POSSIBLE EXPERIMENTAL INVESTIGATION OF PERIODIC STRUCTURES IN SUPERCONDUCTORS OF THE SECOND KIND WITH THE AID OF MUONS

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It is shown that an analysis of the time variation of the polarization of a μ^+ meson in a superconductor makes it possible to determine the character of the periodic structure of the mixed state of superconductors of the second kind, and also to verify the hypothesis of the existence of superconducting filaments in superconductors of the third kind.

As is well known, Abrikosov's solution of the Ginzburg-Landau equations[1] shows that in superconductors of the second kind, the normal and the superconducting phases should form in the mixed state either a quadratic or a triangular periodic two-dimensional lattice. However, the calculation[1,2] shows that the free energy of the triangular lattice is smaller than the free energy of the quadratic one by only 1.7%. Taking into account the fact that both the equations of the theory themselves and their solutions have an approximate character, such a small deviation is clearly insufficient to be able to predict with assurance the structure of the Abrikosov vortices in superconductors of the second kind, even near the critical temperature, where the Ginzburg-Landau equations are valid. As to temperatures much lower than T_c, the question in this case remains in general open. Although the presently reported neutron diffraction experiments[3] seem to indicate that a triangular lattice is realized, further experimental analysis is very desirable. The method proposed here, as expected, will make it possible to determine reliably precisely the type of lattice which is realized in superconductors of the second kind.

Assume that we have a superconductor of the second kind in a mixed state in an external magnetic field H ($H_{c1} < H < H_{c2}$). We aim on the sample, perpendicular to the field direction, a beam of polarized μ^+ mesons. Since the time necessary to decelerate the μ^+ meson to a velocity close to thermal is in any case $< 10^{-10}$, and since no depolarization is observed at all in a metal in the absence of a magnetic field, the time variation of the observed polarization of the μ^+ meson will be determined entirely by the value of the magnetic field at the point of stopping. It is immaterial here whether the μ^+ meson remains in the interstices of the lattice as a defect or whether it forms a chemical compound with the lattice atom.

The proper time variation of the observed polarization is determined by the expression

$$P(t) = \frac{1}{S}\iint_S \exp\left\{\frac{ieH(x,y)}{\mu c}t\right\}dx\,dy. \tag{1}$$

Here S—area of the sample cross section perpendicular to the field direction.

For superconductors of the second kind, the field inside the sample is determined by the relation

$$H(x,y) = H_0 - \frac{1}{2\varkappa}\omega(x,y), \tag{2}$$

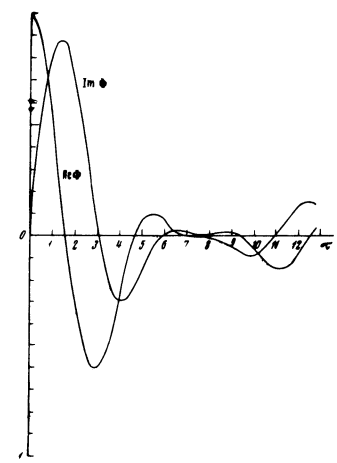

FIG. 1. Real and imaginary parts of $\Phi(i\tau)$ for a triangular lattice.

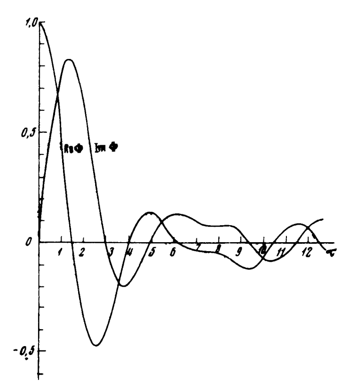

FIG. 2. Real and imaginary parts of $\Phi(i\tau)$ for a quadratic lattice.

where

$$\omega(x,y) = |C_0|^2 3^{-1/2}\sum_{m,n=-\infty}^{\infty}(-1)^{mn}\frac{\exp[2\pi i(mu+nv)]}{\exp[\pi 3^{-1/2}(m^2+mn+n^2)]}$$
$$u = 2x/3^{1/2}L_y, \quad v = (3^{-1/2}x + y)/L_y, \quad L_y = (4\pi/3^{1/2}\varkappa^2)^{1/2} \tag{3}$$

for the triangular lattice and

$$\omega(x,y) = |C_0|^2\sum_{m,n=-\infty}^{\infty}(-1)^{mn}\frac{\exp[2\pi i(mu+nv)]}{\exp[\pi(m^2+n^2)/2]} \tag{4}$$
$$L_1 = (2\pi/\varkappa^2)^{1/2}, \quad u = x/L_1, \quad v = y/L_1$$

for the quadratic lattice[2].

Inasmuch as $\omega(x, y)$ is a periodic function, then neglecting surface effects in (1) we can confine ourselves to integration over the unit cell of the structure. We introduce the symbol $\tau = -(4\pi eM/\mu c)t$, where M—average magnetic moment of the sample. Averaging (2) over the volume, we see immediately that

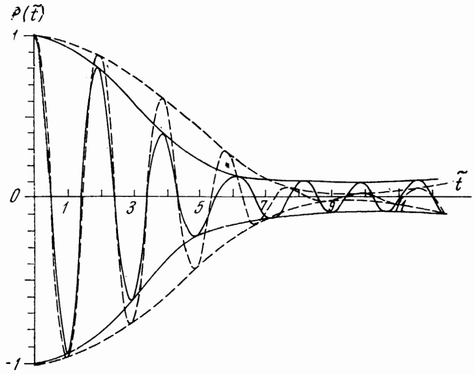

FIG. 3. Time variation of the polarization of the μ^+ meson at $4\pi M/H_0 = 0.2$. Solid curve – quadratic lattice, dashed – triangular lattice; $\tilde{t} = teH_0/\mu c$.

$\tau = e|C_0|^2 t/2\times 3^{1/2}\mu c\kappa$ for a triangular lattice and $\tau = e|C_0|^2 t/2\mu c\kappa$ for a quadratic one. Then

$$P(t) = \exp\left\{\frac{ieH_0}{\mu c}t\right\}\Phi(-i\tau), \tag{5}$$

where

$$\Phi(i\tau) = \int_0^1\int_0^1 \exp[i\tau\Sigma(u, v)]\,du\,dv. \tag{6}$$

Here $\Sigma(u, v)$ denotes the corresponding sums in formulas (3) and (4).

The observed polarization is the real part of expression (1) and can be written in the form

$$P = \cos\frac{eH_0}{\mu c}t\,\mathrm{Re}\,\Phi(i\tau) + \sin\frac{eH_0}{\mu c}t\,\mathrm{Im}\,\Phi(i\tau). \tag{7}$$

The results of the numerical calculation for Re Φ and Im Φ are given in Figs. 1 and 2 for both lattices.

The results of the calculation for P(t) at a ratio $|4\pi M/H_0| = 0.2$, are shown in Fig. 3. As seen from Fig. 3, the time variation of the polarization is essentially different for quadratic and triangular lattices. For the ratio $|4\pi M/H_0| = 0.1$, the polarization curves have a perfectly similar character, and the relative positions of the envelopes duplicates the picture shown in Fig. 3.

In the investigation of the polarization of the μ^+ meson at temperatures much lower than T_c, it must be borne in mind that the picture can become greatly complicated by the fact that at large concentrations of the superconducting electrons forming Cooper pairs it is possible that depolarization will be observed in the superconductor also in the absence of a magnetic field. Indeed, if the μ^+ meson is fully thermalized, then the number of exchanges with the electrons should decrease rapidly, since it is necessary to break the Cooper pair for a single exchange act, and thus lose the binding energy, whereas a decrease of the number of exchanges leads as usual to depolarization. On the other hand, when $T \ll T_c$, as is well known, the equations of the theory have no solution. Therefore any unique interpretation of the results of the observations would be possible only at $T \sim T_c$.

It is also obvious that in the case when the transition region between the superconducting and normal phases is small, the polarization P(t) should have the form

$$P(t) = a + (1-a)\cos\frac{eH}{\mu c}t.$$

Therefore, by observing the polarization of the μ^+ meson in strongly dislocated superconductors of the second kind (now called sometimes superconductors of the third kind), it is possible to verify whether the model of superconducting filaments with a thin transition layer to the normal phase can be realized in them.

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