PROPERTIES OF A LASER WITH AN UNSTABLE RESONATOR

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The angular distribution of the radiation and other characteristics of a neodymium-glass laser with an unstable resonator are studied experimentally. It is shown that the use of an unstable resonator results in effective selection of transverse modes and an appreciable decrease of the angular divergence. Some features and possibilities of the application of such systems are discussed.

1. GENERAL REMARKS

In our previous paper [1] we showed that, in accordance with the ideas of Siegman, [2] there is an effective selection of angular modes in an unstable resonator and hence its use is of considerable interest for the creation of systems with high directivity and coherence of radiation.

The present paper presents additional discussion of the features of a laser with an unstable resonator and the results of corresponding experimental investigations.

Modes of an Unstable Resonator. Geometrical Approximation

Siegman[2] has shown that in the geometrical approximation the modes of an unstable resonator formed by two spherical mirrors with radii of curvature R_1 and R_2 placed at a distance L from one another can be represented by spherical waves with imaginary or real centers propagating in both directions. The position of these centers is such that after a double transformation by reflection of the wave from both mirrors, it coincides with the original one (see Fig. 1). Starting from this condition it is easy to find the basic parameters of the system with the aid of geometrical optics. We shall derive them for the cases that are most important in practice.

If one of the mirrors is planar ($R_2 = \infty$), the center of the wave moving to the plane mirror is located at a distance $(L^2 + LR_1)^{1/2}$ from it; the linear magnification of the transverse dimensions of the spherical wave after a complete traversal of the resonator in both directions[1)] is

$$M = \left|\frac{\sqrt{L^2+LR_1}+L}{\sqrt{L^2+LR_1}-L}\right|$$

(We recall that the losses are $1 - 1/M$ in a two-dimensional resonator and $1 - 1/M^2$ in a three-dimensional one [2]). For $R_1 > 0$ (convex mirror) the center of the spherical wave is imaginary (Fig. 1a). When $R_1 < 0$ (concave mirror) and $L > |R_1|$ the resonator is also unstable; the center in this case is real and is located inside the resonator (Fig. 1b).

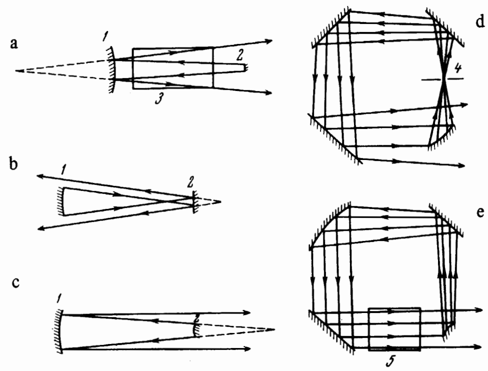

FIG. 1. Different types of resonators with an unstable configuration. 1, 2—mirrors, 3—sample, 4—diaphragm, 5—angular selector.

If one of the mirrors is convex ($R_2 > 0$) and the other concave, and its radius of curvature is $R_1 = -(2L + R_2)$, then the wave propagating toward the convex mirror is planar (Fig. 1c); $M = 1 + 2L/R_2$.

Wave Approximation

Finding the modes of an unstable resonator in the wave approximation by analytical methods is an extremely complex problem. It is quite probable that even the methods developed by Vaĭnshteĭn and used by him with success to calculate other kinds of resonators [3] will not turn out to be very helpful. In fact, these methods are based on the fact that the solution for a certain class of open resonators with mirrors of finite dimensions is well represented in the form of a superposition of two waves that are the solutions for a resonator with infinite mirrors, and these waves are summed with approximately equal amplitudes (the coefficient of reflection of the wave from the edge of the resonator is of the order unity). At the same time, in an unstable resonator the coefficient of reflection from the open edge rapidly decreases with an increase both of M and the Fresnel number N, and one can expect the appearance of transformed waves with greater amplitudes than the reflected one (see [3]). Consequently, the question of the correctness of this approach in the present case is moot, and the method of calculation employed in some papers (e.g., [4]) of the modes of an unstable resonator with infinite mirrors may turn out to be useless for understanding the properties of real systems.

The most valuable information about the modes of unstable resonators was obtained by means of calcula-

[1)]In this paper we consider mainly asymmetrical resonators. Therefore it is more convenient to introduce the amplification and loss for a complete passage of the wave through the resonator, and not just from one mirror to the other, as in [5].

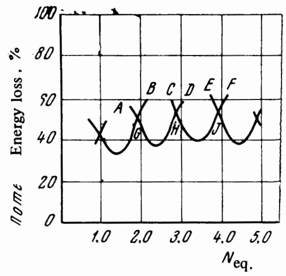

FIG. 2. Dependence of losses in a two-dimensional unstable resonator with ≈ 1.9 on the equivalent Fresnel number N_{eq}.

tions on an electronic computer by the Fox-Li method.[5] However, the treatment of the results presented in that paper seems to be a failure. In fact, the dependence of the losses of the symmetrical modes with highest Q on the equivalent Fresnel number N_{eq} introduced in [5] has the form represented schematically in Fig. 2 (for the construction in Fig. 1a, this number is equal, in our notation, to $(a_2^2/4\lambda L)\times(M - 1/M)$, where $2a_2$ is the transverse dimension of mirror 2). Siegman and Arrathoon [5] showed that the lower "wavy" line GHJ... corresponds to the mode of lowest order, and the V-shaped branches AGB, CHD, EJF, ..., to the closest symmetrical mode. The meaning of the equivalent Fresnel number, as well as the reasons for the "accidental" coincidence of the maxima of the losses of one mode with the minima of the other and the sharp changes of the shape of the mode in going (along the wavy curve) through the points G, H, J, ..., remained unclear. The hope was expressed that, with an increase in N_{eq}, a "smoothing of the waviness" of the line would occur; in the opinion of the authors, this should lead to an increase in the difference between the losses of the lowest symmetrical modes and thus to a strengthening of the discriminating properties of the resonator.

At the same time, all the features of the pattern find a natural explanation if one accepts that the U-shaped lines AGHD, CHJF, ..., correspond to different symmetrical modes, which, in the identical arrangement of points on these lines, have a similar structure that changes smoothly along these lines (and, of course, changes sharply in going from one line to another, i.e., in jumping modes). The individual modes differ from one another chiefly in the magnitude of the change in phase of the field along the mirror of the resonator and can be classified in corresponding fashion (in analogous fashion one classifies the modes of a plane resonator according to the number of nodes in the amplitude of field or current in the plane of the mirror).

We note that N_{eq} is equal to the number of half-periods in the phase change along the mirror for a spherical wave in the geometrical approximation. From the approximate correspondence given in [5] for the phase distribution of modes found in the geometric and wave approximations, it therefore follows that the number of half-periods in the phase change across the mirror for adjacent symmetrical modes differs by approximately 1. For a given N, the mode with the least loss is evidently the one in which the number of periods of phase change, and consequently the wavefront curvature, are the closest to that dictated by purely geometrical considerations.

Two important conclusions follow from the above:

1. Even in the case of large N and M, the values of the losses of adjacent symmetrical modes are comparable for certain values of N_{eq}.

2. As is easy to see, inspite of the opinion expressed in[5], during "smoothing out of the waviness" the differences between the losses of adjacent modes decreases on the average, and the discriminating effect of the resonator is weakened.

We note that the "geometrical modes of high order" considered in[5] have high losses and certainly must not be made to correspond to the wave modes with the least loss. This follows, in particular, from the fact that the given modes are solutions of the corresponding equation for any values of the index n, not only integral ones as was assumed in[5], and it makes no sense to speak of a discrete set of them.

Effect of the Active Medium and Resonator Aberrations

The deformation of modes brought about by the unavoidable presence in real resonators of different kinds of perturbation is an important factor determining the spatial structure of the radiation of solid-state lasers. The character of the deformation depends on the resonator properties and is unique in unstable resonators.

First, we pause to consider the effect of nonuniformity of the distribution of the amplification factor over the resonator cross section. For an unstable resonator an estimate of this effect can be obtained in the framework of the geometrical approximation, which is not possible in the case of a resonator with plane mirrors.

In fact, in a two-dimensional unstable resonator with an intensity amplification factor $k(x)$ that varies slowly over the cross section, the equation of the mode in the geometrical approximation can be written in the form

$$\gamma v(x) = \left\{\frac{k(^1/_2x + {^1/_2}x/M)}{M}\right\}^{1/2} v\left(\frac{x}{M}\right), \tag{1}$$

where γ is the eigenvalue, v is the field amplitude (see Eq. (7) of [5]), and the value of the amplification factor is taken at a point between x/M and x.

The approximate solution of this equation[2)] corresponding to the mode with the least losses has the form

$$v(x) \approx C\exp\left\{\frac{M}{M-1}\int_0^x [\ln\sqrt{k(x')} - \ln(\gamma\sqrt{M})]\frac{dx'}{x'}\right\}, \tag{2}$$

and

$$\gamma = \sqrt{k(0)}/\sqrt{M}. \tag{3}$$

The corresponding expressions for the axially symmetric modes of a three dimensional resonator can be obtained from (2) and (3) by replacing $(M)^{1/2}$ by M.

In the case of steady generation $\gamma = 1$; this occurs for a two-dimensional resonator when $k(0) = M$, and for a three-dimensional resonator when $k(0) = M^2$.

In accordance with (2), a change in k in going from the axis to the edge of the resonator causes a signifi-

2)The analogous equation for the plane resonator (M = 1) has the form $\gamma v(x) = (k(x))^{1/2}v(x)$ and, of course, has no solution.

cant change in the magnitude of v directed to this same side. This in turn creates a favorable condition for the effective removal of the energy of one generating mode over the entire cross section of the resonator.

The effect of phase aberrations in unstable resonators is also different from that in other kinds of resonators. Phase aberrations of the first order, equivalent to an inclination of the mirrors, lead, as is well known, in the case of spherical mirrors only to certain displacements of the resonator axis and hence have much less effect than in resonators with plane mirrors.

Aberrations of the second order are equivalent to a change in the curvature of the mirrors. For resonators with small losses in emission they lead to changes in the volumes of the modes, which significantly affects their competition and the character of the angular distribution of radiation (see, for example, [6]). In unstable resonators the mirror curvature does not affect the volume of the mode with least loss, which practically uniformly fills the entire cross section (for large N and amplification factor M markedly different from 1). The change in curvature of the wave leaving the resonator due to the presence of aberrations can be compensated by tuning a quadratic phase corrector, equivalent to a thin lens with variable focal length, placed outside the resonator. For a fixed L and a given magnitude of the aberrations, the required tuning (in units of optical strength) decreases with increasing M,[3)] which makes resonators with high losses in emission rather insensitive to axially symmetric aberrations, in particular, those due to thermal effects

The effect of aberrations of high order is not subject to analysis in the framework of the geometrical approximation; however, it may be expected that they will lead mainly to an increase of losses in emission. We remark that the magnitude of aberrations of high order is usually significantly less than of first and second order.

Traveling Wave Ring Generators

A high magnitude of losses in emission leads to irreversibility of the path of the light rays. Thus, propagation of waves in the direction opposite to that indicated in Fig. 1a would require supplementary input to the resonator from an external source of radiation from the side of mirror 2. This leads in principle to a new possibility of constructing unidirectional ring generators. Two variants of systems of this kind are illustrated in Fig. 1, d, e. The first of these (Fig. 1d) utilizes a ring resonator with a configuration of mirrors equivalent to that of Fig. 1b; placing a small diaphragm in the plane of the real center of the waves propagating in one of the directions eliminates the possibility of appearance of waves with the opposite direction.[4)] In the second variant (Fig. 1e), the resonator is similar to that shown in Fig. 1c; the unidirectional property is achieved by appropriate disposition of an angular radiation selector, e.g., a selector using total internal reflection. In both cases, introduction of one additional element (diaphragm or selector) provides simultaneously the unidirectional property and angular selection of radiation.

2. EXPERIMENTAL RESULTS

For large Fresnel numbers N the difference in the losses of the lowest modes in resonators of the usual type quickly becomes negligibly small. Hence one may expect that the advantages of unstable resonators, from the point of view of selection of transverse modes, could show up just at these large values of N and at values of M markedly different from 1. The properties of lasers with such resonators have not been fully studied. The present paper presents the results of the corresponding investigations for the neodymium-glass laser.

The resonator had the configuration of Fig. 1a. To avoid the useless scattering of energy use was made of the diffraction output of radiation from the side of the plane mirror, as shown in Fig. 1a. (See also [2,1,5].) Both mirrors were completely reflecting, and a sample with transparent end faces was located in the center of the resonator. The resonator base and the curvature of the convex mirror were chosen so as to make the magnitude of the losses in emission $1 - 1/M^2$ approximately equal to the optimum magnitude of the transmission of the output mirror for the case of a plane resonator. The dimension of the second mirror was determined from the condition that the radiation fill the aperture of the sample, as shown in the figure.

An active rod of diameter 10 mm and length $l = 120$ mm was used in most of the experiments. The separation between the mirrors L_0 was 54 cm, the radius of curvature of the convex mirror R_1 was 20 m, and the diameter of the reflecting portion of the plane mirror was 7.3 mm. From these values one obtains: the effective length of the resonator $L = L_0 - l(1 - 1/\kappa) = 50$ cm (κ is the index of refraction of the sample); distance from the imaginary center of the wave to the plane mirror, 3.2 m; number of Fresnel zones $N \approx 25$; amplification coefficient $M = 1.37$; finally, the magnitude of emission losses $1 - 1/M^2 = 0.47$.

Adjustment of the resonator was effected by the interferrometric method using a gas laser. Pumping was by a pulsed spiral lamp, which provided a high degree of uniformity of sample illumination. Because of the shortness of the pump pulse ($\sim 100\ \mu$sec), thermal deformations of the sample turned out to be negligibly small and had practically no effect on the operation of the laser. At the same time the excess over the generation threshold of the pump intensity was rather high (~ 10 to 20).

The characteristics of the laser with the unstable resonator were compared with the characteristics of the same generator in which both mirrors were replaced by large-diameter plane mirrors. One of them was completely reflecting, while the other had a transmission coefficient equal to the losses in the unstable resonator. The replacement did not produce any per-

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3)In the case of the configurations shown in Fig. 1a, the required tuning is $\frac{1}{2}(R/L)^{1/2}(1 + L/R)^{-3/2}\,\delta$, where δ is the optical strength of the thin lens arising as a consequence of the aberrations and located near mirror 1.

4)At least for waves occupying a large cross section, which are well described in the geometrical approximation.

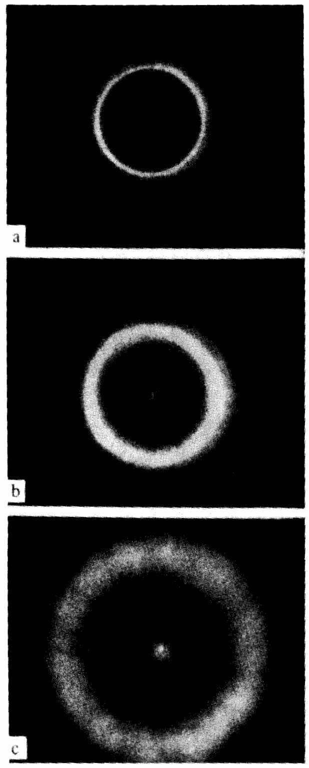

FIG. 3. Cross sections of the beam at different distance d from the exit mirror of an unstable resonator: a–d = 30 cm, b–d = 95 cm, c–d = 320 cm.

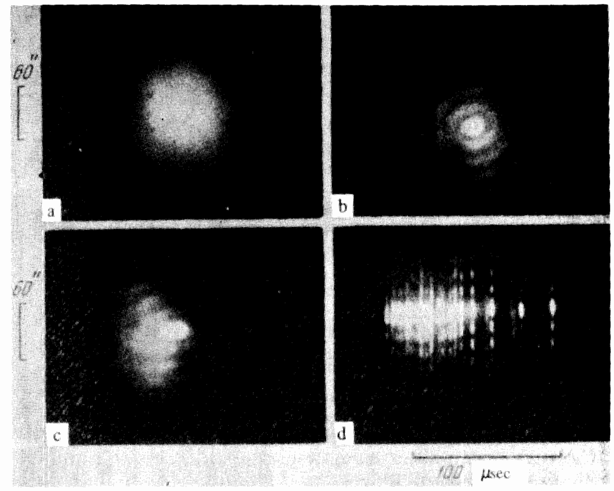

FIG. 4. Photographs of the radiation distribution in the far zone (sample diameter–10 mm, length–120 mm). a–plane resonator, b–unstable resonator, c–unstable resonator misaligned by about 60'', d–time scan of the angular distribution.

ceptible change in the generation threshold. The energy of radiation at different pumping levels was 1.3 to 1.5 times greater in the case of the plane resonator than in the case of the unstable one; the reasons for this will be discussed below.

The results of the investigation of the spatial structure of the radiation are presented in Figs. 3–6. With the aid of the photographs of the cross section of the light beam at different distances from the exit mirror of the unstable resonator (Fig. 3) it is easy to see that the radiation actually has the character of a diverging spherical wave. Photometry gave the distance from the imaginary center of this wave to the exit mirror as about 3.1 m, which is in complete agreement with the value predicted by the geometrical approximation (3.2 m). Of significance is the definition of the rings observed in the center of the section (Fig. 3c).* On the whole, this pattern is analogous to the diffraction pattern of a monochromatic wave falling on an annular aperture (see, for example,[5]).

Figure 4 shows photographs of the distribution of radiation in the far zone, obtained with the aid of a lens with focal length 10 m. In the case of the unstable resonator, the spherical wave was first transformed into a plane wave by means of a compensating lens placed immediately after the exit mirror[5)]; the focus of the lens was at the imaginary center of the wave.

Figure 5 gives the angular distribution of the radiation intensity obtained by photometry from the photographs of Figs. 4a and b. Using the data of Fig. 5, it is easy to find out what fraction of the energy arrives in the solid angle encompassed by a cone with a vertex angle 2φ. The dependence of this quantity on φ, which characterizes the angular distribution of the radiation energy, is given in Fig. 6. The same figure gives the calculated analogous dependences for the cases of diffraction of a plane monochromatic wave by a ring aperture with dimensions equal to that of the beam at exit from the resonator, and by a circular aperture of the same diameter.

As follows from the data of Figs. 4 and 5, the halfwidth of the angular distribution of the radiation intensity for the unstable resonator is approximately one-fifth that of the plane resonator and is equal to the diffraction limit (~20''). This is evidence that a significant portion of the generated energy is concentrated in one transverse mode (we do not, of course, have in mind here the monochromaticity of the regime). However, use of the unstable resonator increased the axial

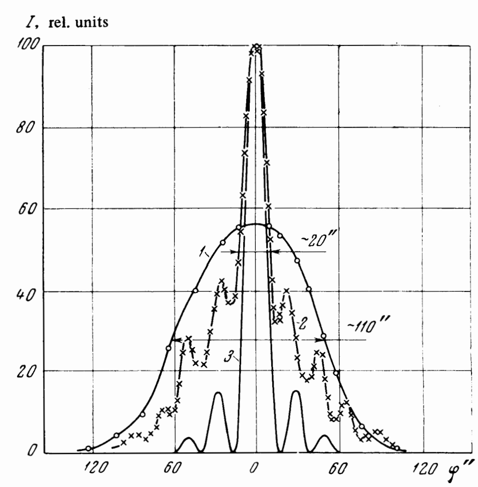

FIG. 5. Angular distribution of radiant intensity (sample diameter–10 mm, length–120 mm). Curve 1–plane resonator; 2–unstable resonator; 3–theoretical curve for diffraction by a ring aperture.

*Editor's Note. Unfortunately, when the photograph Fig. 3c was reduced for publication, these rings were no longer discernible.

[5)] The angular divergence is decreased if a compensating lens of appropriate focal length is placed far from the mirror where the beam has a bigger cross section. Thus, in the case of the unstable resonator, one lens can fulfill the function of a telescopic system.

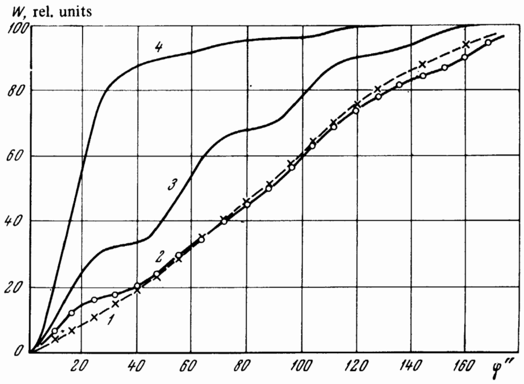

FIG. 6. Angular distribution of radiant energy (sample diameter–10 mm, length–120 mm). Curve 1–plane resonator; 2–unstable resonator; 3–theoretical curve for diffraction by an annular aperture, a_1 = 5.5 mm, a_2 = 3.7 mm; 4–theoretical curve for diffraction by a circular aperture, a_1 = 5.5 mm.

intensity of the radiation at most only 1.8 times, and the angle φ_0 in which half the energy is concentrated (the half-width of the angular distribution of the energy) remained approximately the same as for the plane resonator.

One of the most important reasons for the relatively small gain in directivity appeared to be in this case the small width of the ring through which the output radiation was made to pass. Actually, for equal radiated energy the axial intensity of an ideal ring radiator with outer radius a_1 and inner radius a_2 is less (in the ratio $1 - a_2^2/a_1^2$), and the width of the angular distribution greater than that of a disk radiator with radius a_1. This width quickly increases with decreasing $(a_1 - a_2)/a_1$ and for $a_1/a_2 = M = 1.37$ is extremely large (Fig. 6). The experimentally observed magnitude of φ_0 in the unstable resonator was approximately 1.5 times greater than theoretical. It must also be noted that in spite of the similarity of the experimental and calculated intensity distributions (Fig. 5), the contrast of the lateral maxima in the experimental distribution was significantly less and their intensity greater.

In order to elucidate the possible reasons for the departure of the characteristics of the investigated laser from those of an ideal radiator, a time scan of the angular distribution was photographed. Scanning was effected with a rotating-drum camera. An image of a slit cutting out the center of the distribution pattern was projected onto the surface of the photographic plate. An example of such a scan is shown in Fig. 4d. It is seen that in the process of generation there occur small shifts of the center of symmetry of the angular distribution (within the limits of $\pm 5''$). Such shifts could be caused by small changes in the adjustment of the resonator, by birefringence in the sample (the position of the resonator axis thereby depends on the polarization of the radiation), etc.

It is also interesting that for the majority of generation peaks the intensity of the radiation between the lateral maxima of the angular distribution falls practically to zero, and the distribution itself is very similar to the distribution for an ideal radiator.

At the same time, in the individual peaks the intensity falls with distance more gradually and almost monotonically. The nature of these peaks awaits explanation. Their presence, as well as the shift of the center of symmetry of the angular distribution in the generation process is indeed responsible for the worsening of the integrated characteristics of the radiation.

In the course of these experiments an important feature of lasers with unstable resonators was uncovered--the extremely high stability of all their parameters, among them the shape of the angular distribution. With plane mirrors, in spite of careful adjustment, the angular distribution varied noticeably from flash to flash (this was especially true of the generator with very high N described below); its width increased as the active element aged.

These phenomena were not observed with the unstable resonator. More than this, in accordance with theoretical predictions, when the mirrors were misaligned by 20–30″, which would be sufficient for a very great distortion of the pattern of a laser with a plane resonator, in this case only a corresponding shift of the imaginary center of the wave was observed. The shape of the angular distribution began to change significantly only for such great misadjustments (about 40–50″) that the axis of the mirror system shifted from the center of the sample cross section to its very edge. In this case the core of the distribution stretched out in the direction of the misalignment, and the relative intensity of the "wings" grew (see Fig. 4c; qualitatively the same pattern is given by Fraunhofer diffraction by an aperture that is a correspondingly deformed ring).

Besides the angular distribution, we also investigated the spectral and temporal characteristics of the radiation laser with an unstable resonator. It was observed that they were almost identical to the corresponding characteristics of a laser with plane resonator. One notices only a somewhat smaller duration of the peaks, which is an indirect confirmation of the conclusion made earlier [1] about the expedience of using an unstable resonator in single-pulsed lasers.

Thus, the principal feature of a laser with an unstable resonator is the effective selection of transverse modes. The highest gain in directivity may be obtained through the use of unstable resonators in a system with high gain per pass. In this case, a small feedback coefficient suffices to operate the laser, and consequently diffraction output may be effected through a large portion of the cross section of the end face of the resonator ($a_1/a_2 = M$ large). To check this idea, we measured the energetic and angular characteristics of the radiation of a laser with a rod of diameter 45 mm and length 600 mm. The pump consisted of four ISP-20 000 lamps and was quite intense; the energy of radiation with plane mirrors was about 1000 J (reflection coefficient of the exit mirror ~20%). The use of an unstable resonator with the same distance between the mirrors (L_0 = 97 cm, L = 77 cm, R_1 = 464 cm, M = 2.2, a_2 = 1.1 cm) led to a lowering of the generated energy to 400 J. This lowering can be explained by the fact that a portion of the volume of the sample from the side of mirror 1 is in fact not used (Fig. 1a). Actually, on increasing L_0 to 164 cm and R_1 to 1075 cm, so that the magnitude of M remained as before, the energy rose to 700 J, although usually the energy de-

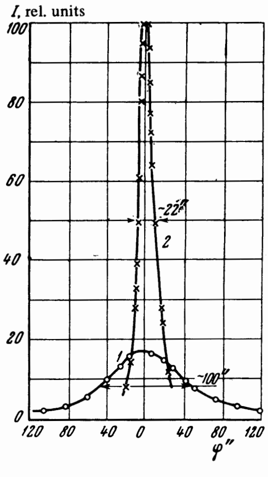

FIG. 7. Angular distribution of radiant intensity (diameter of sample, 45 mm; length, 600 mm). 1—plane resonator, 2—unstable resonator.

creases with increasing resonator base (see, for example,[7]); it is easy to see that the used volume of sample in this case is somewhat greater. From these data it is possible to conclude that for large M and cylindrical active rod, maximum radiated energy will be attained in the configuration of mirrors shown in Fig. 1c.

Figure 7 compares the angular characteristics of the powerful laser with plane and unstable resonators. It is seen that in this case application of the unstable resonator actually leads to a high gain (the axial brightness in spite of less total energy is six times greater; the angle φ_0 diminishes from 4′30″ for the plane resonator to 1′15″ for the unstable resonator). However, the halfwidth of the intensity distribution (~22″) remains substantially greater than the theoretical limit (~5″). Evidently, this circumstance, as well as the absence of "diffraction" structure in the distribution, is associated with the presence of thermal deformations accumulating during the pulse.

3. CONCLUSION

The theoretical analysis and experimental results presented in this paper demonstrate a number of unique properties of systems with unstable resonators which facilitate single-mode generation even for very large volumes of active substance and very large Fresnel numbers (experimentally, resonators with $N \leq 300$ were investigated).

The possibility of formation of a plane front of radiation with uniformly distributed intensity over the end of the radiator make the combination of unstable resonators with angular selectors particularly promising. Along with the limiting directivity, the regime of running waves may be obtained thereby.

These features of unstable resonators, as well as the high stability of their properties under the action of different kinds of perturbation, make this class of resonators the most promising for lasers with high directivity and coherency.

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[1] Yu. A. Anan'ev, N. A. Sventsitskaya, and V. E. Sherstobitov, Dokl. Akad. Nauk SSSR **179**, 1304 (1968) [Sov. Phys.-Dokl. **13**, 351 (1968)].

[2] A. E. Siegman, Proc. IEEE **53**, 277 (1965).

[3] L. A. Vaĭnshteĭn, Otkrytye rezonatory i otkrytye volnovody (Open resonators and open waveguides), Sov. Radio, 1966.

[4] S. R. Barone, Appl. Optics **6**, 861 (1967).

[5] A. E. Siegman and R. Arrathoon, IEEE J. Quant. Electronics **3**, 156 (1967).

[6] A. M. Bonch-Bruevich, N. A. Esepkina, Ya. A. Imas, N. A. Pavelenko, L. N. Pakhomov, V. Yu. Petrun'kin, and S. E. Potapov, Zh. Tekh. Fiz. **36**, 2175 (1966) [Sov. Phys.-Tech. Phys. **11**, 1624 (1967)].

[7] Yu. A. Anan'ev, A. A. Mak, and B. M. Sedov, Zh. Eksp. Teor. Fiz. **52**, 12 (1967) [Sov. Phys. JETP **25**, 6 (1967)].

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