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THE VISCOSITY OF WATER AND MERCURY UNDER SHOCK LOADING

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Results are presented of an investigation of the viscosity of water and mercury at pressures between 40 and 440 kbar. The coefficients of dynamic viscosity for water and mercury ($\sim 10^4$ poise) are approximately the same as those for shock-compressed solids previously studied by the authors. The results of the experiments are ascribed to defects produced during the rapid plastic deformation of the matter behind the front of the shock wave.

INTRODUCTION

THE problem of the true structure of liquids cannot at present be considered to have been finally explained. Any theory of viscous flow starts out to some extent from an assumption about the nature of the structure of the liquid. For this reason investigations of the viscosity of a liquid under extremal conditions, in particular at high densities, temperatures and large rates of loading, such as those produced by means of shock waves, are of great scientific interest.

Metallic liquids, for example mercury, are simplest in their structure and their structures have been better investigated than those of other liquids; they are therefore a suitable object to investigate. The structure of pure liquid metals represents a so-called simple liquid, distinct from the nonsimple liquids, a typical representative of which is water. The viscosity of water and mercury under conditions of high static pressure and temperature has so far been most fully investigated by Bridgman[1] for pressures in the range from zero to 10–12 kbar and a temperature range of 0–75°C. It turned out that both for water and mercury the pressure (the increase of density) and the temperature have an opposing effect on the coefficient of viscosity. The coefficient of viscosity increases with increasing pressure (P) and decreases with increasing temperature. Since in shock loading the density and temperature increase simultaneously, it is rather difficult to predict the results of such an experiment for measuring the coefficients of viscosity of water and mercury.

Let us note yet another circumstance which makes it difficult to predict the results of experiments investigating the viscosity of shock-compressed liquids. As was shown by Kornfel'd,[2] in high-velocity deformation (a velocity of deformation of the order of 10^{14} sec^{-1}) water exhibits what is at first sight an altogether untypical strength. A dependence of the mechanical properties of a substance on the relation between the relaxation time of the medium and the characteristic time of application of the loading is not only a property of liquids but is also valid for all condensed media.[3,4] The value of the relaxation time (τ) for water under normal conditions is according to Debye of the order of 10^{-11} sec.[5]

In this paper we present results of an investigation of the viscosity of water under shock loading in the pressure range of 40–150 kbar. The viscosity of mercury has only been investigated at a pressure of 440 kbar. The method of investigating the viscosity of materials behind the shock-wave front (SWF) was proposed by Sakharov and the authors in[6] and is based on the experimental study of small perturbations on the SWF. The idea of the method consists in a comparison of the laws of the development of perturbations on a SWF in two similar experimental devices of appreciably different scales. The viscosity of a series of solids up to pressures of $\sim$1000 kbar has been investigated by this method.[6,7]

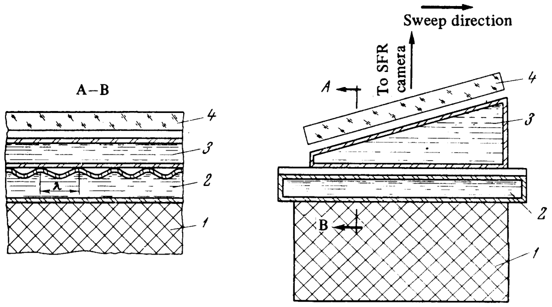

FIG. 1. Experimental setup for investigating the viscosity of shock-compressed mercury. 1—explosive charge, 2, 3—a disc and a wedge filled with mercury, 4—organic glass plate; λ is the wavelength of the perturbations on the SWF.

EXPERIMENTAL SETUP AND RESULTS

The experiments were carried out in a setup described in detail in[6]. The experimental apparatus is shown schematically in Fig. 1. In the case of water we studied the development of perturbations on the SWF of two wavelengths $\lambda_1 = 1$ cm and $\lambda_2 = 2$ cm with a relative initial amplitude $a_0/\lambda = 0.03-0.18$ (where a_0 is the initial amplitude of the perturbations). The development of perturbations on the SWF in mercury was investigated for $\lambda_1 = 0.5$ cm and $\lambda_2 = 1$ cm with an amplitude $a_0/\lambda = 0.22$. In both instances the development of perturbations on the SWF was investigated in two experimental setups which were geometrically similar with respect to λ. The observed difference is caused only by the viscosity of the investigated material.[6]

The experimental setups included: an explosive charge (1 on Fig. 1)—the generator of a plane wave in the investigated material, a disc with grooves (2) with a sinusoidal cross-section profile (to produce perturbations on the SWF), and a sample in the form of a

Substance	P, kbar	σ	D, km/sec	T, °K	ε, %	$\dot{\varepsilon}\cdot10^{-5}$, sec^{-1}	Δx, in units of S/λ	k	$\eta\cdot10^{-4}$, poise	Point on Fig. 2
Water	40	1.44	3.61	470	110	2.5	< 0.01	13	< 0.08	*1*
	80	1.59	4.64	700	44	1,3	0.18±0.02	15	1.8±0.2	*2*
	120	1,70	5.40	980	30	1	0,23±0,03	15	2,9±0.4	*3*
	150	1.76	5.88	1200	19	0,7	0.16±0.03	15	2,2±0,4	*4*
Mercury	440	1.39	3.39		140	6	0.14±0.04	114	0.8±0,2	

wedge (3), in order to record in a single experiment the perturbation at several consecutive times. The water and the mercury were poured into wedge-shaped cells made either of organic glass (in the case of the water) or of copper (in the case of mercury). In the latter case the disc with grooves into which the mercury was poured was also made of copper. In both instances the thickness of the walls of the wedges and of the disc with grooves were chosen to be 0.05λ, i.e., they were chosen small relative to λ and they were therefore not taken into account. In order to produce perturbations on the SWF in water, use was made of a disc with grooves made of paraffin. It follows from the work of Al'tshuler[8] that the dynamic adiabat of paraffin is close to the dynamic adiabat of water. Nevertheless this introduces a certain difference between the experimental conditions and the conditions assumed in the calculations of Zaĭdel'.[9] However, specially set up experiments in which the paraffin was replaced with lead (with constant pressure in the water) showed within experimental error that in both instances perturbations on the SWF in water developed in the same way.

The coefficient of the dynamic viscosity η was calculated according to the formula[6,9]

$$\eta = \frac{\rho D \Delta x}{k(1/\lambda_1 - 1/\lambda_2)},$$

where ρ is the density of the material behind the SWF, D is the wave velocity of the unperturbed SWF, k is a constant calculated by a method indicated in[6,9], Δx is the phase shift between the curves of the development of a perturbation on the SWF with λ_1 and λ_2 plotted in the coordinates $y = a(t)/a_0$ and $x = S(t)/A$ (S is the path traversed by the shock wave along the cold material, and a and a_0 are the initial and current amplitudes of the perturbations respectively).

The obtained experimental $y = f(x)$ curves for water and mercury are of the same nature as those for the solids previously investigated[6,7]: the perturbations oscillate and are damped in time. As in the case of solids, for the investigated λ the experimentally observed value of Δx did not exceed 0.3.

The table lists certain parameters (the pressure P, the compression σ, and the temperature T) of shock-compressed water and mercury and the obtained coefficients of viscosity. The values of η are cited with the maximum error which is the result of the inaccurate determination of the value of Δx. In calculating the parameters of water and mercury behind the SWF use was made of data from the work of Rice and Walsh[10,11]

DISCUSSION OF RESULTS

Let us note the peculiarities of the conditions under which the measurements of the coefficients of viscosity of water and mercury are carried out. The investigated materials are compressed and heated by the shock wave. The artificially imposed perturbations, characterized by a periodicity along the surface of the SWF, constitute on this surface a sort of a "ripple" and bring the substance behind the SWF at a depth of the order of λ into oscillatory motion. Thus the perturbed flow leads to a rapid deformation of the substance behind the SWF; the orders of magnitude of the deformation and the rate of deformation are given respectively by the relations $\epsilon = 2\pi a_0/\lambda$ and $\dot{\epsilon} = 2\pi\epsilon D/\lambda$.[7] It follows from the table that in the experiments carried out the deformation was of the order of 20–140 percent and the rate of deformation was $(1-6) \times 10^5$ sec^{-1}. Under these conditions, in the range of densities ρ of 159–1.76 g/cm^3 (P = 80–150 kbar) and temperatures T = 700–1200°K, the coefficient of the dynamic viscosity of water is practically constant, $\eta \sim 10^4$ poise. We note that the obtained values of η for water and mercury are the same as those of shock-compressed aluminum and lead at pressures of 300–400 kbar.[7] It is characteristic that the values of η for aluminum and lead were obtained at approximately the same values of ϵ and $\dot{\epsilon}$ as those of shock-compressed water and mercury. In our opinion, it is precisely this which can be used to explain the approximate equality of the coefficients of dynamic viscosity of shock-compressed aluminum, lead, sodium chloride, water, and mercury, as well as those of other substances, cited in the work of Sakharov et al.,[6] which are so different under normal conditions.

The noncontradictory nature of the proposed connection of the viscosity of shock-compressed aluminum with point defects appearing in the rapid deformation of the aluminum behind the SWF was shown in[7]. It is well known that the number of defects produced during the deformation of solids depends weakly on the type of material and is basically determined by the values of ϵ and $\dot{\epsilon}$. This is at least correct for metals and ionic crystals.[12] Since in our experiments investigating the viscosity of various shock-compressed solids ϵ and $\dot{\epsilon}$ were about equal, one can expect equal numbers of defects to appear and as a result one can expect approximately equal values of the coefficients of viscosity. If the hypothesis proposed in[7] is also correct for shock-compressed liquids, then one can state that under conditions of rapid deformation the same number of point defects ("holes") is produced in shock-compressed water and mercury as in the shock-compressed solids which we have investigated. The concept of "holes" in a liquid was introduced by Frenkel.[13] Apparently this concept is also valid for shock-compressed liquids. This is confirmed by the results of the experiments of Brish et al.[14] carried out for the purpose of measuring the resistivity of water behind a SWF (100 kbar pressure). It turned out that the initial resistivity of water (10^5–7.6×10^2 ohm-cm) does not influence the resistivity behind the SWF (2 ohm-cm). In our opinion this attests to the fact that the SWF in the water is a powerful generator of defects which determine the conductivity of shock-compressed water.

The applicability of the proposed method of investigation of the viscosity of materials behind the SWF to the estimate of the phase constitution of materials at high pressures (densities) and temperatures has been shown in[6,7]. It turns out that under certain thermodynamic conditions behind the SWF the law of the development of perturbations behind the SWF does not depend on λ; this indicates a sharp decrease in the viscosity of the material. An analogous situation has been determined in this work for water on decreasing the pressure from 80 to 40 kbar, the coefficient of dynamic viscosity of water being then at least by an order of magnitude smaller than the value of η of water at $P = 80$ kbar. In order to understand the connection between the obtained values of η and the phase constitution of water, we turn to Fig. 2 which shows in the T-P plane the curve of the phase equilibrium of water obtained under static conditions[15] and the dynamic adiabat of water[10] on which the points indicate the conditions which correspond to those of the experiments.

It is seen from Fig. 2 that the main part of the experiments with water is carried out under conditions when the water is, in accordance with static experiments, a liquid; only one point at $P = 40$ kbar is in the region of the existence of two phases—liquid and ice. Nevertheless, for $P = 80$–150 kbar water is characterized by the same coefficient of dynamic viscosity as that of unmelted solids behind the SWF. One can therefore conclude that shock-compressed water in the range $P = 80$–150 kbar has a relaxation time $\tau > 1/\dot{\epsilon} \sim 10^{-5}$ sec and for this reason behaves in the indicated experiments effectively like a solid. On the basis of what has been said, it is possible to conclude that on increasing the density of water up to 1.76 g/cm^3 (with a simultaneous increase of the temperature to 1200°K) the relaxation time of water increases from 10^{-11} sec to a value of at least 10^{-5} sec. On the other hand, the coefficient of viscosity is proportional to the relaxation time τ:

$$\eta = \rho\tau(C_\infty^2 - C_0^2),$$

where C_0 and C_∞ are the equilibrium and nonequilibrium speeds of sound.[16] Assuming that $C_\infty^2 - C_0^2 \sim C_0^2$, we find $\eta \sim \rho\tau C_0^2$. Substituting $\rho \sim 1$ g/cm^3, $\tau \sim 10^{-5}$ sec, and $C_0 \sim 10^5$ cm/sec, we obtain $\eta \sim 10^5$ poise, which is in approximate agreement with the results of the experiment.

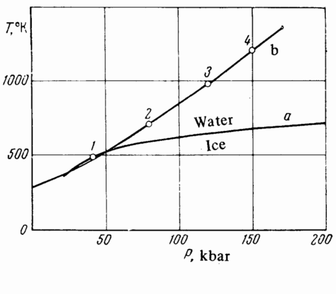

FIG. 2. T-P diagram of shock-compressed water; a—phase equilibrium curve of water under static conditions [15]; b—dynamic adiabat of water. [10]

VISCOSITY AND THE WIDTH OF THE SHOCK-WAVE FRONT

A real SWF is a transition layer of finite thickness on which there occurs a change of all quantities that characterize the state of the material. In order to determine the structure and thickness of the transition layer, one must take into account the viscosity and thermal conduction of the material in the transition zone.

Estimates for gases, confirmed by experiments,[17] showed that for weak shock waves the width of the SWF amounts to several mean free paths. For solids compressed by a shock wave to pressures of about several hundred kilobars, there are no such estimates in the literature, since the coefficients of viscosity and thermal conduction of these solids at high densities and temperatures were not known.

The viscosity of a material plays the principal role in effecting the irreversible shock compression. Zel'dovich and Raĭzer[18] showed that without account of the viscosity one cannot produce a continuous distribution of all thermodynamic quantities in the SWF. It is well known that the flow of a viscous incompressible liquid is characterized by the Reynolds number $R = \rho U\delta/\eta$, where ρ is the density of the material, U and δ—the velocity and characteristic dimension of the flow. It is natural to assume that in the SWF the inertial forces are in equilibrium with the viscous forces, i.e., $R \sim 1$. In this case $\delta = \eta/\rho U$ characterizes the width of the SWF. Such a very simple estimate carried out for water (as well as for the solids which we investigated) indicates that in the pressure range 100–1000 kbar $\delta \sim 10^{-1}$–10^{-2} cm.

Neglecting thermal conduction and volume viscosity, one can make use of the following expression for the width of the front of a weak shock wave in a liquid with a constant coefficient of viscosity[16]:

$$\delta = \frac{16}{3}\frac{V^3}{C^3}\frac{\eta}{(\partial^2 V/\partial P^2)_S P}, \tag{1}$$

where V and C are the specific volume of the material and the velocity of sound behind the SWF respectively, and P is the pressure on the SWF. For a weak shock wave $C \sim D$ and $(\partial^2 V/\partial P^2)_S \sim (\partial^2 V/\partial P^2)_H$. Taking the relationship between the wave (D) and mass (U) velocity behind the SWF in the form $D = C_0 + \alpha U$, we have from (1)

$$\delta = \frac{4}{3}\frac{\eta}{\alpha\rho U}. \tag{2}$$

We note that (2) coincides in order of magnitude with the expression for δ obtained above from dimensionality considerations. The difference is only in the coefficient $4/3\alpha$ which is for the majority of investigated materials of the order of unity.

On the other hand, the experiments of Zel'dovich et al.[19] on the reflection of light by a SWF in water and Plexiglas at pressures of 100–200 kbar show that the width of the SWF $\delta_1 = (3-6) \times 10^{-6}$ cm. In order to explain this apparent contradiction, one should, in our opinion, take into account the fact that in[16] the expression for the width of the SWF was obtained under the assumption that in the transition zone η is constant. As our experiments have shown, the coefficient of dynamic viscosity of water in the range $P = 80-150$ kbar is $\eta \sim 10^4$ poise, whereas on decreasing P from 80 to 40 kbar η decreases at least by an order of magnitude.

Proceeding in analogy with[7], one can by using the Frenkel-Eyring theory of the viscosity of liquids set up an interpolation formula for the dependence of η on σ and T which describes satisfactorily the experimental data presented in this paper and the viscosity of water under normal conditions:

$$\eta = 1{,}5 \cdot 10^5 \sigma \exp\left(\frac{100\sigma^6 - 4600}{T}\right), \qquad (3)$$

where η is in poise and T in °K. Here we have made use of the fact that for water $P \sim \sigma^7$.[18]

The obtained $\eta(\sigma, T)$ dependence allows one to consider the structure of a SWF for any state of the material before and behind the front. The difference between the structure of a SWF with variable $\eta(\sigma, T)$ and the structure of a SWF with constant η can be characterized by values of δ [see (2)] calculated from values of $\eta(\sigma, T)$ corresponding to the state of the material before and behind the SWF.

Thus, for instance in a shock wave with pressures of 0–20 kbar η changes from 10^{-2} to 3 poise [the latter value was obtained from (3)], corresponding to values of δ from $\delta_2 = 10^{-7}$ to $\delta_3 = 10^{-5}$ cm. An increase of the pressure in the shock wave leads only to an increase in δ_3. As a result of this one can represent the structure of the SWF in water in the form of a rapid rise of P (with a width of $\sim 10^{-6}$ cm) from zero to some pressure of 20–30 kbar, followed by a slow increase of P (with a width larger than 10^{-5} cm and up to 10^{-1} cm at P = 100 kbar) to the final pressure. The existence of a section with a steep increase of the pressure and density on the SWF in water has been observed experimentally by Zel'dovich et al. by reflecting light from a SWF[19].

Zel'dovich et al.[19] noted the discrepancy of the refractive indices for shock-compressed water calculated for pressures $P > 40$ kbar by the photometric and geometric methods. In our opinion, this serves as a confirmation of the hypothesis proposed by us in 1961[19] concerning the "breakdown" of the SWF due to the increasing viscosity of water in the SWF.

In this way one can also explain the absence of reflection of light from the front of a second shock wave proceeding in the water compressed by the first shock wave.[20] Thus, for example, in the propagation of two shock waves with pressures of 0–20 and 20– 40 kbar the first shock wave is characterized, as was noted above, by $\delta_2 = 10^{-7}$ and $\delta_3 = 10^{-5}$ cm; for the second shock wave following the first $\delta_2 = 10^{-5}$ cm and $\delta_3 = 10^{-4}$ cm respectively [the latter value was obtained by starting from Eqs. (3) and (2)]. From this it follows that light is reflected by the front of the first shock wave ($\lambda/4\pi \sim 5 \times 10^{-6}$ cm $\sim \delta_2$, where λ is the wavelength of the incident light), whereas it is not reflected by the second shock wave. An analogous conclusion referring to experiments with more intense shock waves will be better grounded, because in this case one will be able to make use of values of η directly measured in the experiment, instead of the interpolation formula (3).

It should be noted that the method of determining the structure of a SWF presented above leads to results which are only of a qualitative nature since the hydrodynamic approach is only justified for weak shock waves.[16]

In conclusion we consider it our duty to express our sincere gratitude to A. D. Sakharov, Ya. B. Zel'dovich, and G. I. Barenblatt for valuable advice and discussions.

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