THEORY OF RESONANCE BRAGG SCATTERING OF γ QUANTA BY REGULAR CRYSTALS

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A detailed analysis is presented of the influence of suppression of inelastic scattering channels[1] on the character and energy dependences of Bragg reflection of γ quanta from perfect single crystals in the presence of resonance nuclear and Rayleigh electron scattering mechanisms. It is shown that if the Bragg condition is satisfied exactly total reflection should be possible (and hence suppression of inelastic reaction channels) even in the case of a pure resonance interaction between the γ quanta and the nuclei, irrespective of the relation between the cross sections for the elastic and inelastic processes. A general analytic expression is derived for the (angular) integral intensity of Bragg reflection for arbitrary relations between the imaginary and real parts of the dynamic equation coefficients. The influence of interference between nuclear and electron scattering on the curves for reflection from perfect crystals is analyzed in detail. A brief analysis of interference is presented for the case of mosaic crystals and the interference curves are compared for perfect and mosaic crystals.

1. INTRODUCTION

THE dynamic theory of γ quanta moving in ideal crystals and interacting in purely resonant fashion with the nuclei was developed in[1]. It was shown there that under certain conditions it is possible to suppress fully (or partially) the inelastic scattering channels in such systems, i.e., to alter radically the character of the nuclear reaction. This effect of suppression of the inelastic channels is connected with the fact that when the particles move in the crystal in a direction close to the Bragg angle, a major change occurs in the particle wave function. As a result, the amplitude of production of the excited nucleus turns out to be close to zero, causing suppression of the inelastic channels of the reaction. This effect remains in force also when the atoms in the crystal oscillate, and also for arbitrary values of the spin of the ground state of the nuclei and the resonant-isotope concentration. A similar phenomenon takes place also in the case of resonant scattering of neutrons in the crystal[2].

In[1] the problem was considered only for the case of passage of γ quanta through the crystal. It is natural to ask how this effect influences the character of the Bragg reflection of Mossbauer resonant radiation. The answer to this question is all the more interesting since experimental studies of resonant Bragg scattering using Mossbauer radiation have been recently initiated. The corresponding problem is analyzed in detail in Sec. 2.

Gamma-quantum scattering in a crystal is characterized by interference between the resonant nuclear and Rayleigh electron scattering. The interference was studied in a number of investigations at the University of Birmingham (see, e.g.,[3-5]). The picture of the interference was clearly outlined in a recent paper by Voĭtovetskiĭ and co-workers[6].

In Sec. 3 we analyze the influence of the dynamic character of scattering in a thick crystal on the picture of Bragg reflection in the presence of interference between the Rayleigh and nuclear scattering.

Scattering differs greatly in thick ideal and mosaic crystals. To display this difference, we present in Sec. 4 a brief analysis of the influence of the mosaic structure. In Sec. 5 we compare the pictures of scattering by ideal and mosaic crystals, using as examples reflections from tin and iron crystals. We emphasize that the experiments performed to date were made on crystals with a highly developed mosaic structure.

2. RESONANT BRAGG SCATTERING OF GAMMA QUANTA BY A THICK CRYSTAL

A. We consider a crystal in the form of a flat plate of thickness l. For the dynamic problem of the motion of gamma quanta inside such a crystal we can use the results of[1] 1). However, bearing in mind the reflection problem ($\beta < 0$), we must modify the boundary conditions:

$$E_s^{(1)} + E_s^{(2)} = \mathscr{E}_{0s}, \quad E_{1s}^{(1)} \exp(iz_s^{(1)} l) + E_{1s}^{(2)} \exp(iz_s^{(2)} l) = 0; \tag{2.1}$$

$$z_s^{(1,2)} = \varkappa \varepsilon_{0s}^{(1,2)} / \gamma_0. \tag{2.2}$$

(We use the notation of I throughout.)

For an electric field in a crystal with polarization s we have the following expression:

$$\mathbf{E}_s(\mathbf{r}) = \mathscr{E}_{0s} e^{i\varkappa\mathbf{r}} [\mathbf{e}_s \psi_0(\mathbf{r}) + \mathbf{e}_{1s} e^{i\mathbf{K}_1\mathbf{r}} \psi_1(\mathbf{r})], \tag{2.3}$$

$$\psi_0(\mathbf{r}) = [(2\varepsilon_{0s}^{(2)} - g_{00}) \exp(iz_s^{(2)} l + iz_s^{(1)} \mathbf{nr}) - (2\varepsilon_{0s}^{(1)} - g_{00}) \exp(iz_s^{(1)} l + iz_s^{(2)} \mathbf{nr})]$$
$$\times [(2\varepsilon_{0s}^{(2)} - g_{00}) \exp(iz_s^{(2)} l) - (2\varepsilon_{0s}^{(1)} - g_{00}) \exp(iz_s^{(1)} l)]^{-1}, \tag{2.3'}$$

$$\psi_1(\mathbf{r}) = -\beta g_{10}^s [\exp(iz_s^{(2)} l + iz_s^{(1)} \mathbf{nr}) - \exp(iz_s^{(1)} l + iz_s^{(2)} \mathbf{nr})] \cdot$$
$$\times [(2\varepsilon_{0s}^{(2)} - g_{00}) \exp(iz_s^{(2)} l) - (2\varepsilon_{0s}^{(1)} - g_{00}) \exp(iz_s^{(1)} l)]^{-1}. \tag{2.3''}$$

The values of the roots $\epsilon_{0S}^{(1,2)}$ are determined by expression I(3.10). The quantities $g_{\alpha\beta}^{S}$, which are fundamental for the problem considered here, can be represented in the presence of a resonant nuclear and electron scattering in the form

1) This paper will henceforth be cited as I.

$$g_{\alpha\beta}{}^{s}=\frac{4\pi}{\varkappa^{2}v_{0}}\sum_{j}\exp\left[i(\mathbf{k}_{\beta}-\mathbf{k}_{\alpha})\boldsymbol{\rho}_{j}\right]\Big\{\eta\exp\left[-\frac{1}{2}z_{j}(\mathbf{k}_{\alpha})-\frac{1}{2}z_{j}(\mathbf{k}_{\beta})\right]$$
$$\times f_{nj}{}^{s}(\mathbf{k}_{\alpha},\mathbf{k}_{\beta})+\exp\left[-\frac{1}{2}z_{j}(\mathbf{k}_{\alpha}-\mathbf{k}_{\beta})\right]f_{ej}{}^{s}(\mathbf{k}_{\alpha},\mathbf{k}_{\beta})\Big\}\quad(\alpha,\beta=0,1). \tag{2.4}$$

Here f_{nj}^{s} and f_{ej}^{s} are the coherent parts of the amplitudes of nuclear and electronic scattering by the j-th atom in the unit cell, with ($\hbar = 1$)

$$f_{nj}{}^{s}(\mathbf{k}_{\alpha},\mathbf{k}_{\beta})=-\frac{1}{2\varkappa}\frac{\Gamma_{1}}{\omega-\omega_{0}+i\Gamma/2}\frac{2I+1}{2(2I_{0}+1)}p_{n}{}^{s}(\alpha,\beta)C_{j},$$

$$f_{ej}{}^{s}(\mathbf{k}_{\alpha},\mathbf{k}_{\beta})=-r_{0}F_{j}(|\mathbf{k}_{\alpha}-\mathbf{k}_{\beta}|)p_{e}{}^{s}(\alpha,\beta), \tag{2.5}$$

where $C_j = 1$ for the unit-cell sites containing resonantly scattering nuclei, and $C_j = 0$ for the remaining sites (η—concentration of the resonant isotope), r_0—classical radius of the electron, $F_j(q)$—atomic factor of the j-th atom in the unit cell, $p_n^s(\alpha, \beta)$ and $p_e^s(\alpha, \beta)$—polarization factors.

Mossbauer nuclei are characterized by transitions of the type E1, M1 and E2. In this case $p_n^s(\alpha, \beta)$ has the following values:

Transition:	$E1$	$M1$	$E2$
$s=1$:	1	$\cos\varphi_{\alpha\beta}$	$\cos\varphi_{\alpha\beta}$
$s=2$:	$\cos\varphi_{\alpha\beta}$	1	$\cos 2\varphi_{\alpha\beta}$

(2.6)

The quantities $p_e^s(\alpha, \beta)$ coincides with the values $p_n^s(\alpha, \beta)$ for the E1 transition.

Using formula (2.3), we can easily find expressions for the intensities of the reflected (P_{1s}) and transmitted (P_{0s}) γ-quantum fluxes:

$$\frac{P_{1s}}{I_{0s}}=|\beta|\,|g_{10}{}^{s}|^{2}\cdot\left|\frac{1-\exp[i(z_{s}^{(2)}-z_{s}^{(1)})l]}{(2\varepsilon_{0s}^{(1)}-g_{00})-(2\varepsilon_{0s}^{(2)}-g_{00})\exp[i(z_{s}^{(2)}-z_{s}^{(1)})l]}\right|^{2}, \tag{2.7}$$

$$\frac{P_{0s}}{I_{0s}}=4\left|\frac{(\varepsilon_{0s}^{(1)}-\varepsilon_{0s}^{(2)})\exp(iz_{s}^{(2)}l)}{(2\varepsilon_{0s}^{(1)}-g_{00})-(2\varepsilon_{0s}^{(2)}-g_{00})\exp[i(z_{s}^{(2)}-z_{s}^{(1)})l]}\right|^{2}. \tag{2.8}$$

B. Let us consider the case of a thick crystal, for which the condition

$$\operatorname{Im}(z_{s}^{(2)}-z_{s}^{(1)})l=\frac{\varkappa l}{\gamma_{0}}\operatorname{Im}(\varepsilon_{0s}^{(2)}-\varepsilon_{0s}^{(1)})\gg 1 \tag{2.9}$$

is satisfied (we assume here and throughout that the index 2 pertains to the root with the larger imaginary part). Then

$$\frac{P_{1s}}{I_{0s}}=\frac{|\beta|\,|g_{10}{}^{s}|^{2}}{|2\varepsilon_{0s}^{(1)}-g_{00}|^{2}} \tag{2.10}$$

or, with allowance for the explicit form for $\epsilon_{0s}^{(1)}$, I(3.10),

$$\frac{P_{1s}}{I_{0s}}=\frac{4|g_{10}{}^{s}|^{2}}{|\alpha-2g_{00}\pm[(\alpha-2g_{00})^{2}-4g_{01}{}^{s}g_{10}{}^{s}]^{1/2}|^{2}} \tag{2.11}$$

where $\alpha = \mathbf{K}_1(\mathbf{K}_1 + 2\boldsymbol{\kappa})/\kappa^2$ characterizes the deviation from the Bragg condition ($\mathbf{K}_1/2\pi$—reciprocal lattice vector). The sign in front of the root is chosen such that the imaginary part of the root is negative. (In obtaining (2.11) we have confined ourselves, in order to simplify the problem, to the symmetrical case of scattering, when $\beta = -1$, and assumed that the condition $g_{00} = g_{11}$ is satisfied).

At a γ-quantum energy close to resonant, the imaginary part of the electronic scattering amplitude can be practically always neglected compared with the imaginary part of the nuclear amplitude.

Let the nuclear transition be of type E1 or M1. Then $p_n^s(\alpha, \beta) = 1$, for one of the polarizations, and the nuclear amplitude for this polarization does not depend on the direction of the vectors $\mathbf{k}_\alpha$ and $\mathbf{k}_\beta$. We shall confine ourselves henceforth to crystals for which we can select the reflection in a manner such that

$$\exp(i\mathbf{K}_{1}\boldsymbol{\rho}_{j})=1,$$
$$\exp[-{}^{1}/{}_{2}z_{j}(\mathbf{k}_{0})]=\exp[-{}^{1}/{}_{2}z_{j}(\mathbf{k}_{1})] \tag{2.12}$$

for all atoms in the unit cell. We then obtain from (2.4) and (2.5) for this polarization

$$g_{01}{}^{s}=g_{10}{}^{s},\quad g_{10}^{s''}=g_{00}''\quad(g_{\alpha\beta}{}^{s}=g_{\alpha\beta}^{s'}+ig_{\alpha\beta}^{s''}). \tag{2.13}$$

A direct analysis of (2.11) shows that when (2.13) is satisfied there always exists an angle

$$\alpha_{0}=2(g_{00}'-g_{10}{}^{s'}), \tag{2.14}$$

at which total reflection takes place, i.e.,

$$P_{1s}/I_{0s}=1.$$

We emphasize that this situation is realized for an arbitrary ratio of the amplitudes of the electronic and nuclear scatterings. The result is a direct consequence of the effect of suppression of inelastic channels in complete analogy with the case of Laue diffraction (see I).

For the other polarization in the case of E1 and M1 transitions, and for both polarizations in the case of an E2 transition, the second relation of (2.13) is not satisfied, and as a result we have

$$P_{1s}/I_{0s}<1$$

for all values of α. The maximum possible value of the reflected intensity depends to a considerable degree on the ratio

$$p^{s}=g_{10}^{s''}/g_{00}''. \tag{2.15}$$

Figure 1 shows the dependence of P_{1S}/I_{0S} on α for the case of purely nuclear scattering and for several values of the parameter p^S, corresponding to different values of the polarization factor (continuous curves). The energy of the γ quanta is assumed to coincide with ω_0, in other words, the parameter

$$V=2(\omega-\omega_{0})/\Gamma \tag{2.16}$$

is assumed to vanish. (The abscissas represent the quantity α/g_0, where $g_0 = |g_{00}(\omega = \omega_0)|$.) The curves

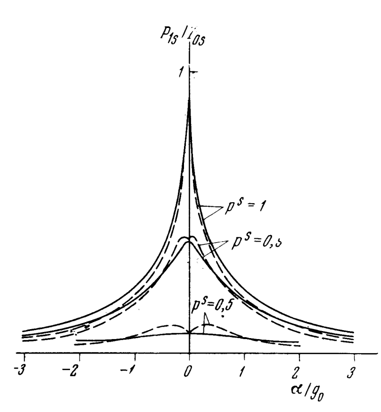

FIG. 1.

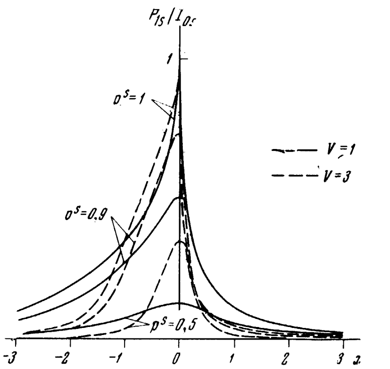

FIG. 2.

demonstrate clearly the difference in the character of the behavior of the reflected intensity as a function of α at small deviations from the Bragg condition, in the case when the inelastic channels are suppressed fully ($p^S = 1$) or partly ($p^S \neq 1$). We note that for small values of α and when $p^S = 1$ we have

$$P_{1s}/I_{0s} = 1 - \sqrt{2|\alpha|/g_0}. \tag{2.17}$$

If $V \neq 0$, then the dependence of P_{1S}/I_{0S} on α becomes asymmetrical. This is clearly seen from the curves of Fig. 2, which shows plots of the reflected intensity against the parameter

$$x = \frac{\alpha - 2(g_{00}' - g_{10}^{s\prime} p^s)}{g_0} \tag{2.18}$$

for $p^S = 1$ and $p^S = 0.9$ and for two values of the parameter V. (It should be noted that the curves remain unchanged when V is replaced by $-V$ and x is simultaneously replaced by $-x$.) This character of the dependence is due to the fact that by varying V we change the ratio of the real and imaginary parts of the scattering amplitude. The sensitivity of the scattering picture to the value of the parameter V leads to a dependence of the reflected intensity on the energy distribution of the incident beam. In the absence of hyperfine splitting in a thin Mossbauer source it can be assumed that this distribution has a Lorentz form

$$I_{0s}(\omega) = I_{0s} \frac{\Gamma'/2\pi}{(\omega - \omega_0')^2 + \Gamma'^2/4} \tag{2.19}$$

where Γ'—width and ω_0'—center of the source line. Figure 1 shows also a plot of P_{1S}/I_{0S} against α, obtained after averaging over the distribution (2.19). The corresponding curves (dashed) were plotted under the very simple assumptions that $\Gamma' = \Gamma$ and $\omega_0' = \omega_0$.

If $\beta \neq -1$, then the picture becomes more complicated. Now in the analysis of (2.10) it is necessary to use the general expression for $\epsilon_{0S}^{(1,2)}$:

$$\varepsilon_{0s}^{(1,2)} = {}^1/_4\{g_{00} - |\beta| g_{11} + |\beta|\alpha \pm [(g_{00} - |\beta| g_{11} + |\beta|\alpha)^2 - 4|\beta|(g_{00}\alpha - \Delta^s)]^{1/2}\}. \tag{2.20}$$

It is easy to see that for arbitrary β the imaginary part of one of the roots vanishes only if

$$\Delta^s = g_{00}g_{11} - g_{01}{}^s g_{10}{}^s = 0. \tag{2.21}$$

The equality (2.21) is satisfied when $p^S = 1$ and the scattering has the purely nuclear character. If, on the other hand, the electronic scattering is significant and $g_{10}^{S\prime} \neq g_{00}'$, then the reflected intensity does not reach unity for any value of α. Consequently, in the case $\beta \neq -1$ the suppression of the inelastic channels is decreased in the simultaneous presence of nuclear and electronic scattering.

It is interesting to note that in a non-cubic crystal the anisotropy of the Mossbauer-effect probability ($\exp[-z(k_0)] \neq \exp[-z(k_1)]$) can lead to $g_{00}'' = |\beta| g_{11}''$ at a certain temperature even when $|\beta| \neq 1$. When $p^S = 1$ we also have $g_{00}'' g_{11}'' = g_{01}^{S\prime\prime} g_{10}^{S\prime\prime}$. (It is assumed throughout that the first of the conditions in (2.12) is satisfied.) An analysis of the expression (2.20) shows that in this case both roots $\epsilon_{0S}^{(1,2)}$ are real when $\alpha_0 = \Delta^{S\prime\prime}/g_{00}''$. Thus, in this case the maximum reflection intensity as a function of the crystal temperature will pass through a maximum, at which $p_{1S}/I_{0S} = 1$.

We have considered so far the case of ideal collimation of incident beam. A real incident beam as a certain finite aperture angle $\Delta\theta$. The integral intensity r_{1S} depends significantly on the ratio of the angular width of the Bragg maximum $\Delta\theta_B$ and of the collimation $\Delta\theta$ of the incident beam.

Let us determine the integral reflected intensity in the case when as before, we confine ourselves to the case of symmetrical reflection ($\beta = -1$) and we assume the following conditions to be satisfied:

$$g_{00} = g_{11}, \qquad g_{01}{}^s = g_{10}{}^s.$$

Since we can put $d\theta = -(2\sin 2\theta_B)^{-1} d\alpha$ near the Bragg angle, it follows that

$$R_{1s} = \int_{-\infty}^{\infty} \frac{P_{1s}(\theta)}{I_{0s}} d\theta = \frac{2|g_{10}{}^s|^2}{\sin 2\theta_B} \int_{-\infty}^{\infty} \frac{d\alpha}{|\alpha - 2g_{00} \pm [(\alpha - 2g_{00})^2 - 4(g_{10}{}^s)^2]^{1/2}|^2}. \tag{2.22}$$

The integral in (2.22) can be expressed in terms of elliptic integrals. As a result we get

$$R_{1s} = \frac{8|g_{10}{}^s|}{3\sin 2\theta_B} F(p^s, u^s), \tag{2.23}$$

where

$$F(p,u) = (u^2 + p^2)^{-3/2}\{(u^2 - 1 - p^2)(u^2+1)^{1/2}\mathrm{E}(k) - {}^3/_4\pi(u^2 - p^2) - (1 - p^2)(u^2+1)^{-1/2}[(2u^2 - 1)\mathrm{K}(k) - 3u^2\Pi_1(-p^2, k)]\}. \tag{2.24}$$

Here $\mathrm{K}(k)$, $\mathrm{E}(k)$, and $\Pi_1(-p^2, k)$ are complete elliptic integrals of the first, second, and third kind, respectively[7,8],

$$u^s = -\frac{g_{10}{}^{s\prime}}{g_{00}''}, \qquad k = \left(\frac{u^2 + p^2}{u^2 + 1}\right)^{1/2}. \tag{2.25}$$

When $p^S = 1$ expression (2.24) reduces to

$$F(1,u) = 1 - \frac{3}{4}\pi \frac{u^2 - 1}{(u^2+1)^{3/2}} - \frac{3}{u^2+1} + \frac{3u}{(u^2+1)^{3/2}} \ln[(u^2+1)^{1/2} + u]. \tag{2.24'}$$

Expressions (2.23) and (2.24) make it possible to analyze the character of the dependence of R_{1S} on V. Let us consider the case of pure nuclear interaction (in this case $u^S = p^S V$). It is easy to see that for large $|V|$ we have

$$R_{1s} \sim 1/|V|.$$

This type of behavior differs strongly from the case of a thin crystal, when the integral intensity, obviously, is proportional to the cross section for the scattering

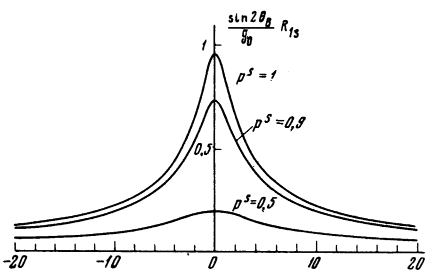

FIG. 3.

by an individual nucleus and when $|V| \gg 1$ we have

$$R_{1s} \sim 1/V^2.$$

Figure 3 shows the corresponding $R_{1S}(V)$ curves for a thick crystal at three values of p^S. It is seen from the figure that in the case of a thick crystal the dependence of the integral intensity $R_{1S}(V)$ on the parameter V is not Lorentzian, and the width of the curve (defined at half the height) increases strongly. Thus, when $p^S = 1$ it is approximately four times larger than in the case of a thin crystal.

3. INFLUENCE OF INTERFERENCE BETWEEN THE RESONANT NUCLEAR AND RAYLEIGH ELECTRONIC SCATTERING

So far we have considered principally resonant nuclear scattering. In this section we shall stop to discuss the features of the Bragg scattering in a thick crystal when interaction between the γ quanta and the electrons is simultaneously present (we shall neglect, however, the imaginary part of the electronic amplitude).

We confine ourselves to the case when conditions (2.12) are satisfied and the scattering has a symmetrical character. To describe the relative rule of the electronic scattering, we introduce the parameter

$$a = -(g_{10}{}^{s\prime})_e / g_0 \tag{3.1}$$

(see (2.4)). When $a \neq 0$, a strongly pronounced interference arises between the electronic and nuclear scattering. This is clearly seen in Fig. 4, which shows $P_{1S}(x)/I_{0S}$ as a function of x for the cases $a = 1$, $p^S = 1$ and 0.9, at three values of the parameter V. The role of the interference is manifest in these curves in their appreciable dependence on the sign of V.

The interference between the electronic and nuclear scattering is clearly pronounced also on the integral

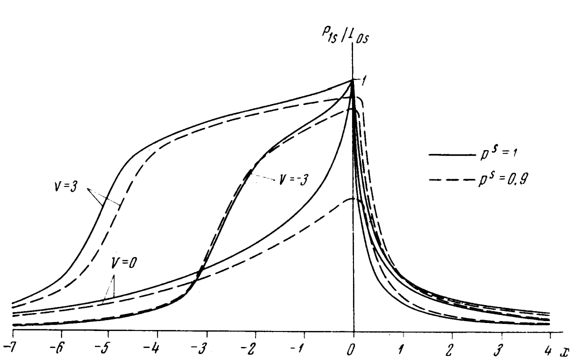

FIG. 4.

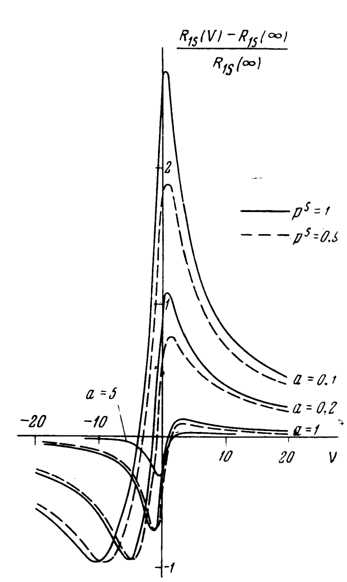

FIG. 5.

intensity of the reflection in the case of broad collimation of the incident beam, as a function of V. The corresponding curves are shown in Fig. 5.

If we compare the presented results with the curves of Fig. 3, then we are struck with the fact that the interference effects are strongly pronounced even at small values of the ratio of the electronic to nuclear amplitude. With increasing a, the maxima of the curves decrease rapidly, and the curves themselves begin to recall more and more the curves of the ordinary Mossbauer absorption. It should be noted, however, that the dip still remains asymmetrical even when $a \gg 1$, and the position of the minimum turns out to be shifted somewhat from $V = 0$ ($V_{min} \approx -0.2$). This result follows directly from formulas (2.23) and (2.24), which for $a \gg 1$ assume the following form:

$$R_{1s} = \frac{8g_0}{3\sin 2\theta_B}\left(a + \frac{p^s V - 3\pi/4}{V^2 + 1}\right) \tag{3.2}$$

4. INFLUENCE OF THE MOSAIC STRUCTURE OF THE CRYSTAL

If the crystal has a noticeable mosaic structure, then the picture of the scattering by a thick crystal changes noticeably compared with the value obtained in the preceding sections. We again consider a crystal in the form of a flat plot consisting of blocks of small dimensions with characteristic thickness $d_0 \ll 1/\kappa|g_{00}|$. The dynamic effects in scattering by an individual block do not play any role here (primary extinction is neglected). Let the characteristic disorientation angle of the blocks δ be large compared with $1/\kappa d_0$. Then all the blocks can be regarded as scattering incoherently and we can use the simplified system of equations for the intensities $P_{0S}(y, \omega)$ (see, for example,[9]):

$$\frac{dP_{0s}(y,\omega)}{dy} = -\left(\frac{\mu(\omega)}{\gamma_0} + r_s(\omega)\right)P_{0s}(y,\omega) + r_s(\omega)P_{1s}(y,\omega),$$

$$\frac{dP_{1s}(y,\omega)}{dy} = \left(\frac{\mu(\omega)}{\gamma_0} + r_s(\omega)\right)P_{1s}(y,\omega) - r_s(\omega)P_{0s}(y,\omega) \tag{4.1}$$

with boundary conditions

$$P_{0s}(0,\ \omega)=I_{0s}(\omega), \qquad P_{1s}(l,\ \omega)=0.$$

(Here it is assumed again that $\beta = -1$.)

Let the collimation angle of the primary beam $\Delta\theta$ be smaller than δ, but much larger than the interval of angles characteristic of reflection from an individual block. We then have for the reflection coefficient $r_S(\omega)$ per unit thickness

$$r_s(\omega)=\frac{\pi\varkappa}{2\gamma_0\delta}\frac{|g_{10}{}^s|^2}{\sin 2\theta_B}. \tag{4.2}$$

The linear absorption coefficient $\mu(\omega)$ is given simply by (see I)

$$\mu(\omega)=\varkappa g_{00}''. \tag{4.3}$$

The solution of the system (4.1) yields for the reflected intensity the expression

$$\frac{P_{1s}(0,\omega)}{I_{0s}(\omega)}=\frac{r_s\gamma_0}{\mu}\left[1-\exp\left(-\frac{2l\mu}{\gamma_0}\sqrt{1+\frac{2r_s\gamma_0}{\mu}}\right)\right]$$
$$\times\left[\left(1+\frac{r_s\gamma_0}{\mu}+\sqrt{1+\frac{2r_s\gamma_0}{\mu}}\right)-\left(1+\frac{r_s\gamma_0}{\mu}-\sqrt{1+\frac{2r_s\gamma_0}{\mu}}\right)\right.$$
$$\left.\cdot\exp\left(-\frac{2l\mu}{\gamma_0}\sqrt{1+\frac{2r_s\gamma_0}{\mu}}\right)\right]^{-1}, \tag{4.4}$$

which depends only on two dimensionless parameters, $r_S\gamma_0/\mu$ and $2l_\mu/\gamma_0$.

In a thick crystal $(2l\mu_0^{-1}(1+2r_S\gamma_0/\mu)^{1/2}\gg 1)$ there remains only a dependence on a single parameter, and expression (4.4) takes the simple form

$$\frac{P_{1s}(0,\omega)}{I_{0s}(\omega)}=\frac{r_s\gamma_0/\mu}{1+r_s\gamma_0/\mu+(1+2r_s\gamma_0/\mu)^{1/2}}. \tag{4.5}$$

In the case of small secondary extinction, corresponding to the condition $r_S\gamma_0/\mu\ll 1$, we get from (4.4)

$$\frac{P_{1s}(0,\omega)}{I_{0s}(\omega)}=\frac{r_s\gamma_0}{2\mu}\left[1-\exp\left(-\frac{2l\mu}{\gamma_0}\right)\right]. \tag{4.6}$$

Such a formula was obtained by O'Connor and Black[5], who also analyzed the interference of the nuclear resonant and electronic scattering essentially for the case of mosaic crystals. We note that in the case of a small block disorientation angle, when $\delta<|g_{10}^S|^2/g_{00}''$, formula (4.6) cannot be used, and the calculation must be carried out in accordance with the general formula (4.4) or (at large values of l) in accordance with (4.5). It must be emphasized that such a situation is perfectly realistic, on the one hand, and on the other hand it leads to an appreciable change of the interference picture in the case of scattering as compared with the case (4.6).

5. DISCUSSION OF RESULTS

An analysis of the results obtained above reveals an appreciable influence of dynamic effects, particularly the effect of suppression of the inelastic channels, on the Bragg scattering of Mossbauer γ quanta. This is particularly easy to trace by comparing the results obtained for ideal and mosaic crystals of the same substance. Figure 6 shows plots of the integral coefficient of reflection $R(V)/R(\infty)$ of unpolarized radiation for the case of a crystal of metallic tin containing 100% of the isotope Sn^{119}. We have considered here the reflections [200] and [600], and the temperature was assumed equal to 77°K. The presence of a sharp peak on the curve, corresponding to reflection from an ideal crystal (the solid curves on Fig. 6, and also on Figs. 7 and 8, correspond to reflection from an ideal crystal, and the dashed curve to a reflection from a mosaic crystal), is connected to a decisive degree with dynamic effects. The increase of the peak on going from the first order of reflection to the third is naturally connected with the decrease of the relative role of the electronic scattering. On the other hand, the dip on both curves has the same nature—interference between the nuclear and electronic scatterings in conjunction with nuclear absorption, and therefore no radical qualitative difference is observed between these curves in this region. By virtue of the effect of the suppression of the inelastic channels, the interference pattern itself is much more distinct in the case of an ideal crystal than in the case of a mosaic crystal.

When the relative role of the nuclear scattering is decreased, as is the case, for example, when the temperature is increased or the concentration of the resonant isotope is decreased, the difference between the pairs patterns in both cases becomes weaker, although in the case of an ideal crystal it turns out to be

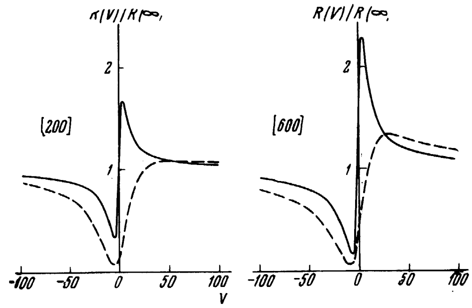

FIG. 6.

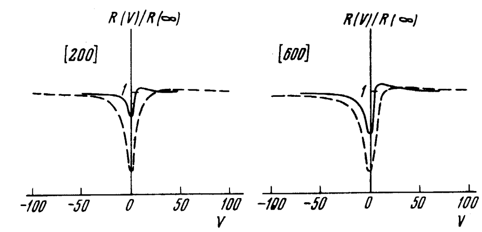

FIG. 7.

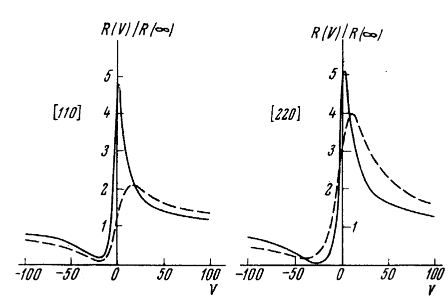

FIG. 8.

more distinct. This can be traced on the curves of Fig. 7, plotted for the same case as the curves of Fig. 6, but for $T = 293°K$.

Similar results were obtained also in the analysis of the Bragg scattering of Mossbauer γ quanta by iron. A 100% content of the isotope Fe^{57} was assumed, and a crystal having the parameters of metallic iron was considered, but without hyperfine splitting. Figure 8 shows curves for ideal and mosaic crystals, for the reflections [110] and [220] and for $T = 293°K$.

It is interesting to note that, unlike the curves of Fig. 6, in this case the transition to the higher order of reflection does not increase the difference in the quality of picture of the interference between the ideal and mosaic crystals, but conversely, weakens it to some degree. Physically this is connected with the fact that on going over to second order of reflection in the case of iron, the nuclear factor p^S (2.15) decreases strongly for one of the polarizations. But it is seen directly from the curves of Fig. 3 that the intensity of the reflection for this polarization decreases strongly at values of T close to zero, this being connected to a considerable degree with the strong attenuation of the effect of suppression of the inelastic channels, and consequently with the intensification of the absorption. As a result, the peak on the curve of Fig. 8 corresponding to an ideal crystal (solid line), is connected to a considerable degree in the case of the the [110] reflection only with one polarization of the quanta, whereas in the case of the [220] reflection the analogous peak is due to γ quanta with both polarizations.

We note in conclusion the interesting circumstance that in the presence of predominantly resonant scattering of the γ quanta (or x-rays) the integral coefficient of reflection from ideal single crystals is close in absolute magnitude to the value of this coefficient for mosaic crystals. In this sense, the picture differs radically from the usual case of non-resonant scattering of x-rays, when the reflection from the mosaic crystal gives a much larger integral intensity as compared with the reflection from the ideal crystal.

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