SPIN-SYSTEM SPECTRUM AND HIGH-FREQUENCY PROPERTIES OF FERROMAGNETIC MONOCRYSTALS CONTAINING PLANE DEFECTS

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It is shown that in ferromagnets containing plane defects, two new types of localized spin-system oscillations (LSSO) are possible. The wave function of one of these is symmetric, of the other antisymmetric, with respect to the operation of reflection in the plane of the defect. These LSSO are of plane-wave character in a plane parallel to the surface of the defect, and they decay with increasing distance from the defect. The spectrum and wave functions of these excitations are found. The density of states of a ferromagnet with a plane defect is calculated, and it is shown to contain spikes due to the presence of LSSO. For certain values of the parameters that describe the plane defect, these spikes occur near the bottom of the spin-wave band of the ideal crystal. The expression obtained for the density of states is correct far from the singular points. The high-frequency magnetic susceptibility of a ferromagnetic monocrystal with a plane defect is found, and it is shown that one of the components of this susceptibility contains resonance terms due to the presence of LSSO. It is shown that in a ferromagnet with a plane defect, as compared with an ideal crystal, there is possible additional resonance absorption of the energy of a high-frequency electromagnetic field, as a result of the excitation of localized modes.

1. INTRODUCTION

AT present much attention is being paid to the study of physical phenomena in ferromagnetic monocrystals that contain defects of various kinds. Thus papers [1-3] studied localized oscillations of the spin system of a ferromagnetic crystal containing point defects in the form of impurity atoms. A point defect destroys the translational invariance of the lattice in all three coordinate directions. In this case the wave vector is not a good quantum number, by means of which the spin states can be enumerated.

If the crystal defect is linear (for example, a dislocation) or planar (for example, a surface of the crystal), then the translational invariance is disturbed only in two directions or in one direction, respectively; this allows introduction of a one- or two-component wave vector of the spin excitations. Consequently, when a linear or plane defect is present in a ferromagnet, we must expect in the spin branch of the energy spectrum of the crystal no longer the appearance of separate local levels, as in the case of point defects, but whole bands of localized states.

Localized excitations of the spin system in the presence of a line defect were found by Savchenko and Tarasenko [4]. In the paper of Wallis, Maradudin, et al. [5] and also in the author's paper [6], various methods were used to find the spectrum of characteristic oscillations of the spin system near the surface of a ferromagnetic crystal. The spin excitations found, called surface spin waves, consist of a band of localized states in the presence of a special type of plane defect. The characteristic feature of this defect is that there are different media on the two sides of it: on one side vacuum, on the other the ferromagnetic material under consideration. In the present communication we shall study the spectrum[1)], the spin-system density of states, and the high-frequency properties of a ferromagnetic crystal containing plane defects of such a type that the same medium (the ferromagnetic material under consideration) is present on both sides of them.

2. FORMULATION OF THE PROBLEM AND INITIAL HAMILTONIAN

We shall restrict ourselves to study of the phenomenon in a crystal with a simple cubic lattice; we direct the x, y, z coordinate axes along the crystallographic axes [100], [010], [001] respectively.

Let the atoms of an ideal infinite ferromagnet (IIF) have spin S, and let the exchange interaction between them, in the nearest-neighbor approximation, be described by the integral J. In the ideal infinite crystal, the wave function of the spin waves with wave vector $\mathbf{k}$ (measured in units of the lattice constant a) is proportional to

$$\exp\left[i(k_x x + k_y y + k_z z)\,/\,a\right]. \tag{2.1}$$

Oscillations localized, for example, about one of the planes perpendicular to y are possible if, for some reason or other, k_y is imaginary. In this case k_y will no longer have the meaning of a component of a wave vector.

If we denote by $\mathscr{E}_{\mathbf{k}}$ the energy of a spin wave of the IIF, then the expression

$$2\sin^2(k_y/2) = \alpha - 2 + \cos k_x + \cos k_z = \mathscr{E} \tag{2.2}$$

with $\alpha = (E - g\mu_0 H)/2JS$, where g is the Landé factor of the IIF and H is the external magnetic field (parallel to z), represents the dispersion law of these

1)This problem was the subject of a preliminary communication [7].

waves, if the energy $E = \mathscr{E}_k$. It follows from this expression that spin-system oscillations localized near any surface parallel [sic] to y are possible if, for some reason or other, $\mathscr{E} \leq 0$ or $E \geq 2$; that is, if

$$E \leqslant \mathscr{E}_{\mathbf{k}}|_{k_y=0} \quad \text{or} \quad E \geqslant \mathscr{E}_{\mathbf{k}}|_{k_y=\pi}. \tag{2.3}$$

For values of the energy determined by the inequalities (2.3), the expression (2.2) will represent a formal expression for the dispersion law of localized spin-system oscillations (LSSO). To find E specifically, it is necessary to determine k_y, which will no longer have the meaning of a component of the wave vector, and which will obviously be connected with the characteristics of the plane defect.

Going over to a specific calculation, we restrict ourselves to the simplest model of a plane defect. We shall describe the position of a spin (a site) by the dimensionless quantities n and α where α is the y-coordinate of the spin and **n** is the position vector in the xz plane. We shall suppose that the plane defect divides the crystal into two halves $\alpha \geq 1$ and $\alpha \leq 0$. Let the exchange interaction of atoms in the plane $\alpha = 0$ with atoms in the plane $\alpha = 1$ be J', so that in general $\beta = 1 - J'/J \neq 0$. We shall also suppose that J'' and g' describe, respectively, the exchange interaction and the Landé factor of atoms in the planes $\alpha = 0$ and $\alpha = 1$, so that in general $\xi = 1 - J''/J \neq 0$ and $\gamma = 1 - g'/g \neq 0$. The assumptions stated insure that the perturbation introduced by the plane defect is local.

Introducing the spin operator $\mathbf{S}_{\mathbf{n}\alpha}$ at the site $\mathbf{n}\alpha$, we represent the Hamiltonian of the initial problem in the form

$$\mathscr{H} = \mathscr{H}_0 + V, \tag{2.4}$$

where

$$\mathscr{H}_0 = -{}^1/_2 J \sum_{\mathbf{n}\alpha}\Big(\sum_{\Delta_1} \mathbf{S}_{\mathbf{n}\alpha}\mathbf{S}_{\mathbf{n}+\Delta_1,\alpha} + \sum_{\Delta_2} \mathbf{S}_{\mathbf{n}\alpha}\mathbf{S}_{\mathbf{n},\alpha+\Delta_2}\Big) - g\mu_0 H \sum_{\mathbf{n}\alpha} S^z_{\mathbf{n}\alpha} \tag{2.5}$$

is the Hamiltonian of the ideal ferromagnet, and where

$$V = {}^1/_2 J\xi \sum_{\mathbf{n}\Delta_1} (\mathbf{S}_{\mathbf{n}1}\mathbf{S}_{\mathbf{n}+\Delta_1,1} + \mathbf{S}_{\mathbf{n}0}\mathbf{S}_{\mathbf{n}+\Delta_1,0})$$

$$+ \beta J \sum_{\mathbf{n}} \mathbf{S}_{\mathbf{n}0}\mathbf{S}_{\mathbf{n}1} + \mu_0 g H\gamma \sum_{\mathbf{n}} (S^z_{\mathbf{n}0} + S^z_{\mathbf{n}1}) \tag{2.6}$$

is the local perturbation due to the presence of the plane defect. The perturbation V in general may be large; therefore in the solution of the problem posed, we shall use its localness but not its smallness [8].

In (2.5) and (2.6), $S^z_{\mathbf{n}\alpha}$ is the z-component of the spin operator. Summation over Δ_1 or Δ_2 signifies, respectively, summation over the Z_1 or Z_2 vectors connecting a given atom with its nearest neighbors in the xz plane or along the y axis.

3. SPECTRUM AND WAVE FUNCTIONS OF LOCALIZED SPIN-SYSTEM OSCILLATIONS IN THE PRESENCE OF A PLANE DEFECT

Following [1], we shall start from the equation

$$\sum_{\mathbf{j}\beta} \langle \mathbf{n}\alpha|\mathscr{H}|\mathbf{j}\beta\rangle \varphi_{\mathbf{j}\beta} = E_0\varphi_{\mathbf{n}\alpha}, \tag{3.1}$$

where $\varphi_{\mathbf{j}\beta}$ is the wave function of the spin deviations of site $\mathbf{j}\beta$ and $\langle \mathbf{n}\alpha|\mathscr{H}|\mathbf{j}\beta\rangle$ are the matrix elements of $\mathscr{H}$ with respect to the wave functions

$$|\mathbf{j}\beta\rangle = (2S)^{-1/2}S_{\mathbf{j}\beta}^-|0\rangle, \tag{3.2}$$

where $|0\rangle$ is the function for the ground state of the spin system and is determined by the equation $S^+_{\mathbf{n}\alpha}|0\rangle = 0$, $S^{\pm}_{\mathbf{n}\alpha} = S^x_{\mathbf{n}\alpha} \pm iS^y_{\mathbf{n}\alpha}$.

In an ideal crystal the matrix elements $\langle \mathbf{n}\alpha|\mathscr{H}|\mathbf{j}\beta\rangle$ depend on the difference $\mathbf{n} - \mathbf{j}$, $\alpha - \beta$. In our case they depend on each index α and β separately, because the plane defect destroys the translational invariance of the crystal along the y axis. In the xz plane, however, the translational invariance is not disturbed. This makes it possible to introduce a two-component wave vector κ in this plane and to represent the wave function $\varphi_{\mathbf{n}\alpha}$ in the form

$$\varphi_{\mathbf{n}\alpha} = e^{i\varkappa\mathbf{n}}\varphi_\alpha. \tag{3.3}$$

On calculating the matrix elements of $\mathscr{H}$ by use of (3.3) and on going over in (3.1) to a Fourier representation with respect to the Latin indices, we have

$$\sum_\alpha \mathscr{H}_{\beta\alpha}{}^0\varphi_\alpha + \sum_\alpha V_{\beta\alpha}\varphi_\alpha = E\varphi_\beta, \tag{3.4}$$

where

$$\mathscr{H}^0_{\beta\alpha} = \delta_{\alpha\beta}\mathscr{E}_\varkappa + JS\Big(Z_2\delta_{\alpha\beta} - \sum_{\Delta_2}\delta_{\alpha,\beta+\Delta_2}\Big), \tag{3.5}$$

$$\mathscr{E}_\varkappa = g\mu_0 H + JS\Big(Z_1 - \sum_{\Delta_1} e^{i\varkappa\Delta_1}\Big) = \mathscr{E}_{\mathbf{k}}|_{k_y=0}, \tag{3.6}$$

and

$$V_{\beta'\alpha} = JS\{\varepsilon(\delta_{\alpha0}\delta_{\beta'0} + \delta_{\alpha1}\delta_{\beta'1}) + \beta(\delta_{\beta'1} - \delta_{\beta'0})(\delta_{\alpha0} - \delta_{\alpha1})\} \tag{3.7}$$

is the matrix of the local perturbation; in it

$$\varepsilon = -(\mu_0 gH\gamma' + \xi\mathscr{E}_\varkappa)/JS, \quad \gamma' = \gamma - \xi; \tag{3.8}$$

$$E = E_0 - E^0, \quad Z = Z_1 + Z_2,$$

$$E^0 = SN\{-({}^1/_2 JSZ + \mu_0 gH) + N^{-1/3}[JS(\xi Z_1 + \beta) + 2\mu_0 gH\gamma]\}. \tag{3.9}$$

Note that for simplicity of notation, we do not indicate the dependence on κ of the quantities that occur in (3.4) and (3.7).

In symbolic form, (3.4) takes the form

$$(E - \mathscr{H}^0)\varphi = V\varphi. \tag{3.10}$$

Introducing the Green's function of the ideal crystal, which in the α, β, κ representation can be written in the form

$$G_{\alpha\beta}{}^0(\varkappa) = (E - \mathscr{H}^0)_{\alpha\beta}{}^{-1} = \frac{1}{N_y}\sum_{k_y}\frac{\exp\{ik_y(\alpha - \beta)\}}{E - \mathscr{E}_{\mathbf{k}}}, \tag{3.11}$$

we write (3.10) in the form

$$((G^0)^{-1} - V)\varphi = 0. \tag{3.12}$$

Hence for the case $\operatorname{Det}(G^0)^{-1} \neq 0$ [2)] we have, on using (3.7),

$$\varphi_\alpha = G_{\alpha0}{}^0[(\varepsilon - \beta)\varphi_0 + \beta\varphi_1]JS + G_{\alpha1}{}^0[(\varepsilon - \beta)\varphi_1 + \beta\varphi_0]JS. \tag{3.13}$$

Hence we get the following system of two equations:

$$\varphi_0[1 - JS(\varepsilon - \beta)G_{00}{}^0 - JS\beta G_{01}{}^0] - \varphi_1[G_{01}{}^0(\varepsilon - \beta) + \beta G_{00}{}^0]JS = 0,$$

$$-\varphi_0[(\varepsilon - \beta)G_{10}{}^0 + \beta G_{00}{}^0]JS + \varphi_1[1 - JS(\varepsilon - \beta)G_{00}{}^0 - JS\beta G_{01}{}^0] = 0. \tag{3.14}$$

Setting the determinant of this system equal to zero

2) The condition $\operatorname{Det}(G^0)^{-1} = 0$ gives the equation that determines the spectrum of the spin waves of the ideal crystal.

leads to the equations

$$\varphi_1^{\pm} = \pm\varphi_0. \tag{3.15}$$

from which we then find the two branches of the LSSO. By using (3.15), we find from (3.14)

$$D \equiv 1 - JS(\varepsilon - \beta)G_{00}{}^0 - JS\beta G_{01}{}^0 \pm JS[G_{01}{}^0(\varepsilon - \beta) + G_{00}{}^0\beta] = 0, \tag{3.16}$$

The last equation enables us to assert that one of the localized modes is symmetric with respect to the operation of reflection in the plane of the defect (SM), the other antisymmetric (AM). In other words, the modes found are spin-system oscillations such that any two spins lying on opposite sides of the planes of the defect and equally distant from it are deviated through the same angle, i.e. the angle between them is zero, for SM; or this angle is π, for AM. From what has been said it can be deduced that the energy and wave function of the SM will be insensitive to the parameter β, i.e., it will not occur in them.

For further calculations it is necessary to know $G^0_{\beta\beta'}$. On going over in (3.11) from summation to integration and on carrying out the integration, we get

$$G^0_{\beta\beta'} = \frac{e^{-i\pi|\beta'-\beta|}}{2JS}\frac{1}{\sqrt{\mathscr{E}(\mathscr{E}-2)}}\begin{cases}-t_1^{|\beta'-\beta|}, & \mathscr{E} \leqslant 0\\ t_2^{|\beta'-\beta|}, & \mathscr{E} \geqslant 2\end{cases} \tag{3.17}$$

where

$$t_1 = \mathscr{E} - 1 + \sqrt{\mathscr{E}(\mathscr{E}-2)}, \qquad t_2 = \mathscr{E} - 1 - \sqrt{\mathscr{E}(\mathscr{E}-2)}. \tag{3.18}$$

On using (3.3), (3.13), (3.17), and (3.18) for the wave functions $\varphi^+_{\mathbf{n}\alpha}$(SM) and $\varphi^-_{\mathbf{n}\alpha}$(AM), we have

$$\begin{aligned}\varphi_{\mathbf{n}\alpha}{}^+ &= \varphi_1^+(-1)^{\eta^+(\alpha-1)}\exp\{i\varkappa\mathbf{n} - (\alpha-1)/\delta_1\}.\\ \varphi_{\mathbf{n}\alpha}{}^- &= \varphi_1^-(-1)^{\eta^-(\alpha-1)}\exp\{i\varkappa\mathbf{n} - (\alpha-1)/\delta_2\}, \quad \alpha \geqslant 1;\end{aligned} \tag{3.19}$$

$$\begin{aligned}\varphi^+_{\mathbf{n}\alpha} &= \varphi_0(-1)^{\eta^+ + \alpha}\exp\{i\varkappa\mathbf{n} + \alpha/\delta_1\},\\ \varphi^-_{\mathbf{n}\alpha} &= \varphi_0(-1)^{-\eta^- - \alpha}\exp\{i\varkappa\mathbf{n} + \alpha/\delta_2\}, \quad \alpha \leqslant 0,\end{aligned} \tag{3.20}$$

where

$$\eta^+ = \begin{cases}0 \text{ for } \varepsilon \leqslant 0\\ 1 \text{ for } \varepsilon \geqslant 2,\end{cases} \quad \eta^- = \begin{cases}0 \text{ for } \varepsilon \leqslant 2(\beta - 1)\\ 1 \text{ for } \varepsilon \geqslant 2\beta\end{cases}. \tag{3.21}$$

The expressions for the wave functions agree with those presented in [7]. The quantities

$$\delta_1 = 1/\ln|1-\varepsilon|, \qquad \delta_2 = 1/\ln|2\beta - 1 - \varepsilon| \tag{3.22}$$

having the meaning of depths of penetration (calculated in dimensionless variables) of the spin excitations that we have found.

It is clear from (3.19)–(3.22) that the excitations found are plane waves localized near the plane defect and propagating along its surface. We shall find the spectrum of these waves. On expressing G^0_{01} in terms of G^0_{00}, we have from (3.15)

$$\begin{aligned}D_1 &= \varepsilon JS(\mathscr{E} - 2)G_{00}{}^0(\varkappa) - (\varepsilon - 2)/2 = 0,\\ D_2 &= -JS\mathscr{E}(\varepsilon - 2\beta)G_{00}{}^0(\varkappa) + [\varepsilon + 2(1-\beta)]/2 = 0,\end{aligned} \tag{3.23}$$

or by use of (3.17)

$$\sqrt{\frac{\mathscr{E}-2}{\mathscr{E}}} = \frac{\varepsilon - 2}{\varepsilon}, \quad \sqrt{\frac{\mathscr{E}}{\mathscr{E}-2}} = \frac{\varepsilon + 2(1-\beta)}{\varepsilon - 2\beta}, \tag{3.24}$$

whence, finally, $\mathscr{E}_1 = \epsilon^2/2(\epsilon - 1)$ and $\mathscr{E}_2 = [\epsilon + 2(1-\beta)]^2/2(\epsilon + 1 - 2\beta)$, or

$$E^+ = \mathscr{E}_\varkappa + Z_2JS\frac{\varepsilon^2}{2(\varepsilon - 1)} \text{ when } \varepsilon \leqslant 0 \text{ or } \varepsilon \geqslant 2, \tag{3.25}$$

$$E^- = \mathscr{E}_\varkappa + Z_2JS\frac{[\varepsilon + 2(1-\beta)]^2}{2(\varepsilon + 1 - 2\beta)} \text{ when } \varepsilon \leqslant 2(\beta - 1) \text{ or } \varepsilon \geqslant 2\beta. \tag{3.26}$$

We note that in the range $0 \leq \mathscr{E} \leq 2$ the Green's function G^0_{00} has a purely imaginary value; consequently, in this case equation (3.23) has no solution, i.e., in the range $0 \leq \mathscr{E} \leq 2$ there are no localized states of the s spin system. This is in full agreement with what was said in Sec. 2.

We note also that, as was mentioned above, the energy and wave function of the symmetric mode do not contain β. When $\beta = 1$, which corresponds to complete elimination of the interaction between the two halves of the crystal separated by the plane defect, $E^+ = E^-$, $\varphi^+_{\mathbf{n}\alpha} = \varphi^-_{\mathbf{n}\alpha}$ for $\alpha \leq 0$ and $\varphi^+_{\mathbf{n}\alpha} = -\varphi^-_{\mathbf{n}\alpha}$ for $\alpha \leq 1$ for given φ_0.

If we suppose that the parameters γ and ξ are different on opposite sides of the plane defect, and if then, following [6], for $\beta = 1$ we reduce the rank of the perturbation twofold, then we find from (3.4) the energy of surface spin waves (SSW); the expression for it agrees with that presented in [6], and in form with E^+. However, the range of applicability of the expression for the energy of SSW is different from that for E^+. The wave function of SSW coincides with the wave function $\varphi^+_{\mathbf{n}\alpha}$, written for the same conditions (see the first formula (3.19)). When

$$\varepsilon + \mu_0\tilde{g}H\gamma \geqslant 1 \tag{3.27}$$

or when

$$0 \leqslant \varepsilon + \mu_0\tilde{g}H\gamma \leqslant \tilde{\mathscr{E}}_\varkappa/(\mathscr{E}_\varkappa + 1),$$

where $\tilde{g} = g/JS$ and $\mathscr{E}_K = (\mathscr{E}_K/JS)|_{H=0}$, it equals

$$\varphi_{\mathbf{n}\alpha} = \varphi_1(-1)^{\eta(\alpha-1)}\exp\{i\varkappa\mathbf{n} + (\alpha-1)/\delta_1\} \quad \text{for } \alpha \geqslant 1, \tag{3.28}$$

where $\eta = 1$ or 0 according as the first or the second inequality (3.27) is fulfilled.

We note that the expression for $\varphi_{\mathbf{n}\alpha}$ given in [6] is valid at large α and also that in it one must write φ_1 instead of φ_0. As is clear from (3.27), the conditions imposed on ϵ in [6] are correct when $H = 0$ or when $g = g'$. We mention also that in [6], everywhere in the conditions imposed on ϵ, instead of the $\mathscr{E}_K$ that was erroneously given, $\tilde{\mathscr{E}}_K$ should be written; instead of the last three terms in E^0, the following should be written:

$$SN^{2/3}\{JS[Z_1(\xi_1 + \xi_2) + 1] + 2\mu_0 gH(\lambda_1 + \lambda_2)\},$$

instead of λ_i in formula (13), $\lambda_i' = \lambda_i - \xi_i$; and, finally, the substitutions $1 \rightleftarrows 2$ should be made in the expressions for Z_1 and Z_2 given in [6].

It is clear from (3.19), (3.20), (3.25), (3.26), and (3.28) that the energy and wave functions of the excitations found depend significantly on ξ, γ, and the wave vector κ. The greater the wavelength λ of the excitations, the greater is the distance to which they penetrate from the defect. In particular, for the SM with $g = g'$, $J'' \leq J$, and $\lambda \to \infty$, $\delta_1 \to \infty$; that is, this mode under the conditions stated goes over to a mode of the ideal crystal. Simultaneously $\mathscr{E}^+$ goes over to $\mathscr{E}_{\mathbf{k}}$, with $k_y = 0$. When $\gamma \neq 0$, however, this mode, even for $\lambda \to \infty$, has a finite though large value of δ_1. For example, if $\mu_0 gH\gamma/JS \ll 1$, then

$$\delta_1 = JS / \mu_0 g H \gamma, \quad \gamma \geqslant 0. \tag{3.29}$$

The SM energy for several values of ξ and for $H = 0$ is shown in Fig. 1 (curves 2, 4, 5, 6).

If $g' = g$, then the AM except in the case $J', J'' \geq J$ and the SM for $\epsilon \geq 2$ can reduce to a spin mode of the ideal crystal, with energy $\mathscr{E}_{\mathbf{k}}|_{k_y = 0}$ or $\mathscr{E}_{\mathbf{k}}|_{k_y = \pi}$ for certain finite values of κ ($\lambda \neq \infty$; see Fig. 1, curves 1, 3, 6, 7). These finite values of κ can be found from the equations $\epsilon = 2(\beta - 1)$, $\epsilon = 2\beta$, $\epsilon = 2$. This is due to the fact that for existence of the excitations under consideration, it is necessary that the energy of interaction of any spin in the defect surface with its immediate surroundings shall be larger than $\mathscr{E}_{\mathbf{k}}|_{k_y = \pi}$ or smaller than $\mathscr{E}_{\mathbf{k}}|_{k_y = 0}$, as follows directly from the inequalities $\epsilon \geq 2$, $\epsilon \geq 2\beta$, $\epsilon \leq 2(\beta - 1)$. But in the cases mentioned above, this is possible only for finite κ.

We note one other interesting peculiarity. If J', $J'' \geq J$ and $H = 0$, then the AM exists for arbitrary finite κ, and its energy at $\kappa = 0$ contains a gap $2Z_2(1 - \beta)^2 JS/(1 - 2\beta$ (see Fig. 1). Finally, if $J' = J'' = J$ and $\gamma \neq 0$, then

$$E^+ = \mu_0 g_+^* H + \mathscr{E}_{\mathbf{k}}|_{k_y = 0,\, \mathbf{H} = 0}, \quad E^- = \mu_0 g_-^* H + \mathscr{E}_{\mathbf{k}}|_{k_y = \pi,\, \mathbf{H} = 0}, \tag{3.30}$$

where

$$g_\pm^* = g\left[1 \mp \gamma\left(\frac{g\mu_0\gamma H}{JS}\right)\right]. \tag{3.31}$$

4. THE GREEN'S FUNCTION OF A FERROMAGNETIC CRYSTAL CONTAINING A PLANE DEFECT

For further calculations it is convenient to use the Green's function of the nonideal crystal, which we shall find in this section. We introduce it, following [3], in the α, β, κ representation in the following form:

$$G_{\alpha\beta}^{-1}(\varkappa) = (E - \widetilde{\mathscr{H}})_{\alpha\beta}, \tag{4.1}$$

where $\mathscr{H} = \mathscr{H}^0 + V$; then from (3.10) we have

$$G^{-1}\varphi = 0.$$

On comparing this equation with (3.12), we find a formal expression for G:

$$G = (1 - G^0 V)^{-1} G_0. \tag{4.2}$$

We note that the quantity $G_{\alpha\beta}$ introduced by means of (4.1) represents the matrix elements of the resolvent operator. However, as is easy to show in our case, they coincide with the Green's function.

Since there is no translational invariance of the crystal along the y axis, G cannot depend on $\alpha - \alpha'$ alone; therefore in going over to a Fourier representation with respect to each of the indices α and α', we must introduce two quantities q and q$'$.

On using the specific form of V and on going over to a Fourier representation with respect to each of the indices α and α', we find from (4.2)

$$G(\omega - i\Delta, q, q', \varkappa) = G^0(q)\delta(q - q') + \frac{1}{N_y} G^0(q) G^0(q') e^{-i(q-q')/2}\, 2JS$$
$$\times \left\{\cos(q/2)\cos(q'/2)\frac{\varepsilon}{D_1} + \sin(q/2)\sin(q'/2)\frac{\varepsilon - 2\beta}{D_2}\right\}, \quad \Delta \to +0, \tag{4.3}$$

where

$$G^0(q) = \frac{1}{N}\sum_{\alpha-\beta} e^{i(\alpha-\beta)q}\, G^0_{\alpha-\beta}(\varkappa)$$

are the Fourier components of $G^0_{\alpha\beta}(\kappa)$.

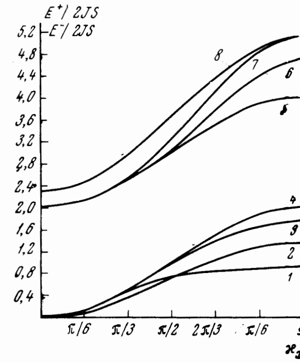

FIG. 1. Dependence of the energy of SM and AM on κ_x ($\kappa_z = 0$) for $\mathbf{H} = 0$. 1–$\xi = 1, \beta = 0.5$; 2–$\xi = 0.5$; 3–$\xi = 0.5, \beta = 0.5$; 4–$\xi = 0$; 5–$\xi = 2$; 6, $\xi = -1$; 7–$\xi = -1, \beta = 0.5$; 8–$\xi = -0.5, \beta = -0.5$. Curves 1, 3, 7, 8, for which the two parameters ξ and β are given, correspond to the energy E^-. Curves 4 and 5 represent the energy $\mathscr{E}_{\mathbf{k}}$ of spin waves of an ideal crystal, with $k_y = 0$, $k_z = 0$ and with $k_y = \pi$, $k_z = 0$, respectively.

5. DENSITY OF STATES OF A FERROMAGNETIC CRYSTAL WITH A PLANE DEFECT

As is known, the density of states can be expressed as follows:

$$g(E) = \frac{1}{\pi N}\,\mathrm{Im}\,\mathrm{Sp}\,G(E - i\Delta). \tag{5.1}$$

By use of (4.2), this expression can be put into the form

$$g(E) = g_0(E) + \Delta g, \quad \Delta g = \frac{1}{\pi N}\,\mathrm{Im}\,\frac{d}{dE}\sum_{\varkappa} \ln D, \tag{5.2}$$

where g_0 is the density of states of the ideal crystal, and where Δg is a correction to the density of states in consequence of the presence of the plane defect.

Remembering that $D = D_1 D_2$ and using the expressions (3.23) for D_1 and D_2, and on the supposition that there are N_i noninteracting defects, we find from (5.2) after a few transformations

$$\Delta g = -\frac{1}{4JS}(N_i/N)\sum_{\varkappa}\{\delta(\mathscr{E}) + \delta(\mathscr{E} - 2)\} + \frac{1}{8JS}(N_i/\pi N)$$
$$\times \sum_{\substack{\varkappa \\ (0 \leqslant \mathscr{E} \leqslant 2)}} \left[\frac{\varepsilon(\varepsilon - 2)}{\varepsilon - 1}\,\mathrm{P}\frac{1}{\mathscr{E} - \mathscr{E}_1} + \frac{[\varepsilon + 2(1 - \beta)](\varepsilon - 2\beta)}{\varepsilon + 1 - 2\beta}\,\mathrm{P}\frac{1}{\mathscr{E} - \mathscr{E}_2}\right]$$
$$+ \frac{1}{2JS}(N_i/N)\left\{\sum_{\binom{\varkappa,\, \mathscr{E} \leqslant 0}{\varepsilon \leqslant 0}} \delta(\mathscr{E} - \mathscr{E}_1) + \sum_{\binom{\varkappa,\, \mathscr{E} \geqslant 2}{\varepsilon \geqslant 2}} \delta(\mathscr{E} - \mathscr{E}_1)\right.$$
$$\left. + \sum_{\binom{\varkappa,\, \mathscr{E} \leqslant 0}{\varepsilon \leqslant 2(\beta - 1)}} \delta(\mathscr{E} - \mathscr{E}_2) + \sum_{\binom{\varkappa,\, \mathscr{E} \geqslant 2}{\varepsilon \geqslant 2\beta}} \delta(\mathscr{E} - \mathscr{E}_2)\right\}. \tag{5.3}$$

First of all, it can be shown that when $\epsilon = 0$ and $\beta = 0$, the value of Δg is zero. Furthermore, from (5.3)

$$\int_{-\infty}^{\infty} \Delta g(E)\, dE = 0, \tag{5.4}$$

that is, as was to be expected, in the approximations adopted in this paper the plane defect leads only to a redistribution of the density of states of the ideal crystal.

From (5.3) it is seen that in the energy range $E \geq \mathscr{E}_{\mathbf{k}}|_{k_y = \pi}$, $E \leq \mathscr{E}_{\mathbf{k}}|_{k_y = 0}$ there can occur in the density of states of the spin system spikes due to excitations localized near the plane defect. Consequently, in the energy range $\mathscr{E}_{\mathbf{k}}|_{k_y = 0} \leq E \leq \mathscr{E}_{\mathbf{k}}|_{k_y = \pi}$ there arises a "deflation" of states, so that (5.4) holds.

The sums inside the curly brackets can be easily calculated; the results are expressed in terms of complete and incomplete elliptic integrals. Since the results are unwieldy, however, we shall not write them out. The second term in Δg can be calculated only numerically. We shall present only qualitative results.

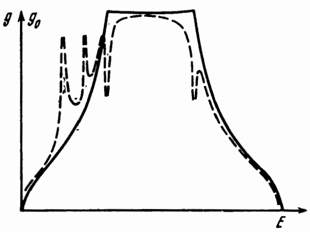

FIG. 2. Qualitative dependence of the density of states of an ideal ferromagnet (solid curve) and of a ferromagnet with a plane defect (dashed curve) on E.

Figure 2 shows the qualitative dependence of g on E, on the background of g_0 vs. E. In the case shown, the spin-state spikes due to both localized modes occur near the bottom of the spin-wave band of the ideal crystal. This is possible if $\epsilon \leq 0$ and $\epsilon \leq 2(\beta - 1)$. Depending on the parameters ϵ and β, other situations are also possible: for example, either both density spikes may be to the right of the plateau in the density of states of the ideal crystal, or one may be to the left and the other to the right.

We shall make a few comments. First of all, we mention that both the "deflation" at singular points (Van Hove points) and the spikes due to inflation of the density of states are infinite. At all these points, however, the expression obtained for the density of states is inapplicable. This is due to the fact that we neglected interaction between different defects. The presence of such interaction, through spin waves, must lead to a washing out of the peaks in the density of states. The height h of the peaks should be related to the concentration C of the defects. If there is a single plane defect, then $C = 1/N_y$, i.e., even in this case $h \neq 0$; but as soon as we set $h = \infty$, i.e., consider that $1/N_y \to 0$, we must remember that Δg, being proportional to $1/N_y$, also $\to 0$. Evaluation of this indeterminancy must lead to the result that at all the points mentioned above, the density of states will be finite also for $1/N_y \to 0$. In particular, in this case, far from these points $\Delta g = 0$.

6. HIGH-FREQUENCY MAGNETIC SUSCEPTIBILITY

Let there be a high-frequency nonuniform magnetic field $h_{l\alpha}(t)$, applied at an angle $\pi/2$ to the external constant magnetic field. The energy of the spin system in such a field can be described in the form

$$E = -\mu_0 \sum_{\alpha\alpha'\varkappa'\varkappa''} \chi^{ik}_{\alpha\alpha'}(\varkappa)\, h^i_\alpha(\varkappa', t)\, h^k_{\alpha'}(\varkappa'', t)\, \delta_{\varkappa\varkappa''}\delta_{\varkappa\varkappa'}. \tag{6.1}$$

Here i and k take the two values x and y, and $h^i_\alpha(\kappa, t)$ are the Fourier components of $h_{l\alpha}(t)$ with respect to the index l. The quantities $\chi^{ik}_{\alpha\alpha'}(\kappa)$ are related as follows to the Green's function $G_{\alpha\beta}$:

$$\begin{gathered}\chi^{xx}_{\alpha\alpha'} = \chi^{yy}_{\alpha\alpha'} = {}^1\!/_2\, \mu_0^2 g(\alpha) g(\alpha') \{G_{\alpha\alpha'}(\omega, \varkappa) + \overset{*}{G}_{\alpha\alpha'}(-\omega, -\varkappa)\},\\ \chi^{xy}_{\alpha\alpha'} = -\chi^{yx}_{\alpha\alpha'} = (1/2i)\, \mu_0^2 g(\alpha) g(\alpha') \{G_{\alpha\alpha'}(\omega, \varkappa) - \overset{*}{G}_{\alpha\alpha'}(-\omega, -\varkappa)\}.\end{gathered} \tag{6.2}$$

On transforming with respect to each of the indices α and α' to the q, q′ representation, we find for the components $\chi^{\pm} = \chi^{xx} \pm i\chi^{xy}$ of the high-frequency magnetic susceptibility

$$\begin{gathered}\chi^+(\omega, \varkappa, q, q') = g^2\mu_0^2 S G(\omega - i\Delta, \varkappa, q, q'),\\ \chi^-(\omega, \varkappa, q, q') = g^2\mu_0^2 S G^*(-\omega - i\Delta, -\varkappa, -q, -q').\end{gathered} \tag{6.3}$$

After using (4.3) and after separation of real and imaginary parts and some transformations, we find for the case of N_i noninteracting plane defects

$$\begin{aligned}\operatorname{Re}\chi^+ = g^2\mu_0^2 S \Big\{ \frac{1}{\hbar} \mathrm{P} \frac{1}{\omega - \omega_3} \delta(q - q')\\ + \frac{\omega_0^2}{\hbar} C \sum_j (\operatorname{Re}\theta_j \operatorname{Re} A_j - \operatorname{Im}\theta_j \operatorname{Im} A_j) \Big\},\end{aligned} \tag{6.4}$$

$$\begin{aligned}\operatorname{Im}\chi^+ = g^2\mu_0^2 S \Big\{ \frac{\pi}{\hbar} \delta(q - q')\delta(\omega - \omega_3)\\ + C\frac{\omega_0^2}{\hbar} \sum_j (\operatorname{Re}\theta_j \operatorname{Im} A_j + \operatorname{Im}\theta_j \operatorname{Re} A_j) \Big\},\end{aligned} \tag{6.5}$$

where $C = N_i/N_y$ is the defect concentration, and where P symbolizes the principal value. The summation over j goes from 1 to 2, where the index 1 corresponds to quantities characteristic of SM, 2 to quantities characteristic of AM. Furthermore

$$\operatorname{Re} A_j = \mathrm{P}[(\omega - \omega_j)(\omega - \omega_3)(\omega - \omega_4)]^{-1},$$

$$\begin{aligned}\operatorname{Im} A_j = \pi \Big[\mathrm{P} \frac{\delta(\omega - \omega_j)}{(\omega - \omega_3)(\omega - \omega_4)} + \mathrm{P} \frac{\delta(\omega - \omega_3)}{(\omega - \omega_4)(\omega - \omega_j)}\\ + \mathrm{P} \frac{\delta(\omega - \omega_4)}{(\omega - \omega_3)(\omega - \omega_j)} \Big],\end{aligned}$$

$$\theta_1 = \nu_1(\varepsilon - 2 + \varepsilon\Gamma), \qquad \theta_2 = \nu_2\{\varepsilon + 2(1 - \beta) + (\varepsilon - 2\beta)\Gamma^{-1}\},$$

$$\nu_1 = e^{-i(q-q')/2} \cos(q/2) \cos(q'/2) \frac{\varepsilon}{2(\varepsilon - 1)} \Big(\frac{\Omega}{\omega_0}\Big),$$

$$\nu_2 = e^{-i(q-q')/2} \sin(q/2) \sin(q'/2) \frac{\varepsilon - 2\beta}{2[\varepsilon + 1 - 2\beta]} \Big(\frac{\Omega - 2\omega_0}{\omega_0}\Big),$$

$$\omega_0 = (2JS/\hbar), \quad \Omega = \omega - \omega_3|_{q=0}, \quad \omega_3 = \mathscr{E}_{\varkappa, q}/\hbar, \quad \omega_4 = \mathscr{E}_{\varkappa, q'}/\hbar,$$

$$\omega_1 = E^+/\hbar, \qquad \omega_2 = E^-/\hbar,$$

$$\Gamma = \begin{cases} \sqrt{1 - 2\omega_0/\Omega} & \text{or } \Omega \leqslant 0 \text{ for } \Omega \geqslant 2\omega_0 \\ -i\sqrt{2\omega_0/\Omega - 1} & \text{for } 0 \leqslant \Omega \leqslant 2\omega_0 \end{cases}$$

The expression for the χ^- component follows directly from (6.4) and (6.5) (compare χ^+ and χ^- in formula (6.3)), and therefore we shall not give it.

First of all we note that if $\epsilon = \beta = 0$, then

$$\chi^+ = \chi_0^+ \delta(q - q'), \tag{6.6}$$

where

$$\begin{gathered}\operatorname{Re}\chi_0^+ = g^2\mu_0^2 S \mathrm{P} \hbar^{-1} (\omega - \omega_3)^{-1},\\ \operatorname{Im}\chi_0^+ = \hbar^{-1} g^2\mu_0^2 S \pi \delta(\omega - \omega_3),\end{gathered} \tag{6.7}$$

this agrees with the known expression for the susceptibility of an ideal ferromagnetic monocrystal. From (6.5) it is seen that on approach of the frequency of the external high-frequency field to ω_1 or ω_2, i.e., to the characteristic frequencies due to the presence of LSSO, there will occur a resonance absorption of the energy of the high-frequency field, because of the excitation of oscillations localized near the plane defect. The contribution to χ^+ that is due to these additional resonances is proportional to the concentration of plane defects. The additional resonances are possible at frequencies such that $\Omega \leq 0$ or $\Omega \geq 2\omega_0$. In the frequency range $0 \leq \Omega \leq 2\omega_0$ there is in χ^+ only a nonresonance contribution from the plane defects; it is proportional to their concentration.

It is interesting to note that the structure of the terms proportional to the defect concentration differs significantly from the structure of $\chi_0^{\mp}$ for the ideal crystal. In fact, resonance terms occur, as is seen from (6.4) and (6.5), both in Re χ^+ and in Im χ^+. In χ^-, as contrasted with χ^+, sums $\omega + \omega_j$ will enter instead of differences $\omega - \omega_j$, and consequently the dependence of χ^- on frequency, as in the case of the ideal crystal, is not of the resonance type.

From (6.1) it is clear that for excitation of spin waves localized near a plane defect, it is necessary that the external high-frequency field shall be nonuniform in a plane parallel to the defect surface (in our case, along x or z). As in [9] it can be shown that the probability of excitation of waves with different $\kappa \neq 0$ is proportional to $|h_\alpha^y(\kappa)|^2$ ($h_\alpha(\kappa)$ directed along y), and that the maximum probability of excitation occurs for magnons localized near the plane defect with $k_x \approx 1/L$; the smaller the extent L of the field nonuniformity, the shorter the waves, in principle, that can be excited. To be sure, the shorter the waves, the less their probability of excitation [9]. In these remarks we have supposed that the high-frequency field is nonuniform along x and has a scale of nonuniformity L.

7. DISCUSSION AND SOME CONCLUSIONS

The results presented in this paper were obtained for the example of a simple cubic lattice and for the case of a certain model of a defect. Of course for real ferromagnets, for example with a body- or face-centered cubic lattice, in general the results will be different. Thus for an arbitrary value of the wave vector κ, the spectrum of spin oscillations localized near a plane defect will be different for crystal lattices of different symmetries—as is true, however, also of the spectrum of an ideal ferromagnetic monocrystal. It is nevertheless possible to draw conclusions valid also for lattices of different symmetry. First of all we note that localized oscillations of the very type found will occur also in ferromagnetic crystals of another symmetry. Furthermore, it may be supposed that the range of energies in which localized oscillations are possible is determined by inequalities of the type (2.3), which can be written in general form thus:

$$E \leqslant \mathscr{E}_{\mathbf{k}}|_{k_\zeta=0}, \quad E \geqslant \mathscr{E}_{\mathbf{k}}|_{k_\zeta=q\pi},$$

here k_ζ are the components of the wave vector, where ζ represents the direction along which the translational invariance of the lattice is upset, and $q\pi$ is the maximum value of k_ζ within the limits of the reduced zone. We note that if in (2.2) we substitute for k_y the quantity $i\delta_1$ or $i\delta_2$, then by taking account of (3.22) we obtain (3.25) or (3.26) respectively; that is, (2.2) represents the dispersion law not only of volume spin waves, but also of those localized near a plane defect. Everything is determined by this alone, whether or not the quantity k_y is a component of a wave vector. Apparently this deduction is correct not only for lattices of symmetry other than that considered in our case, but also for other types of defect.

In this paper a certain hypothetical plane defect was considered. A real plane defect may be, for example, an antiphase boundary, a stacking fault, a twin boundary, etc. The specific form of the spectrum of localized spin excitations will be different from that obtained in this paper; but for spin excitations of long wavelength, this difference will presumably not be very important.

We have shown that the waves found can in principle be excited by means of a nonuniform high-frequency field. This question, however, is very complicated. First of all we note that it is simplest to excite waves whose energy satisfies the inequality $E \leq \mathscr{E}_{\mathbf{k}}|_{k_y=0}$, because the frequencies at which they can be excited will be of the order of the ferromagnetic resonance frequency. To be sure, this can not be asserted with certainty, because in this paper we have not allowed for the correlation between different plane defects. This prevents us from saying how the height and width of the resonance peaks change with approach of κ to zero. Moreover, it is nevertheless apparently to be expected that the peaks mentioned become washed out when $\kappa \approx 0$. This leads to the result that is is necessary to consider excitation of waves with not too large wavelengths. But the frequency of such excitations may be large. From this point of view, it is more convenient to choose materials for which the exchange integral J is as small as possible.

In closing, we shall draw a few conclusions. In a ferromagnetic monocrystal containing plane defects. on both sides of which the same medium is present, in addition to the spin excitations characteristic of the ideal ferromagnetic crystal, there are to be expected two branches of localized excitations, for one of which the wave function is symmetric, and for the other antisymmetric, with respect to the operation of reflection in the plane of the defect.

In the presence of these excitations, the density of states of the spin system, as compared with the density of states of the ideal crystal, may contain additional spikes. For values of the parameters ξ and β determined by the inequalities $\epsilon \leq 0$, $\epsilon \leq 2(\beta - 1)$, these spikes may occur close to the bottom of the spin-wave band; this gives grounds for anticipating that the excitations found will possibly play a role in the consideration of the kinetic coefficients of ferromagnets.

The high-frequency properties of ferromagnets with plane defects are distinguished by the fact that χ^+, as compared with the χ_0^+ of an ideal crystal, has additional resonance terms, due to the existence of spin excitations of ferromagnetic monocrystals with plane defects.

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