CONTRIBUTION TO THE THEORY OF PROPAGATION OF TRANSVERSE SOUND IN SUPERCONDUCTORS

D. P. BELOZOROV and É. A. KANER

Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences

Submitted March 1, 1968

Zh. Eksp. Teor. Fiz. 55, 642–650 (August, 1968)

It is shown that absorption and dispersion of the velocity of transverse sound in superconductors of the first kind at low temperatures and relatively high frequencies is determined to a considerable degree by the macroscopic electromagnetic fields that are produced upon propagation of the sound.

1. INTRODUCTION

A study of the propagation of ultrasound in superconductors makes it possible to obtain important information on their properties. Starting with the appearance of the microscopic theory of superconductivity, this question has been discussed extensively in the literature[1–12]).[1)] However, the discussion was confined either to the internal deformational electromagnetic fields without account taken of the influence of the electrons and the elastic properties of the lattice (see, for example,[1]) or conversely, the analysis was carried out without allowance for the macroscopic fields occurring upon deformation[2–9].

Let us stop to discuss briefly the situation for a normal metal. The complete system of equations describing the propagation of the sound consists of the equations of elasticity theory, Maxwell's equations, and the kinetic equation for the electron distribution function, which is written in a coordinate system moving together with the lattice[13–15]:

$$\rho\frac{\partial^2 u_i}{\partial t^2} = \lambda_{iklm}\frac{\partial^2 \sigma_{lm}}{\partial x_k} + f_i, \qquad (1)$$

$$\operatorname{rot}\operatorname{rot}\mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t}, \quad \frac{dF}{dt} = \hat{I}(F).$$

Here $F(\mathbf{P}', \mathbf{r}', t)$—electron distribution function and I—collision operator. Allowance for the influence of the conduction electrons adds to the right sides of the elasticity equations the volume force f_i, which is a functional of F.[2)] The alternating magnetic field is excluded from Maxwell's equation; the displacement current is neglected, so that $\operatorname{div}\mathbf{j} = 0$. The latter equation is equivalent to the condition of electroneutrality of the metal

$$\rho' = \rho_{el} + \rho_{ion} = 0.$$

The influence of the deformation on the conduction electrons can be taken into account in the electron dispersion law[16], which is specified in the system K′, that moves with the lattice with velocity $\dot{u}_\alpha(\mathbf{r}, t)$:

$$\varepsilon'(\mathbf{P}', \mathbf{r}', t) = \varepsilon_0(\mathbf{P}') - \Lambda_{\alpha\beta}(\mathbf{P}')u_{\alpha\beta}. \qquad (2)$$

Here u_α—displacement vector, $\epsilon_0(\mathbf{P}')$—law of electron dispersion in the absence of external field (for simplicity we assume it to be quadratic and isotropic), $\Lambda_{\alpha\beta}$—symmetrical tensor of the deformation potential, whose mean value is zero on the Fermi surface[3)], and $u_{\alpha\beta}$—deformation tensor. If we replace $\mathbf{P}'$ in $\epsilon_0(\mathbf{P}')$ by the generalized momentum in the electromagnetic field $\mathbf{P}' - e\mathbf{A}'/c$ (where e—electron charge, $e < 0$ and $\mathbf{A}'(\mathbf{r}', t)$—vector potential in the moving reference frame with gauge $\varphi'(\mathbf{r}', t) = 0$), then expression (2) for ϵ' takes full account of the interaction of the conduction electrons with the deformation field and with the electromagnetic field.

It should be noted that in[11,12] they considered also, for the case of a superconductor, a self-consistent system of equations, similar to the system (1). These authors, however, confined themselves to a consideration of only longitudinal oscillations. In the present paper we consider the propagation of transverse sound and show that absorption and dispersion of the velocity of sound are determined not only by the direct deformation interaction, but also by the macroscopic electric fields (a similar effect in a normal metal was considered by Kontorovich[15]).

2. EQUATIONS AND KINETIC COEFFICIENTS

The Hamiltonian of the electron in the system K′ will be written in the form

$$\hat{\mathcal{H}}' = \frac{1}{2m}\int d\mathbf{r}\,\hat{\psi}_\sigma^+(\mathbf{r})\,(i\nabla + e\mathbf{A}'(\mathbf{r}', t))^2\,\hat{\psi}_\sigma(\mathbf{r})$$
$$+ \frac{\lambda}{2}\int d\mathbf{r}\,\hat{\psi}_\sigma^+(\mathbf{r})\,\hat{\psi}_{\sigma_1}^+(\mathbf{r})\,\hat{\psi}_{\sigma_1}(\mathbf{r})\,\hat{\psi}_\sigma(\mathbf{r}) + \frac{1}{2}\int d\mathbf{r}\,\hat{\psi}_\sigma^+(\mathbf{r})\,[u_{\alpha\beta}(\mathbf{r}', t)\,\Lambda_{\alpha\beta}(\hat{\mathbf{k}})$$
$$+ \Lambda_{\alpha\beta}(\hat{\mathbf{k}})\,u_{\alpha\beta}(\mathbf{r}', t)]\,\hat{\psi}_\sigma(\mathbf{r}). \qquad (3)$$

We use a system of units with $\hbar = c = 1$; we took explicit account of the gauge of the electromagnetic field potential $\varphi' = 0$. Here $\Lambda_{\alpha\beta}(\hat{\mathbf{k}})$—operator of the deformation potential tensor, which is symmetrical in the indices α and β and is an even function of $\hat{\mathbf{k}}$. The magnitude of the components of $\Lambda_{\alpha\beta}$ is of the order of the Fermi energy.

The first and second terms of the Hamiltonian des-

[1)]Since our analysis is limited to superconductors of the first kind, all the corresponding literature references pertain only to this cae.

[2)]We neglect here the Stuart-Tolman effect, which arises as a result of the non-inertial nature of the system K′. This effect leads to the appearance of a small additional term in the force, equal to $(m_0/e)\,\partial\mathbf{j}/\partial t$ [15].

[3)]The vanishing of the mean value of $\Lambda_{\alpha\beta}$ is a consequence of the electric quasineutrality of the metal. If it is assumed that the average (over the Fermi surface) energy of interaction of the electrons with the sound $\delta\epsilon = -\lambda_{\alpha\beta}u_{\alpha\beta}$ differs from zero, then it is necessary to include in the expression (2) the energy $\delta\mu$ (the change of the chemical potential), which is connected with the appearance of the volume charge. From the electroneutrality condition it follows that $\delta\mu$ is equal to the average value of $\delta\epsilon$ over the Fermi surface.

cribe a superconductor in an alternating external field; λ ($\lambda < 0$) is a constant connected with the electron-phonon interaction, which is responsible for the superconductivity[17]. We assume that the second term retains its form also in the moving reference frame, since the operators $\hat{\psi}_\sigma(\mathbf{r})$ transform under Galilean transformation: $\hat{\psi}'_\sigma(\mathbf{r}) = \exp[-im\mathbf{v}\cdot\mathbf{r}]\hat{\psi}_\sigma(\mathbf{r})$. Repeated spin indices in H′ imply summation.

Our problem is to find the average force and the average current, which enter in the system (1), and also expressions for the kinetic coefficients relating the current with the fields **A** and **u**. The analysis is carried out in the linear approximation in **A** and **u**. The electron free path is assumed to be infinite.

To determine the kinetic coefficients we use the scheme proposed in[17]. We consider first the Gor'kov equations for a superconductor in the temperature technique and assume that the displacement vector $\mathbf{u}(\mathbf{r}, \tau)$ and the vector potential $\mathbf{A}(\mathbf{r}, \tau)$ depend on the imaginary "time" τ in the interval $(0, \beta)$ ($\beta = 1/T$, T—temperature). Then, solving these equations in the linear approximation, we find the kinetic coefficients. The true physical kinetic coefficients are obtained by analytic continuation in the region of the real frequencies.

We shall need expressions for the current and force in terms of the Green's function. To this end, in accordance with[7], we have

$$\mathbf{j}(x) = \frac{ie}{m}(\nabla_{\mathbf{r}'} - \nabla_{\mathbf{r}})G(x, x')\Big|_{\substack{\mathbf{r}'\to\mathbf{r}\\ \tau'\to\tau+0}} - \frac{2e^2}{m}\mathbf{A}(\mathbf{r}, \tau)G(x, x')\Big|_{\substack{\mathbf{r}'\to\mathbf{r}\\ \tau'\to\tau+0}} \quad (4)$$

The force operator is defined as the variational derivative of the Hamiltonian with respect to the displacement:

$$\hat{f}_\alpha' \equiv -\frac{\delta\hat{\mathscr{H}}'}{\delta u_\alpha} = \frac{1}{2}\frac{\partial}{\partial x_\beta}[\hat{\psi}_\sigma^+(\mathbf{r})\Lambda_{\alpha\beta}(\hat{\mathbf{k}})\hat{\psi}_\sigma(\mathbf{r}) + (\Lambda_{\alpha\beta}(\hat{\mathbf{k}})\hat{\psi}_\sigma^+(\mathbf{r}))\hat{\psi}_\sigma(\mathbf{r})]. \quad (5)$$

We have integrated by parts and used the self-adjoint property of $\Lambda_{\alpha\beta}(\hat{\mathbf{k}})$. The average force can therefore be readily expressed in terms of the single-particle Green's function $G(x, x')$:

$$f_\alpha'(x) = 2\frac{\partial}{\partial x_\beta}\left\{\frac{1}{2}[\Lambda_{\alpha\beta}(\hat{\mathbf{k}}) + \Lambda_{\alpha\beta}(\hat{\mathbf{k}}')]G(x, x')\Big|_{\substack{\mathbf{r}'\to\mathbf{r}\\ \tau'\to\tau+0}}\right\}. \quad (6)$$

The coefficient 2 is the result of the summation over the spins. The Gor'kov system of equations[17] has the usual form

$$\begin{pmatrix} \left\{-\dfrac{\partial}{\partial\tau} - \dfrac{1}{2m}(i\nabla + e\mathbf{A})^2 + \mu - \hat{U}_{ac}\right\}, & \Delta(x) \\ -\Delta^*(x), & \left\{\dfrac{\partial}{\partial\tau} - \dfrac{1}{2m}(-i\nabla + e\mathbf{A})^2 + \mu - \hat{U}_{ac}\right\} \end{pmatrix} \quad (7)$$

$$\times\begin{pmatrix} G(x, x') & F(x, x') \\ \bar{F}(x, x') & -G(x', x) \end{pmatrix} = \begin{pmatrix} \delta(x - x') & 0 \\ 0 & -\delta(x - x') \end{pmatrix}.$$

Here

$$x = (\mathbf{r}, \tau), \qquad \hat{U}_{ac} = {}^1/_2(u_{\alpha\beta}(\mathbf{r}, \tau)\Lambda_{\alpha\beta}(\hat{\mathbf{k}}) + \Lambda_{\alpha\beta}(\hat{\mathbf{k}})u_{\alpha\beta}(\mathbf{r}, \tau)).$$

In the considered case of transverse fields, when div **A** = 0 and div **u** = 0, we can assume that in the linear approximation the gap remains unchanged. Indeed, in this approximation the addition to the gap can depend only on the scalar combinations of div **A** and div **u**, which vanish in our case.

Solving then the system (7) and taking the Fourier transform, we obtain the following expression for the current density:

$$\mathbf{j}(\mathbf{k}, \omega_0) = -\frac{2e^2T}{(2\pi)^3m^2}\sum_\omega\int d\mathbf{p}\,\mathbf{p}(\mathbf{p}\mathbf{A}(\mathbf{k}, \omega_0))\Big[G_0(p_+)G_0(p_-) + F_0(p_+)\bar{F}_0(p_-)\Big] + \frac{2Tie}{(2\pi)^3m}\sum_\omega\int d\mathbf{p}\,\mathbf{p}(\tilde{\Lambda}_{\alpha\beta}(\mathbf{p})k_\beta u_\alpha(\mathbf{k}, \omega_0))$$
$$\times[G_0(p_+)G_0(p_-) - F_0(p_+)\bar{F}_0(p_-)] - \frac{e^2}{m}N\mathbf{A}, \quad (8)$$

where

$$\tilde{\Lambda}_{\alpha\beta}(\mathbf{p}) = {}^1/_2[\Lambda_{\alpha\beta}(\mathbf{p}_+) + \Lambda_{\alpha\beta}(\mathbf{p}_-)],$$

$$p_\pm = \{\mathbf{p}_\pm, \omega_\pm\}, \quad \mathbf{p}_\pm = \mathbf{p} \pm \mathbf{k}/2, \quad \omega_\pm = \omega \pm \omega_0/2.$$

The force density is given by the formula

$$f_\alpha(\mathbf{k}, \omega_0) = -\frac{2ieT}{(2\pi)^3m}\sum_\omega\int d\mathbf{p}(\mathbf{p}\mathbf{A}(\mathbf{k}, \omega_0))\times\tilde{\Lambda}_{\alpha\beta}(\mathbf{p})k_\beta[G_0(p_+)G_0(p_-) + F_0(p_+)\bar{F}_0(p_-)]$$
$$-\frac{2T}{(2\pi)^3}\sum_\omega\int d\mathbf{p}(\tilde{\Lambda}_{\alpha\beta}(\mathbf{p})k_\beta)(\tilde{\Lambda}_{\gamma\delta}(\mathbf{p})k_\delta u_\gamma)[G_0(p_+)G_0(p_-) - F_0(p_+)\bar{F}_0(p_-)], \quad (9)$$

where

$$G_0(p) = -\frac{i\omega + \xi}{\omega^2 + \xi^2 + \Delta^2}, \quad F_0(p) = \bar{F}_0(p) = \frac{\Delta^2}{\omega^2 + \xi^2 + \Delta^2}, \quad \xi = \frac{p^2}{2m} - \mu.$$

In the obtained expressions it is convenient to sum over ω and to carry out an analytic continuation in the upper half-plane of the frequency $\tilde{\omega} = i\omega_0$.

We introduce the kinetic coefficients with the aid of the relations

$$j_\alpha = \sigma_{\alpha\beta}(i\omega A_\beta) + S_{\alpha\beta}u_\beta,$$
$$f_\alpha = S_{\alpha\beta}'(i\omega A_\beta) - l_{\alpha\beta\gamma\delta}k_\beta k_\gamma u_\delta. \quad (10)$$

From a comparison of (10) with (8) and (9), after summing over ω, we obtain expressions for the kinetic coefficients. The conductivity is determined by the usual formula[18]:

$$\sigma_{\alpha\beta}(\omega) = \frac{Ne^2}{m(-i\omega)}\delta_{\alpha\beta} + \frac{e^2}{4(2\pi)^3(i\omega)}\int d\mathbf{p}\,v_\alpha(\mathbf{p})v_\beta(\mathbf{p})L^{(+)}(\varepsilon_+, \varepsilon_-, \omega, \Delta). \quad (11)$$

The tensor $-l_{\alpha\beta\gamma\delta}(\omega)$, which determines the deformation contribution is the force, equals

$$-l_{\alpha\beta\gamma\delta}(\omega) = \frac{1}{4(2\pi)^3}\int d\mathbf{p}(\tilde{\Lambda}_{\alpha\beta}(\mathbf{p})\tilde{\Lambda}_{\gamma\delta}(\mathbf{p}))L^{(-)}(\varepsilon_+, \varepsilon_-, \omega, \Delta). \quad (12)$$

Here

$$L^{(\pm)}(\varepsilon_+, \varepsilon_-, \omega, \Delta) = \left\{\left(1 - \frac{\xi_+\xi_- \pm \Delta^2}{\varepsilon_+\varepsilon_-}\right)\left(\operatorname{th}\frac{\varepsilon_+}{2T} + \operatorname{th}\frac{\varepsilon_-}{2T}\right)\right.$$
$$\times\left(\frac{1}{\varepsilon_+ + \varepsilon_- + \omega + i\delta} + \frac{1}{\varepsilon_+ + \varepsilon_- - \omega - i\delta}\right) + \left(1 + \frac{\xi_+\xi_- \pm \Delta^2}{\varepsilon_+\varepsilon_-}\right)$$
$$\left.\times\left(\operatorname{th}\frac{\varepsilon_+}{2T} - \operatorname{th}\frac{\varepsilon_-}{2T}\right)\left(\frac{1}{\varepsilon_+ - \varepsilon_- + \omega + i\delta} + \frac{1}{\varepsilon_+ - \varepsilon_- - \omega - i\delta}\right)\right\} \quad (13)$$

The tensor $S_{\alpha\beta}$, which determines the deformation current, is given by the expression

$$S_{\alpha\beta}(\omega) = \frac{ik_\gamma e}{4(2\pi)^3}\int d\mathbf{p}\,v_\alpha(\mathbf{p})(\tilde{\Lambda}_{\beta\gamma}(\mathbf{p}))$$
$$\times\left\{\left(\frac{\xi_+}{\varepsilon_+} + \frac{\xi_-}{\varepsilon_-}\right)\left(\operatorname{th}\frac{\varepsilon_+}{2T} - \operatorname{th}\frac{\varepsilon_-}{2T}\right)\left(\frac{1}{\varepsilon_+ - \varepsilon_- + \omega + i\delta} - \frac{1}{\varepsilon_+ - \varepsilon_- - \omega - i\delta}\right)\right.$$
$$\left.+\left(\frac{\xi_+}{\varepsilon_+} - \frac{\xi_-}{\varepsilon_-}\right)\left(\operatorname{th}\frac{\varepsilon_+}{2T} + \operatorname{th}\frac{\varepsilon_-}{2T}\right)\left(\frac{1}{\varepsilon_+ + \varepsilon_- + \omega + i\delta} - \frac{1}{\varepsilon_+ + \varepsilon_- - \omega - i\delta}\right)\right\} \quad (14)$$

where $v_\alpha = p_\alpha/m$—electron velocity, and $\epsilon = (\xi^2 + \Delta^2)^{1/2}$—excitation energy.

The coefficients $S_{\alpha\beta}$ and $S'_{\alpha\beta}$ are connected by symmetry conditions, namely:

$$S_{\alpha\beta}(\mathbf{k}, \omega) = -i\omega S_{\beta\alpha}'(-\mathbf{k}, -\omega). \quad (15)$$

We note that if we put $\widetilde{\Lambda}_{\alpha\beta} = 0$ for the current in (8), then we obtain the well-known expression for the current in an alternating external field[17]. On the other hand, if we neglect the deformation electromagnetic field, i.e., we put $\mathbf{A} = 0$, then the remaining part of the force f_α,

$$f_\alpha(\mathbf{k}, \omega_0) = -\frac{2T}{(2\pi)^3}\sum_\omega \int d\mathbf{p}\,(\widetilde{\Lambda}_{\alpha\beta}(\mathbf{p})k_\beta)$$
$$\times(\widetilde{\Lambda}_{\gamma\delta}(\mathbf{p})u_\gamma k_\delta)[G_0(p_+)G_0(p_-) - F_0(p_+)\bar{F}_0(p_-)],$$

essentially coincides with the expression for the polarization operator[2]:

$$\Pi(\mathbf{k}, i\omega_n) = \frac{T}{(2\pi)^3}\sum_\omega \int d\mathbf{p}[G(p)G(p-k) - F(p)F(p-k)].$$

In the limiting case of a normal metal, expression (9) for the force coincides with that obtained in[14,15]:

$$\lim_{\Delta\to 0} f_\alpha(\mathbf{k}, \omega) = \frac{2}{(2\pi)^3}\frac{\partial}{\partial x_\beta}\int d\mathbf{p}'\,\Lambda_{\alpha\beta}(\mathbf{p}')F(\mathbf{p}', \mathbf{r}', t).$$

Before we proceed to consider the different asymptotic forms of the kinetic coefficients, we must indicate that in the linear approximation the expressions for the current j and force f coincide in the systems K and K'.

3. SOLUTION OF THE SYSTEM OF EQUATIONS

Going over to Fourier components, we write the system (1) in the form

$$\rho\omega^2 u_i = \lambda_{iklm}k_k k_l u_m - f_i(\mathbf{k}, \omega), \qquad k^2\mathbf{A} = 4\pi\mathbf{j}(\mathbf{k}, \omega),$$
$$j_\alpha(\mathbf{k}, \omega) = i\omega\sigma_{\alpha\beta}A_\beta + S_{\alpha\beta}u_\beta,\ f_\alpha(\mathbf{k}, \omega) = i\omega S_{\alpha\beta}'A_\beta - l_{\alpha\beta\gamma\delta}k_\beta k_\gamma u_\delta, \tag{16}$$

whence, bearing in mind the isotropic case (see below), namely $\sigma_{\alpha\beta} = \sigma\delta_{\alpha\beta}$, $S_{\alpha\beta} = S\delta_{\alpha\beta}$, $l_{\alpha\beta\gamma\delta}k_\gamma k_\delta = lk^2\delta_{\alpha\beta}$ ($\alpha, \beta, \gamma, \delta = 1, 2$), we get

$$\mathbf{A} = \frac{Su}{k^2 - 4\pi i\sigma}.$$

The contribution to the force $\mathbf{f}^{em}$, connected with allowance for the macroscopic electromagnetic fields, is given by

$$\mathbf{f}^{em}(\mathbf{k}, \omega) = -\frac{S^2\mathbf{u}}{k^2 - 4\pi i\sigma}. \tag{17}$$

On the other hand, the deformation contribution $\mathbf{f}^d$ is given by

$$f_\alpha{}^d(\mathbf{k}, \omega) = -l_{\alpha\beta\gamma\delta}k_\beta k_\gamma u_\delta = -lk^2u_\alpha(\mathbf{k}, \omega),$$
$$\mathbf{f}^{em} + \mathbf{f}^d = \mathbf{f}. \tag{18}$$

We shall henceforth be interested in the role of the electromagnetic fields arising in the case of sound propagation. It is more convenient to go over immediately to the frequency increments $\delta\omega = \omega - \mathbf{k}s_0$:

$$\delta\omega \approx -\frac{1}{2\rho\omega}\frac{f}{u}, \tag{19}$$

Then

$$\frac{\delta\omega^{em}}{\omega} = \frac{1}{2\rho\omega^2}\ \frac{S^2}{k^2 - 4\pi i\omega\sigma} \tag{20}$$
$$\frac{\delta\omega^d}{\omega} = \frac{1}{2\rho s_0{}^2}l. \tag{21}$$

Here s_0—non-renormalized velocity of transverse sound.

4. ASYMPTOTIC EXPRESSIONS FOR THE KINETIC COEFFICIENT

Let us consider the asymptotic behavior of the kinetic coefficients in the region of the low temperatures and relatively high frequencies, satisfying the inequalities

$$\frac{s}{v}\Delta \ll \omega \ll T \ll \Delta. \tag{22}$$

This region corresponds to temperatures $T \lesssim 1°$K and frequencies $\omega \sim 10^9 - 10^{10}$ sec^{-1}. In this region of frequencies and temperatures it is easy to satisfy the condition $\omega\tau \gg 1$, where τ—electron free path time. The inequality $\omega \gg (s/v)\Delta$ denotes that the wavelength of the sound is considerably smaller than the ordering parameter $\xi_0 = v/\Delta$. Owing to the smallness of the wave vector of the sound k compared with the Fermi momentum, the quantity $\Lambda_{\alpha\beta}(\mathbf{p})$ can be replaced by

$$\widetilde{\Lambda}_{\alpha\beta}(\mathbf{p}) = \Lambda(\delta_{\alpha\beta} - 3n_\alpha n_\beta), \tag{23}$$

where n_α—component of the unit momentum vector $\mathbf{p}/p$ on the Fermi sphere.

The conductivity tensor was investigated in detail in[17,18], and we shall use the asymptotic expressions obtained there. As will be seen below, for the transverse case the quantity $S_{\alpha\beta}$ is proportional to $\sigma_{\alpha\beta}$. For complete clarity, we shall indicate briefly how the calculations are performed, and present the corresponding expressions. As always, in evaluating the integrals we make use of the fact that the region of integration of the momenta is actually a narrow strip near the Fermi surface. In this connection, all the slowly-varying factors such as v^2 etc. are replaced by their values on the Fermi surface. Integrating over the azimuthal angle, we can easily see that the tensors $\sigma_{\alpha\beta}$ and $S_{\alpha\beta}$ are diagonal: $\sigma_{\alpha\beta} = \sigma\delta_{\alpha\beta}$, $S_{\alpha\beta} = S\delta_{\alpha\beta}$ ($\alpha, \beta = 1, 2$) (we recall that we are dealing only with the transverse part).

In the case $\Delta/kv \ll 1$ the main contribution to the integrand is made by the angle region $|\cos\theta| \sim \Delta/kv$. Therefore the terms proportional to $\cos^2\theta$ can be neglected compared with unity. This means that we confine ourselves to the first term of the expansion in the small parameter Δ/kv.

In $l_{\alpha\beta\gamma\delta}(\omega)$ we substitute expression (23) and average over the azimuthal angle:

$$\langle\widetilde{\Lambda}_{\alpha\beta}(\mathbf{p})\widetilde{\Lambda}_{\gamma\delta}(\mathbf{p})\rangle_\varphi = \Lambda^2\{\hat{A}_1 + \hat{A}_2\cos^2\theta + \hat{A}_3\cos^4\theta\}. \tag{24}$$

$\hat{A}_1 = \delta_{\alpha\beta}\delta_{\gamma\delta}$ yields the main term of the expansion. However, in the case of transverse sound this term drops out, since the corresponding term in the Hamiltonian is $\Lambda u_{\alpha\alpha} = \Lambda\,\mathrm{div}\,\mathbf{u} = 0$. Therefore the quantity $l_{\alpha\beta\gamma\delta}$ for the transverse sound turns out to be of higher order of smallness in Δ/kv, with $l_{\alpha\beta\gamma\delta}k_\beta k_\gamma = lk^2\delta_{\alpha\beta}$. At the same time, for the case of longitudinal sound this term plays the principal role.

To improve the convergence of the integrals with respect to ξ, we subtract the values of the integrand at $\omega = 0$, i.e., the stationary conductivity $\sigma_{\alpha\beta}(0)$ and $-l_{\alpha\beta\gamma\delta}(0)$. The term

fo $$-l_{\alpha\beta\gamma\delta}(0) = \frac{2}{(2\pi)^3}\oint_{S_F}\frac{dS}{v}\widetilde{\Lambda}_{\alpha\beta}(\mathbf{p})\widetilde{\Lambda}_{\gamma\delta}(\mathbf{p})$$

gives the constant renormalization of the speed of sound, which can be disregarded if s_0 is replaced by s.

In the expression for $S_{\alpha\beta}$, we integrate over the azimuthal angle, take the slowly-varying terms outside the integral sign, and reduce to a common denominator the terms

$$\left(\frac{1}{\varepsilon_+ - \varepsilon_- + \omega + i\delta} - \frac{1}{\varepsilon_+ - \varepsilon_- - \omega - i\delta}\right)$$
$$\left(\frac{1}{\varepsilon_+ + \varepsilon_- + \omega + i\delta} - \frac{1}{\varepsilon_+ + \varepsilon_- - \omega - i\delta}\right).$$

Introducing one power of $\cos\theta = (\xi_+ - \xi_-)/kv$ under the sign of integration with respect to ξ, and using the identities

$$(\xi_+ - \xi_-)\left(\frac{\xi_+}{\varepsilon_+} \pm \frac{\xi_-}{\varepsilon_-}\right) = (\varepsilon_+ \mp \varepsilon_-)\left(1 \pm \frac{\xi_+\xi_- + \Delta^2}{\varepsilon_+\varepsilon_-}\right), \tag{25}$$

we can show that the following equality holds

$$\frac{3}{2} i\omega \frac{\Lambda}{\mu}\frac{m}{e}\left(i\omega\sigma + \frac{Ne^2}{m}\right) = S(\omega), \tag{26}$$

i.e., S is proportional to the conductivity without the diamagnetic term Ne^2/m. This proportionality takes place only for the transverse case.

Finally, the asymptotic forms of the coefficients in the region of interest to us are determined by the following formulas:

$$\sigma(\omega) = \frac{3\pi^2}{4}\frac{Ne^2}{m(-i\omega)}\frac{\Delta}{kv}\left\{1 - \frac{2i}{\pi}\frac{\omega}{T}\exp\left(-\frac{\Delta}{T}\right)\ln\frac{4T}{\omega\gamma}\right\}$$
$$(\ln\gamma = 0.577)$$
$$S(\omega) = -\frac{3}{2}\frac{\Lambda}{\mu}\frac{m}{e}\omega^2\left\{\sigma(\omega) + \frac{Ne^2}{m(i\omega)}\right\},$$
$$l(\omega) = -\frac{9}{2}\frac{N\Lambda^2}{\mu}\left(\frac{s}{v}\right)^2\left\{1 + \frac{3\pi i}{kv}T\exp\left(-\frac{\Delta}{T}\right)\right\}. \tag{27}$$

5. RESULTS

Let us analyze first the variation of the speed of sound, confining ourselves to the principal terms of the asymptotic expansions. The addition to the speed of sound, due to the direct deformation interaction

$$\frac{\delta s^d}{s} = -\frac{9}{8}\left(\frac{s\Lambda}{v\mu}\right)^2, \tag{28}$$

has the same form as in a normal metal. It is negative, its order of magnitude is $(s/v)^2$, and is independent of the frequency. Consequently, in this approximation the direct deformation interaction leads to a constant renormalization of the speed of sound.

The change of the speed of sound connected with allowance for the electromagnetic fields is determined by the expression

$$\frac{\delta s^{em}}{s} = -\frac{9}{32\pi}\left(\frac{s\Lambda}{v\mu}\right)^2 \frac{1 + O(\Delta/kv)}{(\omega c/\omega_0 s)^2 + 3\pi^2 s\Delta/4\omega v}, \tag{29}$$

where $\omega_0 = (4\pi Ne^2/mc^2)^{1/2}$ is the plasma frequency. The magnitude of this addition to the velocity depends on the frequency. In the frequency region where

$$\omega < \Omega = \left(\frac{3\pi^2}{4}\frac{s^3\omega_0^2\Delta}{c^2 v}\right)^{1/3} \approx 10^{10}\ \mathrm{sec}^{-1}, \tag{30}$$

$$\frac{\delta s^{em}}{s} \approx -\frac{3}{8\pi^3}\left(\frac{s\Lambda}{v\mu}\right)^2 \frac{\omega v}{s\Delta}, \tag{31}$$

the speed of sound changes in proportion to the frequency. When $\omega \approx \Omega$ the change in the speed of sound due to the electromagnetic field is maximal and its order of magnitude is $\delta s^d/s$. In the region of higher frequencies ($\omega > \Omega$) the value of $|\delta s^{em}|$ decreases like ω^{-2}.

The absorption of sound is determined by the imaginary part of the frequency and, as expected, decreases exponentially at low temperatures. In the frequency region under consideration, $\omega \ll \Delta$, there is no absorption due to the pair decay. The deformation absorption is connected with the imaginary part of the quantity $l(\omega)$ in (27) and is given by the formula ($\Gamma \equiv -|\mathrm{Im}\,\omega/\omega|$):

$$\frac{\Gamma_s^d}{\Gamma_n^d} = \frac{4T}{\omega}e^{-\Delta/T}, \tag{32}$$

where the coefficient of deformation absorption in the normal state Γ_n^d is equal to

$$\Gamma_n^d = \frac{27\pi}{32}\left(\frac{\Lambda}{\mu}\right)^2\left(\frac{s}{v}\right)^3. \tag{33}$$

The quantity Γ_s^d/Γ_n^d decreases with increasing frequency, and the damping decrement $\omega\Gamma_s^d$ itself does not depend on ω. The sound absorption connected with the electromagnetic field is given by

$$\frac{\Gamma_s^{em}}{\Gamma_n^{\pi}} = \frac{4}{3\pi^3}\frac{\omega}{T}\frac{v}{s}\frac{e^{-\Delta/T}}{1+(\omega/\Omega)^3}\ln\frac{4T}{\gamma\omega}. \tag{34}$$

In the frequency region $\omega < \Omega$, this part of the absorption increases in proportion to ω, reaching a maximum at $\omega \approx \Omega$, and then decreasing like ω^{-2}. The values of Γ_s^d and Γ_s^{em} at $\omega \approx \Omega$ are comparable in order of magnitude.

Thus, in the region of high sound frequencies (on the order of several dozen GHz) the dispersion of the velocity on the absorption of sound in superconductors are determined to a considerable degree by the macroscopic electric fields which arise when sound propagates. At low frequencies ($\omega \lesssim s\Delta/v \sim 10^8\ \mathrm{sec}^{-1}$) these fields do not play a noticeable role.

In conclusion it should be noted that the role of the macroscopic electromagnetic field is considerable in those cases when the principal term in the deformation direction (the term $\hat{A}_1$ in (24)) gives a zero contribution for the transverse sound. A decisive role is played in this case by $\widetilde{\Lambda}_{\alpha\beta}$ (23). On the other hand, if the contribution connected with $\hat{A}_1$ differs from zero, the electromagnetic fields can be neglected compared with the direct deformation interaction.

[1] T. Tsuneto, Phys. Rev. **121**, 402 (1961).
[2] V. L. Pokrovskiĭ, Zh. Eksp. Teor. Fiz. **40**, 898 (1961) [Sov. Phys.-JETP **13**, 628 (1961)].
[3] V. L. Pokrovskiĭ, ibid. **40**, 143 (1961) [**13**, 100 (1961)].
[4] V. L. Pokrovskiĭ and V. I. Toponovov, ibid. **40**, 1112 (1961) [**13**, 785 (1961)].
[5] V. L. Pokrovskiĭ and M. S. Ryvkin, ibid. **40**, 1859 (1961) [**13**, 1306 (1961)].
[6] I. A. Privorotskiĭ, ibid. **43**, 1331 (1962) [**16**, 945 (1963)].
[7] I. A. Privorotskiĭ, ibid. **42**, 450 (1962) [**15**, 315 (1962)].
[8] B. T. Geilikman and V. A. Kresin, ibid. **41**, 1142 (1961) [**14**, 816 (1962)].
[9] M. Levy, Phys. Rev. **131**, 1596 (1963).
[10] J. C. Callen and R. A. Ferrell, ibid. **146**, 282 (1966).

[11] I. O. Kulik, Zh. Eksp. Teor. Fiz. **47**, 2159 (1964) [Sov. Phys.-JETP **20**, 1450 (1965)].
[12] Yu. I. Balkerei, Fiz. Tverd. Tela **8**, 797 (1966) [Sov. Phys.-Solid State **8**, 797 (1966)].
[13] É. A. Kaner and V. G. Skobov, Zh. Eksp. Teor. Fiz. **45**, 610 (1963) [Sov. Phys.-JETP **18**, 419 (1964)].
[14] V. G. Skobov and É. A. Kaner, ibid. **46**, 273 (1964) [**19**, 189 (1964)].
[15] V. M. Kontorovich, ibid. **45**, 1638 (1963) [**18**, 1125 (1964)].
[16] A. I. Akhiezer, M. I. Kaganov, and G. Ya. Lyubarskiĭ, ibid. **32**, 837 (1957) [**5**, 685 (1957)].
[17] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum-Field-Theoretical Methods in Statistical Physics), Fizmatgiz, 1962 [Pergamon, 1965].
[18] A. A. Abrikosov and I. I. Khalatnikov, in: Fizika nizkikh temperatur (Low Temperature Physics), IIL, 1959, p. 886.

Translated by J. G. Adashko
74