# *SIZE EFFECT AND CYCLOTRON RESONANCE IN CADMIUM ON DRIFTING EFFECTIVE ELECTRONS*

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We investigated resonance effects at frequency $3.6 \times 10^{10}$ Hz and radiofrequency size effects at frequency $5 \times 10^6$ Hz in cadmium in a magnetic field H perpendicular to the surface of the metal. The observed phenomena were due to effective electrons drifting along the magnetic field, which produced a system of field and current spikes inside the metal. It is shown that if the electron orbit has several effective points where the velocity inside the metal is $v_z = 0$, then the cyclotron resonance and the size effect have a number of singularities. Certain characteristics of the Fermi surface of cadmium are obtained from the experimental results.

## INTRODUCTION

MUCH attention has been paid recently, in the study of the high-frequency properties of metals, to the anomalous penetration of the electromagnetic field inside the metal. Such a penetration becomes possible in the presence of a magnetic field and when the mean free path of the electron $l$ is large. The field inside the metal is excited not by all the electrons on the Fermi surface, but by individual groups, and can take place in a magnetic field parallel to the surface of the metal[1], as well as in an oblique field[2-4]. The electrons interacting with the electric field in the skin layer can carry the field to a large depth (on the order of $l$) either along a chain of closed trajectories[1,2] or by drifting along the direction of the magnetic field, if the Fermi surface has elliptical limiting points or sections with an extremum of $\partial S/\partial p_H$[3,4], where S—area of intersection with the plane $p_H$ = const, and $p_H$—projection of the electron momentum on the direction of the magnetic field. The electric field produced inside the metal can either be in the form of sharp spikes[1-3,5] or have a quasi-harmonic distribution[4-6]. The form of the distribution of the field inside the metal is determined by the character of the interaction of the electrons with the field. If the electron trajectory has points in which the velocity is parallel to the surface of the metal ($v_z = 0$, z ∥ n, n—normal to the surface of the sample), then the interaction is "effective" and a system of sharp spikes of field and current are produced inside the metal, and the distance between them is determined by the characteristic dimensions of the electron trajectories in the magnetic field. If the trajectory has no points with $v_z = 0$, the interaction is "ineffective," and the field inside the metal has a quasi-harmonic form.

At low frequencies, when the frequency $\omega$ of the external electromagnetic field is much smaller than the cyclotron frequency $\Omega$ and the frequency of collisions between the electrons and the scatterers $\nu$, the penetration of the field inside a semi-infinite metal does not lead to singularities of the impedance. However, it can be revealed from the size effect on a plane-parallel plane. The field excited on one side of the plane can emerge from the other side and by the same token lead to impedance singularities. This effect was observed experimentally both for "effective" electrons[3-5] and for "ineffective" ones[4-6], and the manner in which the type of the field distribution changes with the character of the interaction was demonstrated in[5].

At high frequencies ($\omega \sim \Omega$, $\omega > \nu$) the occurrence of an electric field inside the metal can lead to singularities of the impedance even for a semi-infinite metal. These singularities have, as a rule, a resonant character and occur when the free path time of the electron between the spikes of the high-frequency field in the metal is a multiple of the period of the external field.

A cyclotron resonance of this type was first observed experimentally on the electrons of the limiting point in Al and was correctly interpreted by Grimes et al.[7] A sufficiently detailed theory of this effect was constructed by Kaner and Blank[8] and by Azbel' and Peschanskiĭ[9]. The condition for observation of the resonance at the limiting point is smallness of the angle of inclination of the field H to the surface of the sample ($\delta_0/u_0 \ll \sin\varphi < (\delta_0/l)^{1/3}$), where $\delta_0$ is the depth of the skin layer at H = 0, $u_0$—displacement of the electron during the cyclotron period. However, small inclination angles are in the general case not essential. In principle, such a resonance can be observed at arbitrary angles of inclination, if at the same time there exists on the Fermi surface a cross section with an extremum of the quantity $\partial S/\partial p_H$ and, in addition, there are on these cross sections points at which $v_z = 0$.

As shown in the present paper, the size effect and cyclotron resonance on the "effective" drifting electrons can be observed even at H perpendicular to the surface of the metal. This can give rise to electron trajectories on which there are several effectiveness points, leading to singularities of both the size effect and of the cyclotron resonance.

## EXPERIMENTAL CONDITIONS

The cyclotron-resonance investigations were made with a superheterodyne spectroscope at a frequency $f = 3.6 \times 10^{10}$ Hz. To study the size effect, we used an autodyne generator operating at a frequency on the order of 5 MHz. In both experiments, we determined the dependence of the derivative of the real part of the surface impedance with respect to the magnetic field (dR/dH) on the magnetic field.

Cadmium samples in the form of discs of 11 mm diameter and d = 0.3 mm thickness and d = 0.2 mm were grown in a dismountable quartz mold. We used in the experiment samples with orientation **n** ∥ [0001]. The deviation of the normal from the indicated axis was 1.5°. The ratio of the resistance at room and helium temperatures for the initial cadmium was $35 \times 10^3$. In the investigation of cyclotron resonance, the samples served as the bottom of a cylindrical resonator operating in the $H_{011}$ mode.

The resonator axis was horizontal, making it possible to establish any angle of inclination between the magnetic field H and the surface of the sample by rotating the magnet in the horizontal plane. In the case of the size effect, the samples were placed in a coil which served as the tank circuit of the autodyne. The magnetic field could be oriented relative to the surface of the sample, just as in the case of cyclotron resonance. All the measurements were made at a temperature T = 1.7°K.

## SIZE EFFECT

A feature of the Fermi surface of cadmium is that at a magnetic field directed normal to the surface of the metal and parallel to the [0001] axis, there exists a group of electrons that have an extremal displacement over the cyclotron period and are effective. Moreover, the orbit contains not one point with $v_z = 0$, but six, leading to a unique distribution of the electric field inside the metal and to singularities of the cyclotron resonance and of the size effect.

Indeed, in the second band there is a Fermi surface, the so-called "monster," which is a complicated corrugated cylinder with axis along [0001][10]. On this surface there are minimal and maximal cross-section areas where $\partial S/\partial p_z = 0$ (see Fig. 1a). Naturally, there should exist between these sections, at certain values of $p_z'$, sections with an extremum of $\partial S/\partial p_H$ (A and A′ on Fig. 1a).

If we now trace the variation of the direction of the electron velocity on such a cross section, i.e., the normal to the Fermi surface, then we can note that the projection of the velocity reverses sign near the edges of the "monster." When moving in real space, this creates a situation wherein, although the displacement of the electron along the field during the cyclotron period is not equal to zero, on certain sections of its trajectory the electron travels from the surface of the metal to its interior, and on others, to the contrary, it travels to the surface. In a total trajectory loop, the velocity projection reverses sign six times. The projection of the trajectory on the xy and xz plane is shown schematically in Fig. 2.

Let us assume that the electron started its motion from the surface of the metal s, at the point corresponding to $B_6$ on the projection of the trajectory on the xy plane (the trajectory is shown by the solid line on Fig. 2). After interacting with the external field in the skin layer and acquiring a velocity increment Δv, such an electron will duplicate this increment at a depth $z_n = n(u_1 - u_2)$ and $z_{1n} = n(u_1 - u_2) + u_1$, where $u_1$ and $u_2$ are the displacements of the electrons to the interior of the metal and to the surface, respectively; n = 0, 1, 2, 3, ...

However, inasmuch as $u_1$ and $u_2$ are functions of $p_z$, the averaging over all the electrons (over all $p_z$) singles out those electrons for which $z_n$ and $z_{1n}$ have extremal values. Since the displacement during the entire period $u = 3(u_1 - u_2)$ is extremal on the sections A and A′, the $z_n$ will also be extremal, and at these distances there should occur spikes of the field. For field spikes to occur also at distances $z_{1n}$, it is necessary that the quantities $u_1$ and $u_2$ also each reach an extremum on this section.

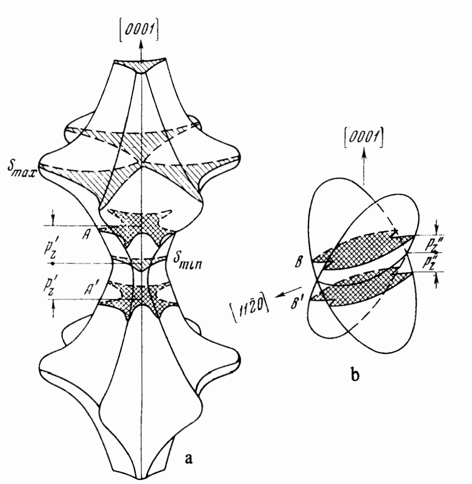


FIG. 1. Fermi surface of cadmium, on which there are sections normal to the [0001] axis and having an extremum of $\partial S/\partial p_H$: a – hole surface in the second band – "monster." $S_{max}$, $S_{min}$ – cross sections of the "monster," normal to the [0001] axis with maximum and minimum area; A, A′ – cross sections with extremum of $\partial S/\partial p_H$ at a certain $p'_z$; b – electronic surface in the third band – "butterfly." B, B′ – "butterfly" cross sections on which $\partial S/\partial p_H$ reaches an extremum at a certain $p_z$ in the almost-free electron approximation.

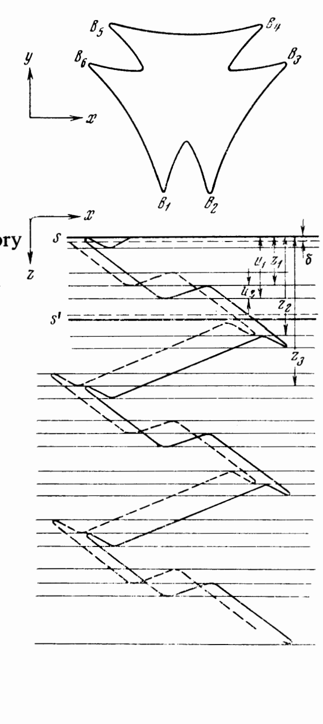


FIG. 2. Projection of the trajectory of the electron on the cross sections A and A′ of the "monster" on the xy and xz plane. $B_1$, $B_2$, etc. – point at which $v_z = 0$. The horizontal lines show the projections of the planes in which spikes of the field are produced in a semi-infinite metal. S′ – hypothetical moving surface of the sample. δ – depth of skin layer.

The field spikes which can occur at distances $z_n$ and $z_{1n}$ are due to electrons that start their motion from the surface of the metal. However, each of the spikes can serve as a new skin layer for electrons moving on such trajectories, in which the effectiveness points are located at the spike locations. Thus, the spike can multiply and several systems of spikes can arise in the metal.

Let us consider schematically the picture that can be expected in the size effect under unilateral excitation, when the spikes emerged to the other side of the plate of thickness d. For convenience we shall fix the magnetic field and the position of one of the surfaces of the sample (s on Fig. 2), and move mentally the other side (s′), increasing the thickness of the sample, starting from zero; this is equivalent to increasing the magnetic field at a constant plate thickness. If the electrons produce primary spikes at distances $z_n$ only ($u_1$ and $u_2$ are not extremal), then these spikes cannot appear, since the trajectory of the electron will be cut off at the ineffective point prior to their appearance. However, at distances $z_{n+1}$ there can occur secondary spikes, produced by the trajectory displaced relative to the surface s by $u_2$ (shown dashed in Fig. 2). These spikes can already emerge to the surface and to become manifest in the size effect. At other values of z, the spikes cannot occur in this case, since in the size effect there should occur singularities of dR/dH = f(H), having one period that is determined by the difference $u_1 - u_2$. In the case when the primary spikes occur at distances $z_n$ and $z_{1n}$, the secondary spikes, multiplying, can occur at distances $z_{1m} = mu_1$, $z_{2m} = mu_2$, $z_{3m} = z_n \pm z_{1m}$, $z_{4m} = z_n \pm z_{2m}$, $z_{5m} = z_{1n} \pm z_{1m}$, and $z_{6n} = z_{1n} \pm z_{2m}$. It is perfectly natural that not all of them may appear in the size effect, and the amplitude of each of the spikes may be different.

So far we have considered electrons that begin their motion from the surface, being situated at the point corresponding to $B_6$ (see Fig. 2). However, field spikes can be produced just as successfully and at the same distances by electrons that start their motion from the surface at the points corresponding to $B_2$ and $B_4$. If furthermore we take account of the fact that the direction of the field in the spike coincides with the direction of the velocity in the xy plane, and the external exciting field has a linear polarization, it becomes clear that both the polarization and the amplitude of the emerging spikes will be different.

In spite of such a complicated picture, we can expect the majority of the spikes observed in experiment to fit within several systems having the same period (determined by the difference $u_1 - u_2$) and shifted relative to one another by an amount as a multiple of $u_1$ or $u_2$.

Figure 3 shows a sample plot of dR/dH against H in the size effect on a Cd sample with d = 0.2 mm. The arrows at the bottom of the figure indicate the locations of the maxima of the singularities of dR/dH. We see that the most intense lines can be grouped in accordance with two identical periods $\Delta H_1$ = 215 Oe. In fields up to H ≈ 2500 Oe, half way between the intense lines, one observes also weak lines which have the same period. The distance from these lines to the intense ones is $\Delta H_2$ = 60 Oe, and is apparently determined by the displacement $u_2$. The existence of the size effect in the form of several systems of lines with identical period indicates that the displacements $u_1$ and $u_2$ have extrema on the sections A and A′ simultaneously. The ratio of these displacements is

$$\frac{u_1}{u_2} = \frac{\Delta H_1 + \Delta H_2}{\Delta H_2} \approx 4.6.$$

From the period of the size-effect lines it is possible to determine the quantity

$$\left(\frac{\partial S}{\partial p_z}\right)_{ext} = 3\frac{ed}{c}\Delta H.$$

At d = 0.2 mm we get

$$\hbar^{-1}\left(\frac{\partial S}{\partial p_z}\right)_{ext} = (2.06 \pm 0.02)\,\text{Å}^{-1}.$$

This value agrees well numerically with the value 2.04 $\text{Å}^{-1}$ obtained from the galvanomorphic oscillations[11]. However, the interpretation proposed by these authors for their own data as being the result of a splitting of the "monster" into three branches is apparently incorrect, all the more since it follows from the results of Tsui and Stark[12] that there is no such splitting.

The quantity $\hbar^{-1}(\partial S/\partial p_z)_{ext}$ was determined also from the oscillations of the sound absorption at **H** ∥ **q** ∥ [0001], where **q** is the wave vector of sound[13]. The obtained value 0.68 $\text{Å}^{-1}$ is one-third as large as our data. However, if we take into account the character of the trajectory of the electron, it can be assumed that the oscillations of the sound absorption will also be determined not by the total displacement per period, but by the difference $u_1 - u_2$, and the true value of $\hbar^{-1}(\partial S/\partial p_z)_{ext}$ should be three times larger. In addition to the size-effect lines with periods $\Delta H_1$, there are also observed lines whose amplitudes, and apparently also the periods, are small, so that against the background of the intense lines they can be poorly resolved. The reduction of a large number of plots of the size effect at different polarizations of the external field and in samples of different thickness make it possible to assume that these lines have a period $\Delta H_3$ ≈ 40 Oe at d = 0.2 mm and are due to the other cross section of the Fermi surface.

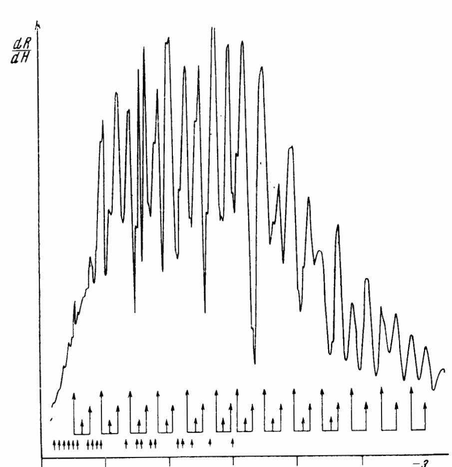


FIG. 3. Plot of dR/dH against H in the size effect. H ∥ n ∥ [0001], radio frequency current I ∥ [1120], d = 0.2 mm. Arrows of equal length denote the positions of the size-effect lines having the same period. The upper role of arrows pertains to the "monster" and the lower one to the "butterfly."

When the magnetic field direction is along [0001], the cross sections with the extremum of $\partial S/\partial p_z$ can be situated on a surface of the "butterfly" type (see Fig. 1b). The electron orbit will then have four effective points and the period of the size-effect lines will be determined also by the difference of the displacements of $u_1 - u_2$, which is half as large as the displacement over the cyclotron period. With such an interpretation of the weak lines, the value of $\hbar^{-1}(\partial S/\partial p_z)$ on the "butterfly" cross section should be approximately 0.24 $\text{Å}^{-1}$. On electrons of such a cross section there should occur also oscillations of sound absorption, but unfortunately there are no such data, this being apparently connected with the insufficiently high sound frequency used by Daniel and Mackinnon[13].

## CYCLOTRON RESONANCE

At high frequencies, excitation of the field spikes inside the metal can lead to the occurrence of cyclotron resonance. However, the resonance conditions can in the general case differ from the ordinary ones. If the field spikes occur at a depth that is a multiple of the displacement of the electron during the period, then the phase of the field in the spike n will differ from the phase of the field in the skin layer by an amount $n\omega T = 2\pi n\omega/\Omega$, where T and $\Omega$ are respectively the period and the frequency of revolution of the electron. The electrons moving towards the surface will carry into the skin layer the field of the n-th spike with phase $4\pi n\omega/\Omega$. The field in the skin layer and the fields of all spikes (with arbitrary n) will be in phase if $4\pi\omega/\Omega = 2\pi q$, i.e., $\omega = q\Omega/2$, where q = 1, 2, 3, ...

In the general case the spikes may not be equidistant in z, and the time of flight between these spikes may not equal the period T. Moreover, the times of flight of the electron between neighboring spikes may also not be equal to one another.

Let us consider in greater detail the phase relations arising for the system of spikes shown in Fig. 2.

We shall assume that the time during which the electron traverses the distance $u_1 = t_1$, and the distance $u_2 = t_2$, i.e.,

$$t_1 = -\frac{c}{eH}\int_{B_6}^{B_1}\frac{dl}{v_\perp}, \quad t_2 = -\frac{c}{eH}\int_{B_1}^{B_4}\frac{dl}{v_\perp}.$$

Here $B_n$—points on the orbit where $v_z = 0$, $v_\perp = \sqrt{v_x^2 + v_y^2}$; $dl$—element of arc on the orbit. Then the phase of the field in the primary spikes, located at distances $z_n$ and $z_{1n}$ from the surface of the metal, will differ from the phase of the external field by an amount respectively $\varphi_n = n\omega(t_1 + t_2)$ and $\varphi_{1n} = n\omega(t_1 + t_2) + \omega t_1$. On the other hand, the phase of the field in the multiplied spikes will differ from the phases $\varphi_n$ and $\varphi_{1n}$ by amounts that are multiples of $\omega t_1$ and $\omega t_2$.

The electrons moving towards the surface will carry away the field of the spikes with double the phase. Therefore, in order for the field emerging from all the spikes to be in phase with the external field, it is necessary to satisfy simultaneously the conditions $\omega(t_1 + t_2) = m\pi$ and $\omega t_1 = q\pi$, where m, q = 1, 2, 3, ... If these conditions are not satisfied, then the impedance will be determined by the conditions of interference between the external field and the emerging field of all the spikes. It is clear from the foregoing that in the general case, at an arbitrary ratio of $t_1$ and $t_2$, the picture of the cyclotron resonance may be very complicated and cannot always be unambiguously interpreted.

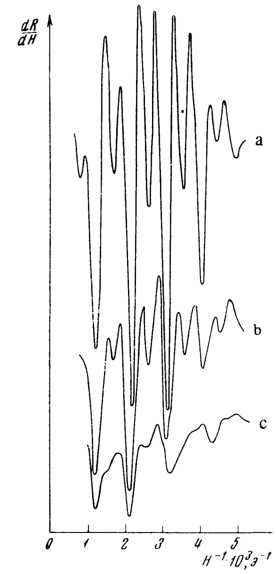

FIG. 4. Plot of dR/dH against $H^{-1}$ in cyclotron resonance and at different angles $\varphi$ between **H** and the [0001] axis: a – $\varphi = 0°$, b – $\varphi = 1.5°$, c – $\varphi = 2.5°$.

Let us assume that $t_1 = 2t_2$. Then the emerging field will be maximal and in phase with the external field at $\omega t_1 = 2r\pi$ (r = 1, 2, 3, ...). At $\omega t_1 = r\pi$, the fields of all the spikes will cancel each other and the emerging field will vanish. At intermediate values of $\omega t_1$, the emerging field can be in phase opposition with the external field, and consequently will be subtracted. One can assume that in this case the impedance will be larger than in the absence of an emerging field. Therefore additional peaks, albeit of smaller amplitude, can arise between the main resonance peaks corresponding to the condition $\omega t_1 = 2r\pi$.

These are precisely the singularities of cyclotron resonance observed at **H** ∥ **n** ∥ [0001], i.e., in the same orientation as the size effect. Figure 4 shows plots of $\partial R/\partial H$ against $H^{-1}$ at different angles of inclination of the field to the [0001] axis. The direction **H** ∥ [0001] was determined accurate to 30′ from the symmetry of the plot. We see that a series of intense resonant peaks is observed, separated by peaks of much smaller amplitude. When **H** deviates from the [0001] axis, when the ratio of the times $t_1$ and $t_2$ apparently changes, the resonance smears out and decreases strongly in amplitude. When the angle between H and [0001] is $\gtrsim 4°$, the resonance is practically inobservable.

The available data do not suffice to identify with assurance the spikes causing the cyclotron resonance. If it is due to spikes produced by the electrons of the "monster," then the total period of revolution of the electron on the sections A and A′ will be approximately $T = 3(t_1 + t_2) = 9t_1/2$, and the effective mass is $m^* \approx 9em_0/2c\omega\Delta H^{-1} = 0.387m_0$. On the other hand, if it is due to spikes which are presumably produced by the "butterfly" electrons, then $T = 2(t_2 + t_1) = 3t_1$ and $m^* \approx 3em_0/c\omega\Delta H^{-1} = 0.256m_0$. Both values of the effective mass are close to those that can be expected respectively for the section of the monster near the narrow neck and the "butterfly" section[10]. It is therefore difficult to give preference to any of these sections.

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