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RAMAN SCATTERING OF LIGHT AT LANDAU LEVELS IN SEMICONDUCTORS

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A theory of light scattering by the electron gas in impurity semiconductors situated in a magnetic field is constructed on the basis of the general theory of Raman scattering in the one-electron approximation. A two-band model is used for a conductor of the InSb type—with a narrow forbidden band. Formulas are derived for the differential scattering cross section at different possible electron excitations. The orders of magnitude of the scattering cross sections are estimated. The results are compared with the available experimental data. In the main, good agreement is observed between theory and experiment. Certain experimental data, whose understanding calls for an allowance for the interaction between the electrons, are discussed.

1. Raman scattering (RS) of light by electrons (and holes) in semiconductors situated in a constant homogeneous magnetic field **H** (henceforth called for brevity RS at Landau levels) was first considered by Wolff[1]. Starting from the simplest two-band model of the semiconductors, in which the conduction and valence bands are degenerate in spin only, Wolff obtained a formula for the RS for a change $\Delta n = n_2 - n_1 = 2$ in the Landau quantum number and a corresponding photon-frequency shift $= \omega_2 - \omega_1 = -2\omega_c$, where ω_c is the cyclotron frequency of the electron in the conduction band. This cross section turned out to be proportional to the square of the magnetic field.

Yafet[2] and Kelley and Wright[3] investigated RS at the Landau levels on the basis of the band model, which is a good approximation of the band spectrum in semiconductors (of the InSb type) with narrow forbidden band. They have shown that besides the process predicted by Wolff[1], in which the change of the Landau quantum number is $\Delta n = 2$ and $\Delta\omega = -2\omega_c$, other processes are possible with $\Delta n = 0$, $\Delta s = -1$, $\Delta\omega = -|g_c|\beta H/\hbar$ or $\Delta n = 2$, $\Delta s = -1$, $\Delta\omega = -2\omega_c - |g_c|\beta H/\hbar$, where s is the spin quantum number, equal to $\pm 1/2$, g_c is the effective g-factor for the electrons in the conduction band, and β is the Bohr magneton. According to[2,3], the cross section of RS with $\Delta n = 2$, $\Delta s = 0$ should be proportional essentially to the square of the magnetic field H, and the cross section for $\Delta n = 0$, $\Delta s = -1$ should depend weakly on H, nevertheless decreasing with increasing H.

RS at Landau levels was recently observed with the aid of a CO_2 laser by Patel and his co-workers in InSb[4] and InAs[5] crystals with electron density in the conduction band $n_e \approx 5 \times 10^{16}$ cm^{-3}. The experimental results obtained in these papers differ fundamentally in two respects from the predictions of the theory[1-3]. First, the cross section of the scattering with $\Delta n = 0$, $\Delta s = -1$ increases with increasing H, whereas according to[2,3] it should decrease slightly with increasing H. The quadratic dependence of the cross section with $\Delta n = 2$, $\Delta s = 1$ on H, predicted in[1-3], is not observed in a large interval of variation of the magnetic field. Moreover, an increase of H from 26 to 30 kOe led in the case of InSb to the complete vanishing of this line in the spectrum of the scattered light[4]. Second, in[4,5] they observed RS with $n = 1$, $s = 0$, which was not predicted by the theory of[1-3].

The reason for such an appreciable discrepancy between the theory[1-3] and experiment[4,5] is the fact that the authors of[1-3] performed all their calculations for a single solitary electron in the conduction band of the semi-conductor. With such a formulation of the problem, in addition to the impossibility of investigating problems connected with the RS line width, many RS lines at the Landau levels drop out of consideration.

It is assumed in the present paper that there are neV electrons (V—volume of the crystal) in the conduction band of the semiconductor with narrow forbidden band (of the InSb type). We derive formulas for the RS cross sections at the Landau levels, corresponding to different possible changes of Δn and Δs. We estimate the widths of the SR lines. The obtained theoretical results are compared with the available experimental data[4,5] and with the deductions of other theoretical works[1-3].

2. The general formula for the differential cross section of RS in which a photon of frequency ω_1 and polarization $\mathbf{e}_1$ is absorbed and a photon with frequency ω_2 and polarization $\mathbf{e}_2$ is emitted in a solid-angle element do_2, has in the dipole approximation the form[1)] (see, for example,[6])

$$n_e V d\sigma = r_0^2 \frac{\omega_2}{\omega_1} \sum_2 |\bar{M}_{21}|^2 do_2, \tag{1}$$

where σ—cross section per electron in the conduction band, $r_0 = e^2/mc^2$—classical radius of the electron, summation is implied over all the final states of the crystal with energy $E_2 = E_1 + \hbar(\omega_1 - \omega_2)$, with E_1—energy of the initial state of the crystal. The composite matrix element is

$$\bar{M}_{21} = \frac{1}{m} \sum_i \left[\frac{(\mathbf{e}_2^* \mathbf{P}_{2i})(\mathbf{P}_{i1}\mathbf{e}_1)}{E_1 - E_i + \hbar\omega_1} + \frac{(\mathbf{e}_1 \mathbf{P}_{2i})(\mathbf{P}_{i1}\mathbf{e}_2^*)}{E_1 - E_i - \hbar\omega_2} \right]. \tag{2}$$

The summation extends here over all the intermediate

[1)] It is assumed that the values of the dielectric constant of the crystal are the same at the frequencies ω_1 and ω_2.

states[2] i of the crystal with energy E_i. In the presence of a magnetic field **H** (see, for example, [8]), we have

$$\mathbf{P}=\sum \mathbf{p}, \quad \mathbf{p}=-i\hbar\nabla+\frac{e}{c}\mathbf{A}, \tag{3}$$

where the sum in the first of the formulas is taken over all the electrons of the crystal, and **A** is the vector potential of the homogeneous magnetic field **H**, for which we use the gauge $\mathbf{A} = \mathbf{H}(0, x, 0)$.

We shall neglect the interaction between the electrons in the sense that in choosing the single-particle $\mathcal{H}_0$ in (7) below we shall take into account only the interaction between the crystal's own electrons, in the self-consistent field approximation. Each single-particle state in the magnetic field **H** is characterized, as will be shown later (see (9)), by the y and z components of the electron wave vector **k**. With the aid of (3) and (9) we readily verify that

$$\langle \mu' n' k_y' k_z' | \mathbf{p} | \mu n k_y k_z\rangle = \langle \mu' n' | \mathbf{p} | \mu n;\ k_z\rangle \delta(k_y' - k_y)\delta(k_z' - k_z), \tag{4}$$

where μ and n are quantum numbers, which together with k_y and k_z determine the single-particle state completely.

Using, besides formula (4), the well known rules for changing over from matrix elements in (2) to the matrix elements of (4) (see, for example, [7], Sec. 65), we can readily transform (1) and (2) into

$$n_e d\sigma = r_0^2 \frac{\omega_2}{\omega_1}\left(\frac{\kappa}{2\pi}\right)^2 \sum_{\mu_1 n_1}\sum_{\mu_2 n_2}\int |M_{21}(k_z)|^2\, dk_z\, do_2, \tag{5}$$

$$M_{21}(k_z)=\frac{1}{m}\sum_{\mu_i n_i}\langle \mu_i n_i | p_\alpha | \mu_1 n_1;\ k_z\rangle \langle \mu_2 n_2 | p_\beta | \mu_i n_i;\ k_z\rangle (-1)^{p_{2i}}$$
$$\times\left(\frac{e_{2\alpha}^{*} e_{1\beta}}{\hbar\omega_1 - \varepsilon_2 + \varepsilon_i}+\frac{e_{1\alpha} e_{2\beta}^{*}}{-\hbar\omega_2 - \varepsilon_2 + \varepsilon_i}\right)+\frac{1}{m}\sum_{\mu_i n_i}\langle \mu_2 n_2 | p_\alpha | \mu_i n_i;\ k_z\rangle$$
$$\times\langle \mu_i n_i | p_\beta | \mu_1 n_1;\ k_z\rangle (-1)^{p_{2i}}\left(\frac{e_{2\alpha}^{*} e_{1\beta}}{\hbar\omega_1 - \varepsilon_i + \varepsilon_1}+\frac{e_{1\alpha} e_{2\beta}^{*}}{-\hbar\omega_2 - \varepsilon_i + \varepsilon_1}\right). \tag{6}$$

where $\epsilon_i \equiv \epsilon_{n_i}^{(\mu_i)}(k_z)$ denotes the energy of the corresponding single-particle state; p_{21}^{i} is an integer determined by the relative placement of the states $\mu_1 n_1$, $\mu_2 n_2$, and $\mu_i n_i$ (see [7], Sec. 65); repeated Greek indices $\alpha, \beta = x, y, z$ imply summation; $\kappa \equiv (eH/\hbar c)^{1/2}$—reciprocal of the so-called magnetic length[3]. The summation over $\mu_1 n_1$ and integration with respect to dk_z in (5) extend over all the occupied single-particle states, and summation over $\mu_2 n_2$ extends over all the unoccupied single-particle states, for which $\epsilon_2 - \epsilon_1 = \hbar(\omega_1 - \omega_2)$. In the first sum in (6) the summation is over all the occupied states $\mu_i n_i$, and in the second sum over all the unoccupied states $\mu_i n_i$. (The occupation of all the single-particle state is defined here with respect to the initial state of the crystal with energy E_1.)

3. The determination of the RS cross section reduces, in accordance with (5), to a calculation of the composite matrix element $M_{21}(k_z)$ which requires, as is evident from (6), knowledge of the spectrum and of the wave functions of the single-electron states. The single-particle Schrödinger equation for an electron in the crystal, in the presence of a magnetic field H at the gauge chosen by us, is of the form

$$\left(\mathcal{H}_0 - \frac{ie\hbar H}{mc}x\frac{\partial}{\partial y}+\frac{e^2H^2}{2mc^2}x^2\right)\psi = \varepsilon\psi, \tag{7}$$

where $\mathcal{H}_0$—Hamiltonian of the electron in the absence of a magnetic field. To solve (7) we shall use the two-band model of a semiconductor, which is known to be a good approximation for crystals of the InSb type[9].

We introduce, following[9], the Bloch functions in the center of the Brillouin zone $\mathbf{k} = 0$ neglecting the spin-orbit interactions, namely S—for the conduction band (a function similar to the atomic s-function), and X, Y, Z—for the valence band (functions similar to the p-functions). When account is taken of the spin-orbit interaction, the Bloch functions at the point $\mathbf{k} = 0$ of the Brillouin zone can be chosen in the following form (see also[9]):

$$u_{c\uparrow} = iS\uparrow, \quad u_{c\downarrow} = iS\downarrow, \quad \varepsilon_c(0) = \varepsilon_g,$$
$$u_{h\uparrow} = \sqrt{1/2}(X+iY)\uparrow, \quad u_{h\downarrow} = \sqrt{1/2}(X-iY)\downarrow, \quad \varepsilon_h(0) = 0,$$
$$u_{l\uparrow} = \sqrt{2/3}Z\uparrow - \sqrt{1/6}(X+iY)\downarrow,$$
$$u_{l\downarrow} = \sqrt{2/3}Z\downarrow + \sqrt{1/6}(X-iY)\uparrow, \quad \varepsilon_l(0) = 0,$$
$$u_{s\uparrow} = \sqrt{1/3}Z\uparrow + \sqrt{1/3}(X+iY)\downarrow,$$
$$u_{s\downarrow} = \sqrt{1/3}Z\downarrow - \sqrt{1/3}(X-iY)\uparrow, \quad \varepsilon_s(0) = -\Delta. \tag{8}$$

Here and further, the symbols c pertain to the conduction band, h to heavy holes, *l* to light holes, and s to the band split by the spin-orbit interaction. By $\uparrow\downarrow$ we denote spin functions corresponding to the electron spin projection on the z axis parallel and anti-parallel to **H**. The same symbols $\uparrow\downarrow$ are used as indices for the Bloch functions u.

Within the framework of the two-band model, when only interaction between the states (8) is taken into account and the interaction with other bands is neglected, the solutions of (7), can be written in the form

$$\psi_{nk_y k_z}^{(\mu)}(\xi) = A_{nk_z}^{(\mu)} e^{i(k_y y + k_z z)}\sum_{\mu'} u_{\mu'}(\xi)\sum_{n'\geqslant 0} B_{nk_z}^{(\mu)}(\mu', n')$$
$$\times\int e^{iq(x+k_y/\varkappa^2)}\Phi_{n'}(q)\, dq, \tag{9}$$

where $\mu, \mu' = c\uparrow\downarrow, h\uparrow\downarrow, l\uparrow\downarrow, s\uparrow\downarrow$; n, n'—integrers, $\Phi_{n'}$—harmonic-oscillator functions normalized to unity, ξ—aggregate of spatial (**r**) and spin coordinates. The coefficients A are obtained from the normalization condition

$$\int \psi_{n'k_y' k_z'}^{(\mu')*}(\xi)\psi_{nk_y k_z}^{(\mu)}(\xi)\, d\xi = \delta(k_y' - k_y)\delta(k_z' - k_z)\delta_{\mu'\mu}\delta_{n'n}. \tag{10}$$

The coefficients B, as well as the energy levels ϵ, are determined by substituting ψ from (9) into (7). The energy levels $\epsilon^{(c)}$, $\epsilon^{(l)}$ and $\epsilon^{(s)}$ are roots of the following algebraic equations:

$$\varepsilon_n(k_z)[\varepsilon_n(k_z) - \varepsilon_g][\varepsilon_n(k_z) + \Delta] - \frac{\hbar^2 P^2}{m^2}[k_z^2 + \varkappa^2(2n+1)]$$
$$\times\left[\varepsilon_n(k_z) + \frac{2}{3}\Delta\right] \pm \frac{\hbar^2 P^2 \Delta \varkappa^2}{3m^2} = 0, \tag{11}$$

[2] If some of the quantum numbers characterizing the states 1, 2, . . ., i acquire a continuous set of values, then the wave functions should be normalized to δ-functions of the corresponding quantities (see, for example, [7], Sec. 38).

[3] The dipole approximation employed by us is valid under the condition that the wavelength of light in the crystal is much larger than $1/\kappa$.

where the upper sign (plus) in the last term on the left pertains to states with projection of the momentum along $\mathbf{H}(\uparrow)$, and the lower (minus) to states with momentum projection opposite to $\mathbf{H}(\downarrow)$. Here

$$P \equiv -\frac{(2\pi)^3\hbar}{v}\int_v S\frac{\partial Z}{\partial \tau}\,d\mathbf{r}, \tag{12}$$

v—volume of unit cell of the crystal. The energy of the states in the heavy-hole band is $\epsilon_n^{(h)}\uparrow\downarrow(k_z) = 0$, i.e., in this approximation the mass of the heavy holes turns out to be infinite, and their g-factor[4] is $g_h = 0$.

Expressions for $\sum_{n'} B_{nk_z}^{(\mu)}(\mu', n')\Phi_{n'}(q)$ were obtained by Yafet (see Table I of[2]), where the energy levels ϵ are denoted by λ, κ^2 is denoted by s, and $\hbar = m = 1$). Knowing B, we can calculate the normalization coefficients A:

$$A = \left\{1 + \frac{\hbar^2P^2[2k_z^2 + \kappa^2(4n+1)]}{3m^2\varepsilon^2} + \frac{\hbar^2P^2[k_z^2 + 2\kappa^2(n+1)]}{3m^2(\Delta+\varepsilon)^2}\right\}^{-1/2}$$

at $\mu = c\uparrow, l\uparrow, s\uparrow$, $n \geq 0$, we have

$$A = \left\{1 + \frac{\hbar^2P^2[2k_z^2 + \kappa^2(4n+3)]}{3m^2\varepsilon^2} + \frac{\hbar^2P^2(k_z^2 + 2\kappa^2 n)}{3m^2(\Delta+\varepsilon)^2}\right\}^{-1/2},$$

$$\text{for } \mu = c\downarrow, l\downarrow, s\downarrow, \quad n \geq 0,$$

$$A = \left\{\frac{2k_z^2 + \kappa^2(n+1)}{2k_z^2 + \kappa^2(4n+1)}\right\}^{1/2} \quad \text{for } \mu = h\uparrow, \quad n \geq 1,$$

$$A = \left\{\frac{2k_z^2 + \kappa^2 n}{2k_z^2 + \kappa^2(4n+3)}\right\}^{1/2} \quad \text{for } \mu = h\downarrow, \quad n \geq -1, \tag{13}$$

where for brevity we have left out the indices μ, n, and k_z of A and ϵ.

Knowing the single-electron wave functions (9) and the corresponding energy levels (11), we can calculate the matrix elements (4). To calculate the scattering cross sections (5) it is necessary also to know the position of the Fermi level ϵ_0 relative to the Landau levels in the conduction band[5]. In the general case of arbitrary concentrations n_e of electrons in the conduction band and of arbitrary magnetic fields H, the Fermi level ϵ_0 is determined from the formula

$$\left(\frac{\kappa}{2\pi}\right)^2 \sum_{n=0\,1,\ldots}\left(\int_{\varepsilon_{n\uparrow}^{(c)}(0)}^{\varepsilon_{0\uparrow}^{(c)}(0)+\varepsilon_0}\left|\frac{dk_z}{d\varepsilon}\right|d\varepsilon + \int_{\varepsilon_{n\downarrow}^{(c)}(0)}^{\varepsilon_{0\uparrow}^{(c)}(0)+\varepsilon^0}\left|\frac{dk_z}{d\varepsilon}\right|d\varepsilon\right) = \frac{n_e}{2}. \tag{14}$$

This formula pertains to the case of a negative g-factor, when the level $n\uparrow$ lies below the level $n\downarrow$, just as in the InSb crystal. In the opposite case, the upper limit of integration in (14) should be replaced by $\epsilon_{0\downarrow}^{(c)}(0) + \epsilon_0$.

We confine ourselves to the case when only two levels, $0\uparrow$ and $0\downarrow$, or else only one level $0\downarrow$, are located below the Fermi level ϵ_0. It is assumed henceforth that $\hbar\omega_c > |g_c|\beta H$ and $g_c < 0$, just as in InSb. Assuming further that the $\epsilon(k_z)$ dependence can be regarded as quadratic, we obtain the Fermi energy

$$\sqrt{\varepsilon_0} = \frac{\pi^2\hbar n_e}{\sqrt{2m_c}\,\kappa^2}\left(1 + \frac{1}{2}\left|\frac{m_c}{m_c'}\right|\frac{\kappa^6}{\pi^4 n_e^2}\right), \tag{15}$$

where we introduce the "spin" electron mass in the conduction band, connected with the g-factor by the relation $m_c' = 2m/g_c$. Expression (15) determines the Fermi energy under the condition that only two Landau levels $0\uparrow$ and $0\downarrow$ are located below the ϵ_0 level. At a specified electron density n_e, this condition imposes the following limitations on the magnetic field H or on κ^2:

$$\left(2\pi^4 n_e^2\left|\frac{m_c'}{m_c}\right|^2\right)^{1/3}\left(1 - \sqrt{1 - \left|\frac{m_c}{m_c'}\right|}\right)^{2/3} \leq \kappa^2 \leq \left(2\pi^4 n_e^2\left|\frac{m_c'}{m_c}\right|^2\right)^{1/3}. \tag{15'}$$

If only one level $0\uparrow$ lies below ϵ_0, i.e., if

$$\kappa^2 \geq \left(2\pi^4 n_e^2\left|\frac{m_c'}{m_c}\right|\right)^{1/3}, \tag{16'}$$

then

$$\sqrt{\varepsilon_0} = 2\hbar\pi^2 n_e/\sqrt{2m_c}\,\kappa^2. \tag{16}$$

For the InSb crystal, where $m_c = 0.013m$, $g_c = -55$ ($\epsilon_g = 0.23$ eV, $\Delta = 0.9$ eV)[9], at the ordinary concentrations $n_e \approx 5\times10^{16}$ cm^{-3} [4], the case (15) is realized at magnetic fields 35 kOe < H < 70 kOe, and the case (16) is realized at H > 70 kOe. Magnetic fields up to 55 kOe were used in the experiments of[4].

4. To calculate the RS cross sections we presented the matrix elements $\langle \mu' n' | p | \mu n; k_z\rangle$ and the energies $\epsilon_n^{(\mu)}(k_z)$ in the form of series in powers of $\hbar\omega_c/\epsilon_g$ and $\hbar^2k_z^2/m_c\epsilon_g$, confining ourselves in the corresponding series for $M_{21}(k_z)$ to terms of order κ^3 and k_z^3. Omitting cumbersome but fundamentally straightforward calculations, we present only the results for different (Stokes) RS lines. The differential scattering cross section is

$$\frac{d\sigma}{d\omega_2} = r_0^2\frac{\omega_2}{\omega_1}\left(\frac{m}{m_c}\right)^2 G, \tag{17}$$

where G is determined by the type of the transition.

A. The transition $0\uparrow \rightarrow 0\downarrow$, $\Delta n = 0$, $\Delta s = -1$, $\omega_1 - \omega_2 = |g_c|\beta H/\hbar$:

$$G = \left(\frac{\sqrt{2m_c\varepsilon_0}\kappa^2}{\pi^2\hbar n_e} - 1\right)|e_{1z}e_{2-}^* - e_{1+}e_{2z}^*|^2C^2, \tag{18}$$

$$C = \frac{a}{2}\frac{\hbar\omega}{\varepsilon_g}(\varphi_1 - \varphi_2). \tag{19}$$

In these and in subsequent formulas,

$$a \equiv \frac{2\Delta + 2\varepsilon_g}{2\Delta + 3\varepsilon_g}, \quad b \equiv \frac{\Delta^2 - 3\varepsilon_g^2}{\Delta(\Delta+\varepsilon_g)}, \quad e_\pm \equiv e_x \pm ie_y, \quad \omega \equiv \frac{\omega_1 + \omega_2}{2},$$

$$\varphi_1 \equiv \varphi_1(\omega) = \frac{\varepsilon_g^2}{\varepsilon_g^2 - (\hbar\omega)^2}, \quad \varphi_2 \equiv \varphi_2(\omega) = \frac{\varepsilon_g^2}{(\Delta+\varepsilon_g)^2 - (\hbar\omega)^2},$$

$$f_1 \equiv 1 + \frac{1}{8}\frac{\varepsilon_g}{\Delta+\varepsilon_g} + \frac{3}{8}\frac{\varepsilon_g}{\Delta}, \quad f_2 \equiv 1 - 3\frac{\varepsilon_g}{\Delta} + 5\frac{\varepsilon_g}{\Delta+\varepsilon_g},$$

$$f_3 \equiv 1 - 3\frac{\varepsilon_g}{\Delta} + 2\frac{\varepsilon_g}{\Delta+\varepsilon_g}. \tag{20}$$

If we substitute in (18) ϵ_0 from (16'), then we get Yafet's formula[2], which is thus valid only in sufficiently strong fields, when $|g_c|\beta H \geq \epsilon_0$.

According to (17)—(20), the cross section in an

[4] Since we have neglected terms $\sim m_c/m$ ($m_c \approx 0.01m$ – electron mass in the conduction band in InSb), this means that in fact $|m_h| \sim m$ and $|g_h| \sim 1$.

[5] It is assumed that the electrons in the conduction band of the semiconductor form a completely degenerate Fermi gas.

InSb sample with $n_e = 3 \times 10^{16}$ cm^{-3} is approximately three times larger than the cross section in a sample with $n_e = 5 \times 10^{16}$ cm^{-3}. In[4] they observed a five-fold increase of the cross section following the indicated change of n_e from 5×10^{16} to 3×10^{16} cm^{-3}, but unfortunately the authors did not state precisely the value of H at which this dependence of the cross section on n_e was observed. An estimate of the cross section with a radiation polarization $\mathbf{e}_1 \perp \mathbf{H}$, as in[4,5], gives for InSb at $n_e = 3 \times 10^{16}$ cm^{-3} and H = 50 kOe a value $d\sigma/d\omega_2 \approx 4 \times 10^{-23}$ cm^2. Slusher, Patel, and Fleury[4] give for this concentration a value $d\sigma/d\omega_2 \approx 10^{-3}$ cm^2, without indicating, to be sure, the corresponding value of the field H. At $n_e = 5 \times 10^{16}$ cm^{-3}, the cross section (17) approximately quadruples when H changes from 35 to 70 kOe, and this does not contradict the experiment[4], in which the cross section nearly doubled when H changed from 26 to 52 kOe.

As to the width Δn of the RS line, it is easy to estimate here that part which is connected with the nonparabolicity of the conduction band. The corresponding estimates yield 4 cm^{-1} for $n_e = 1 \times 10^{16}$ cm^{-3} and 25 cm^{-1} for $n_e = 6 \times 10^{16}$ cm^{-3} in InSb at T = 30°K and H = 50 kOe. Experimental values of approximately 5 and 30 cm^{-1} for the total line width are given in[4] without indicating the value of H.

B. The transition $0\uparrow\downarrow \to 1\uparrow\downarrow$; $\Delta n = 1$, $\Delta s = 0$, $\omega_1 = \omega_2 = \omega_c$:

$$G = \frac{\pi^4 n_e^2}{3\varkappa^6}\left\{\left(\frac{\sqrt{2m_c\varepsilon_0}\varkappa^2}{\pi^2\hbar n_e}\right)^3 |C_+ e_{1z}e_{2+}^* + C_- e_{1-}e_{2z}^*|^2 + \left(2 - \frac{\sqrt{2m_c\varepsilon_0}\varkappa^2}{\pi^2\hbar n_e}\right)^3 |C_- e_{1z}e_{2+}^* + C_+ e_{1-}e_{2z}^*|^2\right\}, \quad (21)$$

$$C_\pm = \frac{\hbar\omega_c}{\varepsilon_g}a^2\left[\varphi_1\left(f_1 \pm \frac{\hbar\omega}{16\varepsilon_g}f_2\right) \pm \pm\frac{3\hbar\omega}{8\varepsilon_g}\left(1 \pm \frac{\hbar\omega}{\varepsilon_g}\right)\varphi_1^2 + \frac{b}{8}\varphi_2\left(1 \mp \frac{\hbar\omega}{\Delta + \varepsilon_g}\right)\right]. \quad (22)$$

This line was not predicted by the earlier theories[1-3]; this is perfectly natural, since it vanishes in the limit as $n_e \to 0$, which is precisely the limit considered in[1-3] (at $n_e \to 0$ the value of ϵ_0 is determined by (16) and $G \to 0$ like n_e^2). The fact that $d\sigma/d\omega_2 \to 0$ as $n_2 \to 0$ is directly connected with the fact that the composite matrix element $M_{21}(k_z)$ in this case, as in cases D and E, is proportional to k_z, and consequently $M_{21} = 0$ is we confine ourselves, just as in[1-3], to one electron in the conduction band in the state $n = 0$, $k_z = 0$.

In the experiments[4,5], the line considered after the line $\Delta n = 0$, $\Delta s = -1$ is the strongest and narrowest. For InSb at $n_e = 5 \times 10^{16}$ cm^{-3} and H = 50 kOe, our formulas yield $d\sigma/d\omega_2 \approx 4 \times 10^{-24}$ cm^2. In[4] is given an experimental value $d\sigma/d\omega_2 \approx 10^{-24}$ cm^2 at the indicated concentration $n_e = 5 \times 10^{16}$ cm^{-3}, but the corresponding value of H is again not given. An increase of the concentration n_e from 3×10^{16} to 5×10^{16} cm^{-3} should, in accordance with (21) and (22), lead to an increase of the cross section by approximately eight times. For comparison we note that in the experiment[4] it was observed (at the same change of n_e), an increase of the cross section by about ten times (unfortunately, it is again not clear from[4] at which value of H this was observed).

C. The transition $0\uparrow\downarrow \to 2\uparrow\downarrow$; $\Delta n = 2$, $\Delta s = 0$, $\omega_1 = \omega_2 = 2\omega_c$:

$$G = |e_{1-}e_{2+}|^2 C^2,$$

$$C = \sqrt{2}\,a^2\frac{\hbar\omega_c}{\varepsilon_g}\left[\varphi_1 f_1 + \frac{3}{8}\left(\frac{\hbar\omega}{\varepsilon_g}\right)^2\varphi_1^2 + \frac{1}{8}\varphi_2 f_3\right], \quad (23)$$

which coincides with the formulas obtained for this RS line by Yafet[2]. Already noted at the beginning of the article, the experimental results of[4,5] on the dependence of the cross section of RS with $\Delta n = 2$, $\Delta s = 0$ on the concentration n_e and on the magnetic field H cannot be explained by means of formulas (17) and (23). A possible cause of this disparity between theory and experiment will be discussed at the end of the paper.

D. The transition $0\downarrow \to 1\uparrow$; $\Delta n = 1$, $\Delta s = 1$, $\omega_1 - \omega_2 = \omega_c - |g_c|\beta H/\hbar$. The RS cross section should have in this case a polarization dependence in the form $|e_{1-}e_{2+}|^2$. However, it should be smaller by at least four orders of magnitude than the RS cross section for $\Delta n = 0$ and $\Delta s = -1$. At sufficiently large magnetic fields (16), when $|g_c|\beta H > \epsilon_0$, there is of course no scattering. In the experiments[4,5] this RS line was not observed, and we shall not present the corresponding formulas.

E. The transition $0\uparrow \to 1\downarrow$; $\Delta n = 1$, $\Delta s = -1$, $\omega_1 - \omega_2 = \omega_c + |g_c|\beta H/\hbar$:

$$G = \frac{\pi^4 n_e^2}{6\varkappa^6}\left(\frac{\sqrt{2m_c\varepsilon_0}\varkappa^2}{\pi^2\hbar n_e}\right)^3 |[\mathbf{e}_1\mathbf{e}_2^*]_z|^2 C^2, \quad (24)$$

$$C = \frac{a^2}{4\sqrt{2}}\frac{\hbar\omega_c}{\varepsilon_g}\frac{\hbar\omega}{\varepsilon_g}\left(\varphi_1 f_2 + 6\varphi_1^2 - \frac{\varepsilon_g}{\Delta + \varepsilon_g}\varphi_2 f_3\right). \quad (25)$$

This RS line is present in principle also in the $\mathbf{e}_2 \parallel \mathbf{H}$ polarization when $\mathbf{e}_1 \parallel \mathbf{H}$, but the scattering cross section is in this case smaller by two orders of magnitude than the cross sections (17), (24), and (25). An estimate for Insb at H = 35 kOe and $n_e = 5 \times 10^{16}$ cm^{-3} yields $d\sigma/d\omega_2 \approx 6 \times 10^{-27}$ cm^2, which is almost three orders of magnitude smaller than the cross section for the process $\Delta n = 1$, $\Delta s = 0$. In spite of this, the $\Delta n = 1$, $\Delta s = -1$ line was observed in[4], although to be sure in not all samples.

F. The transition $0\uparrow \to 2\downarrow$; $\Delta n = 2$, $\Delta s = 1$, $\omega_1 - \omega_2 = 2\omega_c - |g_c|\beta H/\hbar$. Within the framework of the two-band model, this transition is strictly forbidden. It was not observed in the experiments[4,5].

G. The transition $0\uparrow \to 2\downarrow$; $\Delta n = 2$, $\Delta s = -1$, $\Delta_1 - \Delta_2 = 2\omega_c + |g_c|\beta H/\hbar$:

$$G = \frac{\sqrt{2m_c\varepsilon_0}\varkappa^2}{2\pi^2\hbar n_e}|e_{1z}e_{2+}^* - e_{1-}e_{2z}^*|^2 C^2, \quad (26)$$

$$C = \frac{a^2}{4\sqrt{2}}\frac{\hbar\omega_c}{\varepsilon_g}\frac{\hbar\omega}{\varepsilon_g}\left(\varphi_1 f_2 + 3\varphi_1^2 - \frac{\varepsilon_g}{\Delta + \varepsilon_g}\varphi_2 f_3\right). \quad (27)$$

For InSb at $n_e = 5 \times 10^{16}$ cm^{-3}, H = 35 kOe, and $\mathbf{e}_1 \perp \mathbf{H}$ we get from formulas (17), (26), and (27) $d\sigma/d\omega_2 \approx 4 \times 10^{-25}$ cm^2, which is smaller by one order of magnitude than the cross section at $\Delta n = 1$, $\Delta s = 0$, but almost two orders of magnitude larger than the RS cross section at $\Delta n = 1$, $\Delta s = -1$, which, as already noted above, was experimentally registered in[4]. Nevertheless. this transition was not observed in the RS spec-

trum in[4,5], and this experimental fact must be explained. We shall attempt to do so in the next section.

5. It was shown above that the theory proposed in the present paper is essentially in good agreement with the available experimental data[4,5] on RS at the Landau levels. The same pertains to the nature of the transitions in the RS, the polarization dependences of the RS cross sections, the dependence of the RS cross sections on the electron density and on the magnetic field, and also to the orders of magnitude of the cross section. Exceptions are the two transitions $\Delta n = 2$, $\Delta s = 0$ and $\Delta n = 2$, $\Delta s = -1$, with the largest values of the electron excitation energies—$2\hbar\omega_c$ and $2\hbar\omega_c + |g_c|\beta H$ respectively (see Secs. 1, 4C, and 4G).

The discrepancy between theory and experiment can be explained, our opinion, only by taking into account the interaction between the electrons. In the one-electron approximation assumed in the present paper and in[1-3], the electron "lives" in the excited states for an infinitely long time, if we neglect the interaction with the phonons, scattering by the impurity atoms, and the spontaneous radiative transitions. Allowance for the interactions (collisions) between the electrons changes the energy spectrum of the electronic excitations and, in particular, limits the lifetimes of these excitations. Insofar as the author knows, no many-electron theory of an electron gas in a magnetic field has been constructed, but it is obvious that the larger the excitation energy, the stronger its attenuation.

Let, as before, $\epsilon_0 < h_c$, i.e., let the electrons in the ground state fill the sub-bands corresponding to the Landau levels $0\uparrow$ and $0\downarrow$. It is then clear that there can be present in the RS spectrum only those of the lines considered in Sec. 4, for which

$$1/\tau < \sigma F_1, \tag{28}$$

where σ—total scattering cross section, calculated from the formulas of Sec. 4, F_1—flux density of photons with frequency ω_1, and τ has the meaning of the average lifetime of the corresponding electronic excitations. At an excitation energy on the order of $2\hbar\omega_c$ for $\Delta n = 2$, $\Delta s = 0$ and $2\hbar\omega_c + |g_c|\beta H$ for $\Delta n = 2$, $\Delta s = -1$, the inequality (27) may cease to be satisfied starting with a certain magnetic field value H when $\hbar\omega_c$ and $|g_c|\beta H$ (and with them also $1/\tau$) become sufficiently large. At larger values of H, these lines will be missing from the RS spectrum.

This qualitative explanation agrees with the experimental data. Indeed, in InSb[4] the line $\Delta n = 2$, $\Delta s = 0$ drops out at $H = 30$ kOe if $n_e = 5 \times 10^{16}$ cm^{-3}, but this line is not observed at all if $n_e = 1 \times 10^{16}$ cm^{-3}, i.e., when the collisions in the electron gas become more significant[6)]. (We recall that the larger the electron density n_e, the more "ideal" the degenerate electron gas, see, for example,[10], Sec. 56.). In InAs[5], where the effective electron mass m_c is double the value of m_c in InSb, the line $\Delta n = 2$, $\Delta s = 0$ was observed up to $H = 55$ kOe. Finally, the value of H at which the lines $\Delta n = 2$, $\Delta s = 0$ (-1) vanish from the RS spectrum should depend also on the intensity of the scattered light, or, at a given $\hbar\omega_1$, on F_1. This dependence was not investigated experimentally. It follows from (27) that the line should vanish at smaller H with decreasing F_1.

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[6)] At small frequency shifts $\omega_1 - \omega_2$, it is difficult to separate the radiation of frequency ω_2 from the background of the laser radiation elastically scattered by the crystal. Therefore the RS cannot be observed at arbitrarily small magnetic fields [5]