ENERGY SPECTRUM OF FERMI EXCITATIONS IN FERROMAGNETIC METALS

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Quantum field theory methods are used to consider the properties of the energy spectrum of the conduction electrons in a ferromagnetic metal. It is shown that the interaction with the spin waves leads to an appreciable renormalization of the Fermi-excitation spectrum and to an additional damping, due to the processes of radiation of spin waves.

1. As is well known, the conduction electrons exert an appreciable influence on the magnetic properties of ferromagnetic metals. Together with the electrons that originate from the internal incomplete shells of the transition-element metals forming excitation bands with high level density (d- or f-bands) in the metals, the conduction electrons "responsible" for the ferromagnetic ordering mechanism itself contribute to the magnetic moment of the ferromagnet and lead to singularities in the spin-wave spectrum[1].

It is natural to assume that the ferromagnetic ordering itself, in turn, affects the properties of the conduction electrons. The purpose of the present paper is to consider the properties of the energy spectrum of the conduction electrons, due to the interaction with the spin waves in a ferromagnetic metal. It will be shown below that this interaction results in a strong renormalization of the law of conduction-electron dispersion and in an additional damping due to spin-wave radiation.

To simplify the calculations, we shall use the isotropic model of the conduction-electron spectrum, according to which the ground state of the ferromagnetic metal at $T = 0$ is characterized by two Fermi spheres with radii p_+ and p_- for the electrons with $s_z = 1/2$ and $s_z = -1/2$ respectively (s_z—quantum number of the projection of the electron spin on the direction of the total spin momentum of the system). The distance between the Fermi surfaces $\Delta \equiv p_+ - p_-$ determines the contribution M_s of the s-electrons to the total magnetic moments M_0 of the system and, as shown in[1], determines the intensity of their interaction with the spin waves.

The calculations will be carried out for the case of a zero temperature, although obviously the main properties of the result remain in force also on going to $T \neq 0$, if $T \ll \Theta$ (Θ—Curie temperature). We shall neglect, in addition, the effects of the magnetic anisotropy, since allowance for them, in view of their smallness compared with the exchange interaction, entails corrections of little significance to the main result.

2. In determining the spectrum and the attenuation of the Fermi excitations, we shall start from the fact that they are determined by the poles of the electronic Green's function[2]:

$$G_\pm(\varepsilon, \mathbf{p}) = [\varepsilon - \varepsilon_0(\mathbf{p}) - \Sigma_\pm(\varepsilon, \mathbf{p})]^{-1} \quad (1)$$

($\epsilon_0(\mathbf{p})$—energy of interacting electrons, $\Sigma_\pm(\epsilon, \mathbf{p})$—self-energy part, the plus signs and minus signs denote the spin polarization, and we shall henceforth take G to mean the Green's-function component corresponding to the conduction-electron band).

We separate from the general expression for $\Sigma_\pm$ that part which corresponds to a non-analytic contribution of diagrams containing the interaction of electrons with spin waves, denoting it by $\Sigma'_\pm(\epsilon, \mathbf{p})$. Since we are interested in the spectrum of the conduction electrons in the region of energies $\sim \Theta$, and since the characteristic feature determining the dependence of $\Sigma'_\pm$ on ϵ is the maximum spin-wave energy $\sim \Theta$, and the scale of the dependence of $\Sigma'_\pm$ on p is the Fermi momentum, in the main region of variables ϵ and $\mathbf{p}$ considered here the function $\Sigma'_\pm(\epsilon, \mathbf{p})$ depends little on p, and we therefore assume that

$$\Sigma_\pm'(\varepsilon, |\mathbf{p}|) \cong \Sigma_\pm'(\varepsilon, p_\pm) \equiv \Sigma_\pm'(\varepsilon).$$

Further calculations will show that

$$\left|\frac{\Sigma_\pm'(\varepsilon) - \Sigma_\pm'(0)}{\varepsilon}\right| \ll 1 \text{ when } |\varepsilon| \gg \Theta.$$

In this connection we introduce the function

$$\tilde{G}_\pm(\varepsilon, \mathbf{p}) = [\varepsilon - \varepsilon_0(\mathbf{p}) - (\Sigma_\pm(\varepsilon, \mathbf{p}) - \Sigma_\pm'(\varepsilon) + \Sigma_\pm'(0))]^{-1}, \quad (2)$$

which is thus equal to the Green's function $G_\pm(\epsilon, \mathbf{p})$ when $|\epsilon| \gg \Theta$, accurate to quantities $\sim \Theta/|\epsilon|$. We assume further that the difference $\Sigma_\pm(\epsilon, \mathbf{p}) - \Sigma'_\pm(\epsilon)$ is analytic in the lowest order in ϵ/ϵ_F. Using the pole expression for $\tilde{G}$:

$$\tilde{G}_\pm(\varepsilon, \mathbf{p}) = \frac{a_\pm}{\varepsilon - v_\pm(|\mathbf{p}| - p_\pm) + i\delta \operatorname{sign} \varepsilon}, \quad \delta \to +0, \quad (3)$$

we represent the G-function near the pole in the following form:

$$G_\pm(\varepsilon, \mathbf{p}) = \frac{a_\pm}{\varepsilon - v_\pm(|\mathbf{p}| - p_\pm) - a_\pm(\Sigma_\pm'(\varepsilon) - \Sigma_\pm'(0))} \quad (4)$$

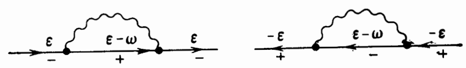
The quantity $v_\pm$ which enters in (3) and (4) is the velocity of the Fermi excitations when $\Theta \ll |\epsilon| \ll \epsilon_F$

As is well known, the self-energy part has the same analytic properties as $G(\epsilon, \mathbf{p})$, and there exists for it a dispersion relation connecting to the real and imaginary parts (see, for example,[3]):

$$\operatorname{Re}(\Sigma_\pm(\varepsilon) - \Sigma_\pm(0)) = \frac{\varepsilon}{\pi} \int_{-\infty}^{\infty} \frac{d\varepsilon' \operatorname{Im} \Sigma_\pm(\varepsilon') \operatorname{sign} \varepsilon'}{\varepsilon'(\varepsilon' - \varepsilon)}. \quad (5)$$

The problem of determining the spectrum of the Fermi excitation thus reduces to a finding of $\operatorname{Im} \Sigma'_\pm(\epsilon)$.

We note that the imaginary part of Σ is determined by the real processes of decay and scattering of the quasiparticles, the initial product of which is one Fermi excitation. When $|\epsilon| \ll \epsilon_F$, owing to the smallness of the statistical weight per electron or per hole, the maximum probability is possessed by processes in



whose final state is contained only one Fermi excitation. Such processes are the emission of a spin wave by an electron with $s_z = -1/2$ or a hole with $s_z = 1/2$. The corresponding contribution to $\Sigma'_\pm$ is shown graphically in the figure (the straight lines in the figure represent the electronic Green's function, and the wavy lines the Green's function of the spin waves).

We note that, owing to the finite distance between the Fermi surfaces, these turn out to be threshold processes, and accordingly $\mathrm{Im}\,\Sigma'_-(\epsilon) = 0$ when $\epsilon < \epsilon_0$ and $\mathrm{Im}\,\Sigma'_+(\epsilon) = 0$ when $\epsilon > -\epsilon_0$ ($\epsilon_0 \approx \omega(\Delta)$, $\omega(k)$—energy of spin wave with momentum k).

Following reasoning similar to that used in[4] to obtain the damping of the quasiparticles as a result of fermion-fermion interaction, we arrive at the following equations for $\Sigma'_\pm$:

$$\mathrm{Im}\,\Sigma_-'(\varepsilon) = -\int_0^\varepsilon \frac{d\varepsilon'}{\pi}\int\frac{d\mathbf{p}'}{(2\pi)^3}|g(\mathbf{p},\mathbf{p}-\mathbf{p}')|^2 \times \mathrm{Im}\,G_+(\varepsilon',\mathbf{p}')\,\mathrm{Im}\,D(\varepsilon-\varepsilon',\mathbf{p}-\mathbf{p}'), \tag{6}$$

$$\mathrm{Im}\,\Sigma_+'(\varepsilon) = -\int_0^{-\varepsilon} \frac{d\varepsilon'}{\pi}\int\frac{d\mathbf{p}'}{(2\pi)^3}|g(\mathbf{p}',\mathbf{p}'-\mathbf{p})|^2 \times \mathrm{Im}\,G_-(\varepsilon',\mathbf{p}')\,\mathrm{Im}\,D(\varepsilon'-\varepsilon,\mathbf{p}'-\mathbf{p}) \tag{7}$$

(here $D(\omega, \mathbf{k})$ is the Green's function of the spin waves and $g(\mathbf{p}, \mathbf{k})$ is the vertex part corresponding to the scattering of an electron by a spin wave).

Using (4) and the expression for the spin-wave Green's function

$$D(\omega,\mathbf{k}) = [\omega - \alpha k^2 + i\delta]^{-1}, \quad \delta \to +0$$

($\alpha \sim \Theta a^2$, a—interatomic distance), which is valid if we neglect magnetic anisotropy, the interaction of the spin waves with one another, and the singularities due to the magnon-fermion interaction, we get

$$\mathrm{Im}\,\Sigma_-'(\varepsilon) = -\pi\int_0^\varepsilon d\varepsilon'\int\frac{d\mathbf{p}'}{(2\pi)^3}|g(\mathbf{p},\mathbf{p}-\mathbf{p}')|^2 a_+ \times \delta(\varepsilon' - v_+(|\mathbf{p}'|-p_+) - a_+(\Sigma_+'(\varepsilon')-\Sigma_+'(0)))\,\delta(\varepsilon-\varepsilon'-\alpha(\mathbf{p}-\mathbf{p}')^2), \tag{8}$$

$$\mathrm{Im}\,\Sigma_\pm'(\varepsilon) = \pi\int_0^{-\varepsilon} d\varepsilon'\int\frac{d\mathbf{p}'}{(2\pi)^3}|g(\mathbf{p}',\mathbf{p}'-\mathbf{p})|^2 a_- \times\delta(\varepsilon' - v_-(|\mathbf{p}'|-p_-) - a_-(\Sigma_-'(\varepsilon')-\Sigma_-'(0)))\,\delta(\varepsilon'-\varepsilon-\alpha(\mathbf{p}'-\mathbf{p})^2). \tag{9}$$

The weakness of the damping due to the processes not considered here, and the vanishing of $\mathrm{Im}\,\Sigma'_+(\epsilon)$ when $\epsilon > 0$ and of $\mathrm{Im}\,\Sigma'_-(\epsilon)$ when $\epsilon < 0$, has enabled us to represent $\mathrm{Im}\,G_\pm$ in the last formulas in the form of δ functions.

It is easy to see that, accurate to quantities $\sim \Theta/\epsilon_F$, the right-hand sides of (8) and (9) do not depend on the concrete form of $\Sigma'_\pm(\epsilon)$. Indeed, carrying out integration with respect to $|\mathbf{p}'|$, we obtain

$$\mathrm{Im}\,\Sigma_-'(\varepsilon) = -\pi\int_0 d\varepsilon'\int\frac{do_{\mathbf{p}'}}{(2\pi)^3}|g(\mathbf{p},\mathbf{p}-\mathbf{p}_+(\varepsilon'))|^2 a_+ \cdot\delta(\varepsilon-\varepsilon'-\alpha(\mathbf{p}-\mathbf{p}_+(\varepsilon'))^2), \tag{10}$$

$$\mathrm{Im}\,\Sigma_+'(\varepsilon) = \pi\int_0^{-\varepsilon} d\varepsilon'\int\frac{do_{\mathbf{p}'}}{(2\pi)^3}|g(\mathbf{p}_-(\varepsilon'),\mathbf{p}_-(\varepsilon')-\mathbf{p})|^2 a_- \times\delta(\varepsilon'-\varepsilon-\alpha(\mathbf{p}_-(\varepsilon')-\mathbf{p})^2) \tag{11}$$

($|\mathbf{p}_\pm(\epsilon)|$—roots of the equations

$$\varepsilon - v_\pm(|\mathbf{p}_\pm(\varepsilon)| - p_\pm) - a_\pm(\Sigma_\pm'(\varepsilon) - \Sigma_\pm'(0)) = 0,$$

$do_{p'}$—element of solid angle in the direction of $\mathbf{p}_\pm(\epsilon')$). But since the region of variation of $|\mathbf{p} - \mathbf{p}_\pm(\epsilon)|$ is of the order of p_F, and when $\epsilon \sim \Theta$ we have $|\mathbf{p}_\pm(\epsilon)| - p_\pm \sim \Theta p_F/\epsilon_F$, the substitution $|\mathbf{p}_\pm(\epsilon)| \to p_\pm$ in (10)–(11) leads to an error $\sim \Theta/\epsilon_F$ in the final result.

Carrying out the integration with respect to the energy and the angular variables in (10) and (11), and also using estimates for $g(\mathbf{p}, \mathbf{k})$[1], we get

$$a_-\mathrm{Im}\,\Sigma_-'(\varepsilon) = \begin{cases} 0, & \varepsilon < \varepsilon_0 \\ -\pi b(\varepsilon-\varepsilon_0), & \Theta' > \varepsilon > \varepsilon_0, \\ -\pi b\Theta', & \varepsilon > \Theta' \end{cases} \tag{12}$$

$$a_+\mathrm{Im}\,\Sigma_+'(\varepsilon) = \begin{cases} \pi b\Theta', & \varepsilon < -\Theta' \\ -\pi b(\varepsilon+\varepsilon_0), & -\Theta' < \varepsilon < -\varepsilon_0. \\ 0, & \varepsilon > -\varepsilon_0 \end{cases} \tag{13}$$

Here

$$\Theta' = \min(\Theta, 4\alpha p_F^2), \qquad \varepsilon_0 = \alpha(p_+ - p_-)^2 \sim \Theta^2/\varepsilon_F,$$
$$b = a_+a_-g^2p_F^2/8\pi^2\alpha p_F^2 \sim 1.$$

From this we get with the aid of (5)

$$a_-[\mathrm{Re}\,\Sigma_-'(\varepsilon) - \Sigma_-'(0)] = -b\left\{(\varepsilon-\varepsilon_0)\ln\left|\frac{\Theta'-\varepsilon}{\varepsilon_0-\varepsilon}\right| + \varepsilon_0\ln\frac{\Theta'}{\varepsilon_0} - \Theta'\ln\left|\frac{\Theta'-\varepsilon}{\Theta'}\right|\right\}, \tag{14}$$

$$a_+[\mathrm{Re}\,\Sigma_+'(\varepsilon) - \Sigma_+'(0)] = -b\left\{(\varepsilon+\varepsilon_0)\ln\left|\frac{\Theta'+\varepsilon}{\varepsilon_0+\varepsilon}\right| - \varepsilon_0\ln\frac{\Theta'}{\varepsilon_0} + \Theta'\ln\left|\frac{\Theta'+\varepsilon}{\Theta'}\right|\right\}. \tag{15}$$

Formulas (12)–(15) retain their form when the sign of the difference of the Fermi momenta $p_+ - p_-$ is reversed, thus demonstrating that they are independent of the mutual orientation of the total magnetic moment of the system and the total magnetic moment of the conduction electrons ($M_S \upuparrows M_0$ or $M_S \uparrow\downarrow M_0$)[1].

It should be noted, however, that the obtained expres-

[1] A different situation takes place in the analysis of the singularities of the spin-wave spectrum. In [1] the spin-wave polarization operator $\Pi(\omega, \mathbf{k})$ was set in correspondence with the diagram representing the splitting of the spin wave into an electron with $s_z = 1/2$ and a hole with $s_z = -1/2$. This is actually not true. Since the magnetic moment of the spin waves is antiparallel to M_0, the operator $\Pi(\omega, \mathbf{k})$ is determined by a diagram describing the decay of a spin wave into an electron with $s_z = -1/2$ and a hole with $s_z = 1/2$ (the author is grateful to A. A. Abrikosov for calling his attention to this circumstance). The spin-wave spectrum obtained in [1] corresponds formally to the case $M_S \uparrow\downarrow M_0$. In order to go over to a description of systems with $M_S \upuparrows M_0$ it is necessary to make the substitution $\Delta p_0 \to -\Delta p_0$, which is equivalent to $\omega \to -\omega$, in the expression for $(\omega, \mathbf{k})$ from [1]. The graphic representation of the spin-wave spectrum near the singularity is then, for the case $M_S \upuparrows M_0$, the mirror reflection of the spectral curve for systems with $M_S \uparrow\downarrow M_0$ relative to the line $\omega = \alpha\Delta^2$. The sign of the singularity in the spin-wave spectrum thus affords an experimental possibility of determining the sign of M_S/M_0.

sions for $\Sigma(\epsilon)$ become meaningless in the vicinity of the point ϵ_0, which is close to the threshold energy of the decay processes. The non-applicability of the obtained formulas for Σ when $\epsilon \sim \epsilon_0$ is connected with the fact that in their derivation we used an expression for the Green's function of the spin wave, not renormalized by the interaction with the conduction electrons. Yet it is precisely in the region $\omega \sim \omega_0$ that the renormalization of $D(\omega, \mathbf{k})$ becomes significant (its consequence is the singularity of the spin-wave spectrum, obtained in[1]).

It can be shown that those diagrams for Σ which contain line complications connected with the interaction between the electrons and the spin waves introduce in the calculated $\Sigma(\epsilon)$ corrections are not small compared with $(|\epsilon| - \epsilon_0)\ln(|\epsilon| - \epsilon_0)$ in the vicinity of $|\epsilon| \sim \epsilon_0$. This points to the need for simultaneously considering the singularities of the spectrum of the spin waves and the Fermi excitations. Separating in the diagrams for Σ the singular element—the loop of Green's function GD $(D(\omega, \mathbf{k}) = [\omega - \alpha\mathbf{k}^2 - \Pi(\omega\mathbf{k})]^{-1}$—we arrive at $|\epsilon| \sim \epsilon_0$ to the following system of equations:

$$\Sigma_-(\varepsilon, \mathbf{p}) = -i\int \frac{d\varepsilon'\, d\mathbf{p}'}{(2\pi)^4} |g(\mathbf{p}, \mathbf{p} - \mathbf{p}')|^2 G_+(\varepsilon', \mathbf{p}') D(\varepsilon - \varepsilon', \mathbf{p} - \mathbf{p}'), \quad (16)$$

$$\Sigma_+(\varepsilon, \mathbf{p}) = -i\int \frac{d\varepsilon' d\mathbf{p}'}{(2\pi)^4} |g(\mathbf{p}', \mathbf{p}' - \mathbf{p})|^2 G_-(\varepsilon', \mathbf{p}') D(\varepsilon' - \varepsilon, \mathbf{p}' - \mathbf{p}), \quad (17)$$

which must be supplemented by the corresponding equation for $\Pi(\omega, \mathbf{k})$:

$$\Pi(\omega, \mathbf{k}) = -i\int \frac{d\varepsilon\, d\mathbf{p}}{(2\pi)^4} |g(\mathbf{p}, \mathbf{k})|^2 G_-(\varepsilon + \omega, \mathbf{p} + \mathbf{k}) G_+(\varepsilon, \mathbf{p}). \quad (18)$$

Inasmuch as $\partial\epsilon_+/\partial p > \partial\epsilon_-/\partial p$ in the case of momenta lying near the corresponding Fermi surfaces (for concreteness we assume that $M_S \uparrow\uparrow M_0$, i.e., $p_+ > p_-$), we can show that the threshold energy of the spin wave $\omega = \epsilon_-$ corresponds to its decay into a hole with $s_z = 1/2$, $\epsilon = 0$, $p = p_+$ and an electron with $s_z = -1/2$, an energy $\epsilon = \epsilon_-$ (we denote its momentum by $\tilde{p}_- > p_-$, the threshold value of the spin-wave momentum is $\Delta_- = p_+ - \tilde{p}_- < \Delta$). The threshold of the process for an electron with $s_z = -1/2$ also takes place at $\epsilon = \epsilon_-$, and corresponds to a decay into an electron with $s_z = 1/2$, $\epsilon = 0$, $p = p_+$ and a spin wave with $\omega = \epsilon_-$ and $k = \Delta_-$. The threshold energy of a hole with $s_z = 1/2$ is larger than ϵ_-.

We take further account of the fact that the singularity in $\Pi(\omega, \mathbf{k})$ and $\Sigma(\epsilon, \mathbf{p})$ is the result of integration in (16)—(18) over the region of variables lying in the immediate vicinity of their threshold values, and we also make the natural assumption that near the threshold Σ and Π depend on their variables via linear combinations of their deviation from the threshold values (it can be verified directly that such a dependence cannot be stronger than $x \ln x$). As a result, the system of integral equations can be reduced to a system of first-order differential equations. Omitting these straightforward but rather cumbersome manipulations, we present the final result:

$$\Sigma_-(\varepsilon, p) - \Sigma_-(\varepsilon_-, \tilde{p}_-) \sim [\varepsilon - \varepsilon_- - \tilde{v}_-(p - \tilde{p}_-)] \times \{\ln[\varepsilon - \varepsilon_- - \tilde{v}_-(p - \tilde{p}_-)]\}^{\Theta/\varepsilon_F}, \quad (19)$$

$$\Pi(\omega, k) - \Pi(\varepsilon_-, \Delta_-) \sim [\omega - \varepsilon_- + \tilde{v}_-(k - \Delta_-)] \times \{\ln[\omega - \varepsilon_- + \tilde{v}_-(k - \Delta_-)]\}^{1-\Theta/\varepsilon_F}, \quad (20)$$
$$\tilde{v}_- \approx (\partial\varepsilon_-/\partial p)_{p=p_-}.$$

Its meaning consists in the fact that the singularity of the spin-wave spectrum remains the same as calculated in[1] without allowance for the renormalization of the electron spectrum. On the other hand, the singularity of the Fermi-excitation spectrum is practically nonexistent, since $\{\ln[\epsilon - \epsilon_- - \tilde{v}_-(p - \tilde{p}_-)]\}^{\Theta/\epsilon_F}$ differs noticeably from unity in an exponentially narrow interval of the variables ϵ and p.

Let us consider the consequences for the Fermi-excitation spectrum, following from the obtained formulas (12)—(15) and (19) for Σ. When $|\epsilon| \gg \Theta$, in accordance with the initial assumption, we have

$$\left|\frac{\Sigma'(\varepsilon) - \Sigma'(0)}{\varepsilon}\right| \gg 1$$

and the excitation energy is $\epsilon_\pm(p) = v_\pm(p - p_\pm)$. In the region $|\epsilon| \lesssim \epsilon_0$ we have

$$\varepsilon_\pm(p) = \tilde{v}_\pm(p - p_\pm), \quad \tilde{v}_\pm = \frac{v_\pm}{1 + b\ln(\varepsilon_F/\Theta)}.$$

When $\epsilon_0 \ll |\epsilon| \ll \Theta$, the excitation velocity depends on the energy and is equal to

$$\frac{\partial\varepsilon_\pm}{\partial p} \cong \frac{v_\pm}{1 + b\ln(\Theta/\varepsilon)}.$$

The damping due to the mechanism considered here takes place for electrons with $s_z = -1/2$ when $\epsilon \gtrsim \epsilon_0$ and for holes with $s_z = 1/2$ when $\epsilon \lesssim -\epsilon_0$. Its magnitude in the region $\epsilon_0 \ll |\epsilon| \ll \Theta$ is equal to

$$\gamma \sim \frac{\pi b|\varepsilon|}{1 + b\ln(\Theta/\varepsilon)}.$$

When $|\epsilon| \sim \Theta$, the electrons with $s_z = -1/2$ and the holes with $s_z = 1/2$ have a damping that is comparable with their energy, and consequently the description of the electronic excitations in terms of quasiparticles is meaningless in this region.

Thus, the interaction with the spin waves leads to a strong renormalization of the velocity of the Fermi excitations of the conduction band in the direct vicinity of the Fermi surface. This renormalization leads to an additional growth of the specific heat of the system compared with a nonferromagnetic metal and its dependence $\sim T \ln T$ in the temperature region $\Theta^2/\epsilon_F \ll T \ll \Theta$.

3. The properties of the energy spectrum of the conduction electrons in ferromagnetic metals, considered by us here, are a manifestation of the general properties of interacting quasi particles in solids. We note, for example, the analogy between the system considered here and a system of interacting electrons and phonons[5]. In both cases there take place two regions of the spectrum $|\epsilon| \ll \omega_0^2/\epsilon_F$ and $|\epsilon| \gg \omega_0$ (ω_0—limiting frequency of spin waves or phonons), where quasiparticles exist with a linear dispersion law and with weak damping, determined essentially by the electron-electron interaction. In either system, when $|\epsilon| \sim \omega_0$ (in an electron-magnon system this takes place for electrons with $s_z = -1/2$ and holes with $s_z = 1/2$), the attenuation is comparable with the energy, and the Fermi excitations do not reduce to long-lived quasiparticles.

We call attention to the fact that, in view of the complexity of the dispersion of conduction electrons near the Fermi surface, it is possible to introduce, besides the limiting values of the two-particle vertex part $\Gamma(p_1, p_2; k)$ with respect to the momentum transfer $k = (\omega, \mathbf{k})$:

$$\Gamma^{\omega}(p_1, p_2) = \lim_{k=0,\ \omega\to 0} \Gamma(p_1, p_2; k),$$
$$\Gamma^{k}(p_1, p_2) = \lim_{\omega=0,\ k\to 0} \Gamma(p_1, p_2; k),$$

which are known from the theory of Fermi liquids[6], also the additional quantity[2)]

$$\widetilde{\Gamma}^{\omega}(p_1, p_2) = \Gamma(p_1, p_2; k)|_{k=0,\ \Theta\ll\omega\ll\epsilon_F},$$

which would coincide with the usual limit Γ^{ω}, accurate to terms of order Θ/ϵ_F, were the loops of the Green's functions $G(p)G(p + k)$ in the diagrams for Γ to be replaced by $\widetilde{G}(p)\widetilde{G}(p + k)$. The convenience of the function $\widetilde{\Gamma}^{\omega}$ lies in the fact that the quantity Γ^k is expressed in terms of $\widetilde{\Gamma}^{\omega}$ and parameters characterizing the Fermi excitations when $\Theta \ll |\epsilon| \ll \epsilon_F$ with the aid of an equation identical to that obtained in[6] for the connection between Γ^k and Γ^{ω}. It can be shown that as a result of this circumstance the magnetic susceptibility of the para process, as well as the statistical limits of other quantities characterizing the kinetic properties of ferromagnetic metals, are determined by the function $\widetilde{\Gamma}^{(\omega)}$ and by the density of the electronic states in the region $\Theta \ll |\epsilon| \ll \epsilon_F$.

In conclusion, I am deeply grateful to I. E. Dzyaloshinskiĭ, whose interest and advice were of great help in the work.

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[2)] The advisability of introducing $\widetilde{\Gamma}^{\omega}$ was pointed out by I. E. Dzyaloshinskiĭ.