the notation of (1.4), this equation takes the form

$$\psi''(\tau) + [p + 1/2 - 1/4(\tau - a \operatorname{sign} X)^2]\psi(\tau) = 0, \tag{3.2}$$

where $\tau = x/l$ is the dimensionless distance from the surface of the metal.

Equation (3.2) has a solution, which is finite as $\tau \to \infty$, in the form of a parabolic cylinder D_p (τ—a sign X). The energy levels should be obtained from the boundary condition (2.4), i.e.,

$$D_{p_0}(-a \operatorname{sign} X) = 0. \tag{3.3}$$

Let us find the solution of this equation, using different asymptotic representations of the function $D_p(z)$.

1. We consider trajectories whose centers are located at large distances inside the metal (Fig. 3), when the X coordinate of the center of revolution is much larger than l and $R_\perp$. In our notation, these inequalities take the form

$$a \gg 1, \qquad a \gg 2p^{1/2}. \tag{3.4}$$

We use the following asymptotic form of the function $D_p(-a)$:

$$D_p(a \exp \pi i) \approx a^p \exp\left(i\pi p - \frac{a^2}{4}\right) + \frac{(2\pi)^{1/2}}{\Gamma(-p)} a^{-p-1} \exp\left(\frac{a^2}{4}\right), \tag{3.5}$$

where $\Gamma(x)$ is the Euler gamma function. From (3.3) we obtain the energy levels

$$p_0 = n + \frac{a^{2n+1}}{(2\pi)^{1/2} n!} \exp\left(-\frac{a^2}{2}\right).$$

In the standard notation, the electron energy is

$$\varepsilon_n^{(0)} = \frac{p_z^2}{2m} + \hbar\Omega\left(n + \frac{1}{2}\right) + \hbar\Omega \frac{a^{2n+1}}{(2\pi)^{1/2} n!} \exp\left(-\frac{a^2}{2}\right). \tag{3.6}$$

We have obtained a natural result: an exponentially small term due to the finite distance from the center of the electron orbit to the metal surface is added to the Landau formula (1.1). The exponential correction is connected with the interaction of the exponentially attenuating "tail" of the wave function with the surface of the metal.

2. As the X coordinate of the revolution center decreases, the orbit approaches the boundary of the metal and is tangent to the surface at $a = 2\sqrt{p}$ (see Fig. 3). Let us find the unperturbed energy levels in the following limiting case

$$p^{-2/3} \ll 1 - a^2/4p \ll 1. \tag{3.7}$$

The left-hand inequality is the condition for quasi-classical motion. It signifies that the segment $R_\perp - X$ spans a large number of "wavelengths" $1/k_x$, i.e., $k_x(R_\perp - X) \gg 1$. The right-hand inequality in (3.7) is the condition that the metal surface "cut off" a small segment of the electron orbit. The asymptotic form of the parabolic-cylinder function, from which we should find the energy levels, was obtained by Gyunninen and Makarov[7] and in this case has a rather cumbersome form. We shall therefore not present here, and write down immediately the spectrum of the surface states:

$$\varepsilon_n^{(0)} = \frac{p_z^2}{2m} + n\hbar\Omega\left[1 + \frac{2}{3\pi}\left(1 - \frac{a^2}{4n}\right)^{3/2}\right], \qquad n \gg 1. \tag{3.8}$$

The correction to the Landau condition is due to the obvious decrease of the area of the classical orbit S as a result of collisions between the electrons and the surface. Unlike the exponential correction to formula (3.6), the increment to (3.8) can be obtained also directly from the Born–Sommerfeld quantization condition (1.2).

3. There exists an asymptotic representation of the parabolic-cylinder functions at large values of the index

$$p \gg 1, \qquad p \gg a^2/4. \tag{3.9}$$

These inequalities correspond to orbits of electrons with $R_\perp \gg l$, $|X|$, when the center of revolution is near the surface of the metal (see Fig. 3). According to [7], the asymptotic form of $D_p(z)$ is in this case

$$D_p(z) = \sqrt{2}\left(p + \frac{1}{2}\right)^{p/2} \exp\left(-\frac{p}{2} - \frac{1}{4}\right) \cos\left[z\left(p + \frac{1}{2}\right)^{1/2} - \frac{\pi p}{2}\right]. \tag{3.10}$$

From (3.3) we get the spectrum

$$\varepsilon_n^{(0)} = \frac{p_z^2}{2m} + 2n\left(1 - \frac{2}{\pi} \frac{a \operatorname{sign} X}{(2n)^{1/2}}\right)\hbar\Omega. \tag{3.11}$$

The condition for the applicability of the quasi-classical approximation is in this case simply $n \gg 1$. Formula (3.11) can be readily obtained from the quantization condition (1.2), if it is recognized that in this case one quantizes half the area of the circular orbit ($p_0 \approx 2n$). The correction to the levels (the second term in the brackets in (3.11) is due to the finite value of X coordinate of the center of revolution. This correction is positive if the center of the orbit is outside the metal (sign X = −1) and negative if the center of the orbit is inside the metal.

4. Finally, let us consider the most interesting case of "glancing" orbits, whose centers are outside the metal (sign X = −1). The inequalities corresponding to this limiting case are given by formula (3.7). Their physical meaning has already been discussed above in case 2. The asymptotic expression for the parabolic-cylinder function has in this case the form[7]

$$D_p(a) \approx \sqrt{2}\left(p + \frac{1}{2}\right)^{p/2} \left(1 - \frac{a^2}{4p}\right)^{-1/4} \exp\left(-\frac{p}{2} - \frac{1}{4}\right) \times \cos\left[\frac{2}{3} p\left(1 - \frac{a^2}{4p}\right)^{3/2} - \frac{\pi}{4}\right]. \tag{3.12}$$

From (3.3) we get the energy spectrum of the "glancing" electrons

$$\varepsilon_n^{(0)} = \frac{p_y^2}{2m} + \frac{p_z^2}{2m} + \left[\frac{3\pi a}{4}\left(n - \frac{1}{4}\right)\right]^{2/3} \hbar\Omega. \tag{3.13}$$

Consequently, only the energy of motion along the x axis is quantized here, since the electrons under con-

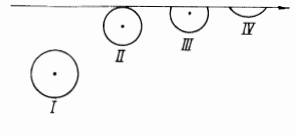

FIG. 3. Electron trajectories. I—trajectories inside the metal (sign X = +1, $|X| \gg R_\perp, l$); II—trajectories tangent to the surface (sign X = +1, $|X| \leqslant R_\perp \gg l$); III—trajectories with center of the orbit close to the metal surface; IV—"glancing" electrons (sign X = -1).

sideration drift not only in the direction of the magnetic field but also along the surface of the metal in the direction of the y axis. Apart from the notation, formula (3.13) coincides with (1.5).

4. WIDTH AND SHIFT OF LEVELS FOR AN UNEVEN BOUNDARY

In order to determine the energy level shift and the attenuation of the surface states of the electrons scattered by an uneven surface, let us consider the time-dependent factor $\Phi(t)$ in the reflected wave. According to (2.5), after single scattering we have

$$\Phi(t) = V \exp(-i\varepsilon^{(0)}t/\hbar). \tag{4.1}$$

In the case of a smooth boundary, the reflection coefficient is $V = -1$, i.e., the phase of the reflected wave is changed by π. Since this phase shift does not lead to a shift of the energy levels $\varepsilon^{(0)}$, it is convenient to introduce in lieu of $\Phi(t)$ the function

$$\Phi_1(t) = -V \exp(-i\varepsilon^{(0)}t/\hbar), \tag{4.2}$$

which describes the real change of the spectrum and the damping of the surface states. After m collisions, the phase factor takes the form

$$\Phi_m(t) = (-V)^m \exp(-i\varepsilon^{(0)}t/\hbar), \tag{4.3}$$

where

$$m = E(t/T), \tag{4.4}$$

E(x) is the integer part of the ratio x, and T is the time between two successive collisions with the surface $x = 0$. The shift and attenuation of the levels can be obtained with the aid of the spectral density of the function $\Phi_m(t)$, which we define as follows:

$$\rho(\varepsilon) = |\varphi(\varepsilon)|^2 \Big/ \int_{-\infty}^{\infty} d(\varepsilon)\,|\varphi(\varepsilon)|^2, \tag{4.5}$$

where

$$\varphi(\varepsilon) = \frac{1}{2\pi\hbar}\int_0^{\infty} dt\, \exp(i\varepsilon t/\hbar)\Phi_m(t) \tag{4.6}$$

is the Fourier component of the function $\Phi_m(t)$. After simple calculations we obtain the following expression for $\rho(\varepsilon)$:

$$\rho(\varepsilon) = \frac{2\hbar}{\pi T}\frac{\sin^2(\varepsilon-\varepsilon^{(0)})T/2\hbar}{(\varepsilon-\varepsilon^{(0)})^2} \times \frac{1-|V|^2}{(1-|V|)^2+4|V|\sin^2[(\varepsilon-\varepsilon^{(0)})T/2\hbar-\alpha/2]}, \tag{4.7}$$

where

$$-\alpha = \arg(-V) \tag{4.8}$$

is the change of the phase upon reflection of a plane wave from an uneven boundary.

Formula (4.7) allows us to find the level shift $\delta\varepsilon$ and the attenuation γ in the general case of an arbitrary reflection coefficient. It is of interest to consider two limiting cases: small reflection coefficients $|V|$, and those close to unity. In the case

$$|V| \ll 1 \tag{4.9}$$

the modulus of the wave function $\Phi_m(t)$ is a rectangular "step" of width T, and the spectral function

$$\rho(\varepsilon) = \frac{2\hbar}{\pi T}\frac{\sin^2((\varepsilon-\varepsilon^{(0)})T/2\hbar)}{(\varepsilon-\varepsilon^{(0)})^2} \tag{4.10}$$

has a maximum at $\varepsilon = \varepsilon^{(0)}$, the width of which at the 0.5 level equals

$$\gamma \approx \frac{2{,}8\hbar}{T} \approx \frac{\pi\hbar}{T}. \tag{4.11}$$

In this case there is no level shift.

If the reflection is close to specular

$$1 - |V| \ll 1, \tag{4.12}$$

then the wave function $\Phi_m(t)$ is described by a slowly damped plane wave. Therefore the spectral density $\rho(\varepsilon)$ has a Lorentz shape:

$$\rho(\varepsilon) = \frac{\hbar}{\pi T}\frac{1-|V|}{(\varepsilon-\varepsilon^{(0)}-\hbar a/T)^2+[\hbar(1-|V|)/T]^2}. \tag{4.13}$$

This expression is valid if $|\varepsilon-\varepsilon^{(0)}|T/2\hbar \ll 1$. The attenuation and the level shift are equal to

$$\gamma = \frac{\hbar}{T}(1-|V|), \qquad \delta\varepsilon = \frac{\hbar a}{T}. \tag{4.14}$$

All formulas of this section contain the time T between two successive collisions with the surface. From simple calculations it follows that

$$T = \frac{\pi + 2\theta\,\operatorname{sign} X}{\Omega}, \tag{4.15}$$

where θ is the angle of incidence of the electron on the surface (the angle between the electron velocity and the normal to the surface).

The coefficient V of reflection of the average field from a statistically uneven surface was calculated in [5,6], the results of which we shall use.

1. In the geometrical-optics approximation (the Kirchhoff approximation), the reflection coefficient, without allowance for shadowing, was obtained by Isakovich,[5] namely $V = -\exp(-2\sigma^2 k_x^2)$. In our notation this formula becomes

$$-V = \exp[-2s^2(p - {}^1\!/_4 a^2)], \tag{4.16}$$

where s is the average height of the roughnesses in units of the magnetic length l. Formula (4.16) is valid when the characteristic radius of curvature of the uneven surface is much larger than the de Broglie wavelength, i.e.,

$$\frac{L^2}{\sigma l}p^{1/2} \gg 1, \qquad \sigma \leqslant L. \tag{4.17}$$

2. The reflection coefficient V under the condition

$$|1+V| \ll 1 \tag{4.18}$$

was calculated by Bass.[6] It is necessary here to distinguish three cases.

When the correlation radius L is much larger than the de Broglie wavelength ($p^{1/2}L/l \gg 1$) and the glancing angles $\varphi = \pi/2 - \theta$ are sufficiently large, $\varphi \gg (2l/p^{1/2}L)^{1/2}$, the expression for V is obtained by expanding the exponential in (4.16) in powers of s^2:

$$1 + V = 2s^2(p - a^2/4). \tag{4.19}$$

If the glancing angles φ are small ($\varphi \ll (2l/p^{1/2}L)^{1/2}$),

and the correlation length L is as before large compared with the de Broglie wavelength, then

$$1+V=\exp\left(\frac{\pi i}{4}\right)s^2\left(\frac{l}{L}\right)^{1/2}p^{3/2}\left(1-\frac{a^2}{4p}\right)^{1/2}C_1, \qquad (4.20)$$

where the constant C_1 is given by

$$C_1=-2^{1/2}\pi^{-1/2}\int_0^\infty dx\,x^{-1/2}\frac{dW(x)}{dx}\sim 1.$$

Finally, in the case of small values of L compared with the wavelength

$$1+V=i\frac{s^2}{L}l\left(p-\frac{a^2}{4}\right)^{1/2}C_2, \qquad (4.21)$$

$$C_2=-2\int_0\frac{dx}{x}\frac{dW}{dx}\sim 1.$$

We shall not write out the formulas obtained as a result of substitution of (4.16), (4.19)–(4.21) in formulas (4.14) for the level shift and for the damping. We note only that in those cases when the reflection coefficient V is real, there is no level shift $\delta\varepsilon$, and when the quantity 1 + V is imaginary, the damping γ vanishes in the approximation linear in $|1+V|$.

5. DISCUSSION OF RESULTS

Let us discuss the damping of the surface state in the most important case, when the electron scattering can be regarded in the Kirchhoff approximation. For electrons colliding with the surface at an angle close to $\pi/2$, the damping depends on the magnitude of the parameter $2s^2p \approx (4\sigma^2m/\hbar^2)2n\hbar\Omega$. This parameter is the ratio of the transverse energy $\varepsilon_\perp \approx 2n\hbar\Omega$ to the characteristic "energy" $\varepsilon_\sigma = \hbar^2/4\sigma^2m$, which is connected with the random roughnesses. (The mean square of the momentum component $\hbar^2 \approx \hbar^2/\overline{k_x^2}\sigma^2$). In the region of relatively weak magnetic fields, when

$$\varepsilon_\perp = 2n\hbar\Omega \ll \varepsilon_\sigma, \qquad (5.1)$$

the attenuation is given by

$$\gamma_n=\frac{\hbar\Omega}{\pi}\frac{\varepsilon_\perp}{\varepsilon_\sigma}=\frac{8}{\pi}n(\sigma\Omega)^2m. \qquad (5.2)$$

It increases quadratically with the magnetic field. When $\varepsilon_\perp \gg \varepsilon_\sigma$, it follows from (4.11) and (4.15) that

$$\gamma_n \approx \hbar\Omega. \qquad (5.3)$$

In this case the attenuation is comparable with the distance between the neighboring levels and the discreteness of the surface levels becomes meaningless.

From physical considerations it is quite obvious that analogous conclusions are valid not only for normally incident electrons, but also at incidence angles θ on the order of unity.

Thus, the diffuse character of the scattering of the electrons by an uneven surface does not lead to an appreciable smearing of the surface levels, provided the transverse electron energy is small compared with ε_0. For roughnesses of atomic scale ($\sigma \sim 10^{-8}$ cm), the energy ε_0 is of the order of the Fermi energy ε_F. In this case it is meaningful to speak of discrete surface levels only for electrons with values of $|p_z| \approx p_F$, i.e., near the limiting points of the Fermi surface. For larger inhomogeneities, discrete surface states exist in a still narrower vicinity of the limiting points, where $\varepsilon_\perp \ll \varepsilon_\sigma \ll \varepsilon_F$. In other words, the majority of the electrons on the Fermi surface experience diffuse scattering and the spectrum of their surface states is practically continuous.

The conclusion formulated in similar form concerning the influence of the random roughnesses on the spectrum of the surface states is actually valid in the general case, and not only for angles of incidence on the order of unity. However, for "glancing" orbits, the condition for the "resolution" of the discrete structure of the surface states with respect to the magnetic field is much more favorable. This is connected with the fact that the distance between the different levels (1.6) is much larger for the "glancing" electrons, and therefore the surface quantization can appear in the region of weak fields (H ~ 1–10 Oe). In fact, when

$$2s^2\left[\frac{3\pi a}{4}\left(n-\frac{1}{4}\right)\right]^{2/3}\ll 1 \qquad (5.4)$$

the attenuation, in accordance with (4.14)–(4.16) is determined by the expression

$$\gamma_n \approx \frac{\hbar\Omega}{\varphi_n}s^2\left[\frac{3\pi a}{4}\left(n-\frac{1}{4}\right)\right]^{2/3}. \qquad (5.5)$$

Recognizing that

$$\varphi_n \approx \frac{2}{a}\left[\frac{3\pi a}{4}\left(n-\frac{1}{4}\right)\right]^{1/3},$$

we get

$$\gamma_n=\frac{\hbar\Omega}{2}s^2\left[\frac{3\pi}{16}\frac{p_y^4}{(\hbar\Omega m)^2}\left(n-\frac{1}{4}\right)\right]^{1/3}. \qquad (5.6)$$

In stronger fields, when the inequality (5.4) is reversed, we get in accordance with (4.11)

$$\gamma_n \approx 0{,}7\hbar\Omega a\left[\frac{3\pi a}{4}\left(n-\frac{1}{4}\right)\right]^{-1/3}. \qquad (5.7)$$

In this case the ratio of the attenuation to the distance between the neighboring levels is

$$\frac{\gamma_n}{\Delta\varepsilon_n}=\frac{2{,}8}{3\pi(n-1/4)^{1/3}}\frac{1}{(n+3/4)^{2/3}-(n-1/4)^{2/3}}. \qquad (5.8)$$

At all values of n, this ratio is approximately 0.5. Consequently, discrete surface states exist only in the case when the inequality (5.4) is satisfied. This inequality can be represented in the form analogous to (5.1)

$$\left[\frac{3\pi}{2}\hbar\Omega\left(n-\frac{1}{4}\right)\right]^{2/3}\left(\frac{p_y^2}{2m}\right)^{1/3}\ll\varepsilon_\sigma. \qquad (5.9)$$

Assuming the energy of the drift motion along the surface $p_y^2/2m$ to be on the order of ε_F, we can rewrite (5.9) in the form

$$\frac{3\pi}{2}\hbar\Omega n\ll\varepsilon_\sigma\left(\frac{\varepsilon_\sigma}{\varepsilon_F}\right)^{1/2}. \qquad (5.10)$$

For H $\lesssim$ 10 Oe and n ~ 1–3, this condition is satisfied for average roughness heights $\sigma \lesssim 10^{-6}$ cm. Thus, for "glancing" electrons at low values of nH, the uneven surface of the metal is a practically specularly reflecting plane. This conclusion is perfectly obvious physically: the more effective the "wavelength" k_x^{-1}, the

closer to specular is the reflection of electrons ($|V| = \exp(-2k_x^2\sigma^2)$) from the rough surface.

Besides the attenuation of the surface state as a result of the diffuse scattering, there is also the usual volume damping $\gamma_\nu = \hbar\nu$, connected with the volume collisions (ν—average frequency of collisions with the scatterers). It follows from simple estimates that at $nH \lesssim 30$ Oe and $\nu \sim 10^9$ sec^{-1}, the ratio $\gamma_n/\gamma_\nu \sim (\sigma/10^{-6})^2$. Consequently, when $\sigma < 10^{-6}$ cm the attenuation of the surface states for "glancing" electrons will be determined essentially by the volume collisions, and not by the scattering from the uneven surface.

We have considered above the case of a magnetic field that is strictly parallel to the surface of the metal. Obviously, when the vector H is inclined relative to the surface x = 0, electron drift to the inside of the metal takes place; this drift leads to a change in the period of the motion in the direction of the 0x axis, i.e., to an effective damping of the surface state. Let us estimate the value of this attenuation at small inclination angles α for "glancing" orbits. A change of the X coordinate of the center of revolution by an amount

$$\Delta X = \int_0^T v_{cn}\,dt \approx \alpha v_z T \tag{5.11}$$

leads to an attenuation on the order of (5.12)

$$\gamma_\alpha \approx \frac{p_y eH}{mc}\Delta X \approx \alpha p_y \Omega v_z T \approx 4\alpha\hbar\Omega\left(\frac{p_z^2}{2m\hbar\Omega}\right)^{1/2}\left[\frac{3\pi a}{4}\left(n-\frac{1}{4}\right)\right]^{1/3}.$$

The ratio of γ_α to the quantized energy of motion along the x axis will be small if

$$4\alpha\left(\frac{2}{3\pi}\right)^{1/3}\frac{|n_z|}{|n_y|^{1/3}}\frac{N^{1/6}}{n^{1/3}} \ll 1. \tag{5.13}$$

Here n_y and n_z are the projections of the electron momentum on the coordinate axis, $N = \varepsilon_F/\hbar\Omega \gg 1$. It follows therefore that the attenuation of the surface states of the electrons as a result of the inclination of the magnetic field will be small for small $|n_z|$, i.e., near the central (extremal) sections of the Fermi surface.

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