THE INFLUENCE OF DISLOCATIONS ON SUPERCONDUCTIVITY

R. O. ZAĬTSEV

Submitted December 1, 1967

Zh. Eksp. Teor. Fiz. 54, 1445–1454 (May, 1968)

We show that vibrations of dislocation lines may be a possible source of an increase in the transition temperature of a superconductor with residual deformation. We use the simplest model to evaluate the shift in the transition temperature. We find in the framework of this model the upper critical field and the residual resistivity. We compare the results obtained with experimental data and find qualitative agreement.

IT is well known that the properties of superconductors with defects differ in many ways from the properties of pure superconductors. For instance, impurities decreasing the correlation radius of Cooper pairs increase the critical field without changing the transition temperature. Experiments on superconductors with finite size defects show the following peculiarities:

1. The external field dependence of the magnetic moment has a strong hysteresis.[1]
2. The jump in the temperature-dependence of the resistivity is strongly smeared out.[2-4]
3. The critical field and critical current increase and in many cases they are anisotropic.[5,6]
4. The critical temperature increases starting with some deformation.[2-4]

The first two peculiarities are apparently connected with the non-uniform distribution of one-dimensional defects, and also the existence in metals of two- and three-dimensional defects, so that they shall not be considered in the following. One can explain the anisotropy of the critical fields and currents by the presence of a large number of parallel dislocations in the metal, but it is not possible to find the magnitude of the critical current.

The last property, the positive shift of the transition point, is of most interest. One can show that any static defects except metallic three-dimensional formations, do not change the transition temperature in an isotropic superconductor. On the other hand, corrections to the dimensional formations, do not change the transition temperature in an isotropic superconductor. On the other hand, corrections to the transition temperature depend linearly on the residual resistivity[4] so that we must assume that this phenomenon is determined by the dynamic properties of the defects. It is then clear that vibrations of one-dimensional defects are most effective as they have the largest statistical weight. From all possible one-dimensional defects the largest contribution comes from edge defects. It is well known that the interaction of electrons and defects is determined by the divergence of the displacement vector $\mathbf{u}$. For an edge dislocation $\operatorname{div}\mathbf{u} \sim b/r$, for a screw dislocation $\operatorname{div}\mathbf{u} = 0$, while for a chain of point defects this quantity decreases exponentially.[7]

The increase of the transition temperature of an isotropic superconductor with defects can thus be explained by the existence of an additional interaction between the electrons through the vibrations of edge dislocations. We note that for such a consideration the dislocation degrees of freedom must be subtracted from the total number of degrees of freedom of the crystal. One sees easily that this leads to an insignificant change in the Debye cut-off parameter Θ by a quantity of the order of magnitude $\Theta Da^2 \lesssim 10^{-4}\Theta$ (D is the dislocation density and a the size of a cell in the crystal).

MODEL DESCRIPTION OF DISLOCATIONS

According to modern ideas at low temperatures a dislocation performs oscillations around its equilibrium position which is determined by its interaction with the lattice. The equation for the vibrations which was first proposed by Seeger[8] has the form

$$-\frac{\partial^2\xi}{s^2\partial t^2}+\frac{\partial^2\xi}{\partial z^2}=\frac{a\beta}{2\pi}\sin\left(\frac{2\pi}{a}\xi\right), \tag{1}$$

$\beta = 2\pi b\sigma^p/aF$, z is the direction of the dislocation axis, $b\sigma^p$ the amplitude of the force acting on the dislocation, F the energy per unit length, s a quantity of the order of magnitude of the sound velocity.

For an edge dislocation ξ is the deviation from the equilibrium position in the direction of the Burgers vector b (for two-dimensional vectors we drop the vector indices)

$$s^2 = 2c_t^2\left(1-\frac{c_t^2}{c_l^2}\right)\left(1+\frac{c_t^4}{c_l^4}\right),$$

c_l and c_t the longitudinal and transverse sound velocities.[9]

It is natural to assume that at sufficiently low temperatures one can neglect non-linear effects so that we have instead of (1)

$$-\frac{\partial^2\xi}{s^2\partial t^2}+\frac{\partial^2\xi}{\partial z^2}=\beta\xi. \tag{2}$$

The energy of the oscillations of the dislocation is

$$H=\frac{F}{2}\int_0^L\left[\left(\frac{\partial\xi}{s\partial t}\right)^2+\left(\frac{\partial\xi}{\partial z}\right)^2+\beta\xi^2\right]dz. \tag{3}$$

We write down the quantum solution of Eq. (2):

$$\xi=\sum_n\left(\sqrt{\frac{\hbar s^2}{2FL\omega_n}}a_n e^{ik_n z-i\omega_n t}+\text{h. c.}\right), \tag{4}$$

$$k_n = 2\pi n/L,\quad \omega_n=\sqrt{s^2k_n^2+\omega_0^2},\quad \omega_0^2=\beta s^2,$$

a_n is a Bose annihilation operator.

We assume that there are in the metal a large number of parallel, (in the x0y plane) randomly distributed edge dislocations with random directions of the Burgers vector (we are dealing with an isotropic model). We shall assume that the average distance between the dislocation lines is much larger than that between the atoms. In that case one can neglect the interaction between the dislocations as compared with their interaction with the lattice. In this approximation the oscillations occur independently so that the state of the dislocation (c) is determined by the coordinate $r_c = (x_c, y_c)$ and the operator ξ_c (the operators ξ_c with different indices c commute).

INTERACTION OF ELECTRONS WITH DISLOCATIONS

The Hamiltonian of the interaction of the electrons with the deformed crystal has the form[10]

$$H_{int} = \sum_c \frac{2}{3}\varepsilon_f \int \operatorname{div} \mathbf{u}_c(\mathbf{r})\psi^+(x)\psi(x)\,d\mathbf{r},$$

ε_f is the Fermi energy, $\mathbf{u}_c(\mathbf{r})$ the vector of the displacement of the medium near the defect number c, $x = (\mathbf{r}, t)$.

We expand the quantity div $\mathbf{u}_c(\mathbf{r})$ in powers of the deviation (ξ) of the dislocation line from the equilibrium position (r_c). As a result the energy is split into a static and a dynamic part

$$H_{st} = \sum_c \frac{2}{3}\varepsilon_f \int \operatorname{div} u^0(r - r_c)\psi^+(x)\psi(x)\,d\mathbf{r}, \tag{5}$$

$$H_d = \sum_c \frac{2}{3}\varepsilon_f \int \Phi_c(z, t; r - r_c)\psi^+(x)\psi(x)\,d\mathbf{r}, \tag{6}$$

$$\Phi_c(z, t; r) = \sum_n \left(\sqrt{\frac{\hbar s^2}{2FL\omega_n}}\, a_{nc}(t) e^{ik_n z} \int V(k_n, b_c, p) e^{ipr} \frac{d^2p}{(2\pi)^2} + \text{h.c.} \right).$$

We find the quantity V in Appendix A; it has the form

$$V(k_n, b, p) = \left(\frac{c_t}{c_l}\right)^2 \frac{2p_\alpha b_\alpha e_{\beta\gamma} b_\beta p_\gamma}{b(p^2 + k_n^2)}, \tag{7}$$

$a_{nc}(t)$ is the annihilation operator in the interaction representation, $e_{\beta\gamma}$ the completely antisymmetric second rank tensor, b_c the two-dimensional Burgers vector with number c. All quantities with a Greek index are two-dimensional vectors.

To evaluate the different physical quantities with the Hamiltonian (5), (6) one must average all final results (a) over a Gibbs distribution of non-interacting electrons and dislocations, (b) over a random distribution of equilibrium positions of dislocation lines, and (c) over the directions of the Burgers vector. The diagram technique is developed in the usual way.[11] Each static line then gives a factor

$$\left(\frac{c_t}{c_l}\right)^4 \frac{16\pi D b^2 \varepsilon_f^2}{9(p_\perp^2 + \delta^2)}\,\delta(p_z) \tag{8}$$

(D—the two-dimensional density of dislocations, δ an infinitesimally small screening parameter disappearing from the final answers). To a dislocation corresponds a D-function

$$D_{\omega_n}(p) = -\frac{s^3}{\omega_n^2 + (sp_z)^2 + \omega_0^2}\left(\frac{p_\perp^2}{p_\perp^2 + p_z^2}\right)^2, \tag{9}$$

and to each electron-dislocation vertex a factor g:

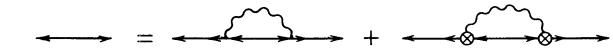
$$g^2 = \left(\frac{c_t}{c_l}\right)^4 \frac{2Db^2\varepsilon_f^2}{9\hbar s F}. \tag{10}$$

The quantity $g^2/D \sim 10^{-13}$ to 10^{-14} cm^2 so that for moderate dislocation concentrations ($D \lesssim 10^{12}$ cm^{-2}) it is sufficient to restrict ourselves to the first order of perturbation theory. We can find the frequency ω_0 by a comparison with experiments about the critical shear stress σ^p. From (1) and (4) we get the estimate $\omega_0 \sim s/b\sqrt{\sigma^p/\mu} \sim 10^{10}$ sec^{-1} ($\hbar\omega_0 \sim 1°$K).

INCREASE IN THE TRANSITION TEMPERATURE

It is well known that static defects appreciably change the correlation radius of the electrons but do not change the transition temperature. For this reason we include the static part of the interaction in the zeroth-order Hamiltonian when determining T_c. The remaining part is the interaction of the electrons with the phonons and with the oscillations of the dislocations.

To find the transition temperature it is necessary to solve the superconductivity equations linearized in the number of paired electrons (see [11]):



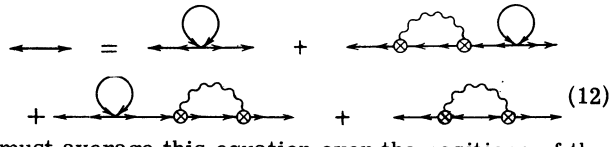
(11)

A line with arrows in one direction is the exact G-function of the normal metal, a line with arrows in different directions a F^+-function of the condensate particles, a wavy line a phonon line, and a wavy line ending in circles with crosses is a D-function of a dislocation.

We solve this equation in the approximation which is linear in g^2. Neglecting corrections of order $(T_c/\Theta)^2$ we can replace the phonon D-function by the number $-g_0^2 \sim -\nu^{-1}$. We neglect also all phonon corrections to the G-function since near the transition temperature they are of order T_c^3/Θ^2.[12] When the interaction with dislocations is taken into account the correction to the G-function will be of order $T_c g^2$, so that the whole calculation is valid for not too small concentrations, since we assume that

$$(T_c/\Theta)^2 \ll g^2 \ll \nu g_0^2 \lesssim 1/2 \quad (\nu = mp_0/2\pi^2).$$

Here p_0 is the momentum on the Fermi surface. Under this condition the final equation for F^+ has the following form:



(12)

We must average this equation over the positions of the impurity atoms and of the dislocations. We show in Appendix B that we are led after averaging to the same equation where now we must take as the G-function the free electron G-function.

We make the substitution $F^+(p) = G(p)G(-p)\Sigma_\omega(p)$ and then write down Σ in the zeroth and first approximations in g^2:

$$\Sigma^{(0)} = \frac{T}{(2\pi)^3} g_0^2 \sum_{\omega'} \int G_{\omega'}(\mathbf{p}') G_{-\omega'}(\mathbf{p}') \Sigma_{\omega'}(\mathbf{p}')\,d\mathbf{p}',$$

$$\Sigma_\omega^{(1)} = -\Sigma^{(0)} g^2 G_{-\omega}(-\mathbf{p}) \frac{T}{(2\pi)^3} \sum_{\omega'} \int D_{\omega+\omega'}(\mathbf{p} + \mathbf{p}') G_{\omega'}(\mathbf{p}')\,d\mathbf{p}'$$

$$- \Sigma^{(0)} g^2 G_\omega(\mathbf{p}) \frac{T}{(2\pi)^3} \sum_{\omega'} \int D_{\omega-\omega'}(\mathbf{p} - \mathbf{p}') G_{\omega'}(\mathbf{p}')\,d\mathbf{p}'$$

$$-\Sigma^{(0)}g^2\frac{T}{(2\pi)^3}\sum_{\omega'}\int D_{\omega-\omega'}(\mathbf{p}-\mathbf{p}')G_{\omega'}(\mathbf{p}')G_{-\omega'}(\mathbf{p}')\,d\mathbf{p}'.$$

Substituting this expression into the initial equation and integrating in the vicinity of the Fermi surface we find the shift in the transition temperature

$$T_c-T_{c0}=\frac{g^2mp_0T_{c0}^2}{32\pi^2}\sum_{\omega'\omega''}\left[\left(\frac{\operatorname{sign}(\omega'\omega'')}{\omega'^2}-\frac{1}{|\omega'\omega''|}\right)\times\int D_{\omega'-\omega''}(\mathbf{n}'-\mathbf{n}'')\,d\mathbf{n}'\,d\mathbf{n}''\right].$$

Here T_{c0} is the transition temperature of the superconductor without dislocations. Substituting here the D-function from (9) we observe that the important region of integration is the region of small $|\cos\theta'-\cos\theta''|\lesssim\omega_0/\Theta\ll 1$. We can thus at once take $p_\perp^2/(p_\perp^2+p_z^2)\to 1$ and with the same accuracy integrate over $\cos\theta'-\cos\theta''$ from $-\infty$ to $+\infty$. After changing to a summation over positive frequencies and subsequent integration we find

$$T_c-T_{c0}=\frac{g^2ms^2}{2\pi^2}W\left(\frac{\omega_0}{2\pi T_{c0}}\right),$$

$$W(\alpha)=\sum_{n',\,n''\geq 0}\frac{1}{(1+2n')^2(1+2n'')^2}\left[\frac{(1+n'+n'')^2}{[(1+n'+n'')^2+\alpha^2]^{1/2}}-\frac{(n'-n'')^2}{[(n'-n'')^2+\alpha^2]^{1/2}}\right] \qquad (13)$$

We show in Appendix C that the additional resistivity arising from the dislocations is proportional to their concentration. The change in the transition temperature is thus proportional to the residual resistivity of a metal if the latter is mainly determined by the dislocations.

In concluding this section we estimate the transition temperature connected with the vibrations of the dislocations only. Putting in Eq. (11) $g_0^2=0$, $g^2\ll 1$ we have

$$\Sigma_\omega(\mathbf{n})=-\frac{g^2mp_0T}{8\pi^2}\sum_{\omega'}\frac{1}{|\omega'|}\int D_{\omega-\omega'}(\mathbf{n}-\mathbf{n}')\Sigma_{\omega'}(\mathbf{n}')\,d\mathbf{n}'.$$

Neglecting corrections of order ω_0/Θ we find

$$\Sigma_\omega=\frac{g^2ms^2}{16\pi^2T}\sum_{n'}\frac{\Sigma_{\omega'}}{|n'+1/2|[(n-n')^2+(\omega_0/2\pi T)^2]^{1/2}}$$

We assume that g^2 is so small that $\omega_0\gg 2\pi T_c$. In that case we can restrict the summation to the quantity $\omega_0/2\pi T_c$ and neglect the frequency-dependence of Σ. As a result we get

$$T_c=\frac{g^2ms^2}{8\pi^2}\ln\left(\frac{16\pi\gamma\omega_0}{g^2ms^2}\right). \qquad (14)$$

Hence it follows that the transition point from dislocation oscillations is in any case less than $1/10$ of a degree.

CRITICAL FIELD

The upper critical field depends appreciably on the impurity concentration. To find this quantity we must thus completely neglect dislocation oscillations since the field will be mainly determined by the electron mean free path due to static defects. One checks easily that in that case we neglect effects of order $p_0s/\mu b^3\sim 10^{-2}$. We shall assume that there are apart from the edge dislocations a small number of impurities in the metal. We write down the Hamiltonian for the interaction of the electrons with defects:

$$H=\sum_c{}^{2}/_{3}\varepsilon_f\int\operatorname{div}u^0(r-r_c)\psi^+(x)\psi(x)\,d\mathbf{r}+\sum_a\int U\delta(\mathbf{r}-\mathbf{r}_a)\psi^+(x)\psi(x)\,d\mathbf{r}; \qquad (15)$$

U is a constant ($U\sim{}^{2}/_{3}\varepsilon_f(c_t/c_l)^2b^3$ [13]).

For the sake of simplicity we assume that the arrangement of the impurities is independent of the position of the dislocations so that the averaging over the positions of the impurity atoms and of the dislocations is performed independently. If we assume that the temperature is close to the critical one, our problem is reduced to finding the coefficients in the linearized Ginzburg–Landau equation. All calculations are the same as those in Gor'kov's paper [14] so that we give at once the answer (see also Appendix C):

$$\left[\frac{\chi_{sk}}{2m}\left(\frac{\partial}{\partial x_s}+2ieA_s\right)\left(\frac{\partial}{\partial x_k}+2ieA_k\right)+\frac{T_c-T}{T_c\eta}\right]F^+(\mathbf{r})=0. \qquad (16)$$

For our choice of axes the non-vanishing components χ_{sk} (principal values) have the form

$$\chi_{xx}=\chi_{yy}=\chi_\perp=\frac{12}{7\zeta(3)}\sum_{n=0}^{\infty}\frac{1}{(1+2n)^2}\int_0^1\frac{(1-x^2)\,dx}{1+2n+\rho_{\text{imp}}+\rho_d/(1-x^2)}$$

$$\chi_{zz}=\chi_\parallel=\frac{8}{7\zeta(3)}\sum_{n=0}^{\infty}\frac{1}{(1+2n)^2(1+2n+\rho_{\text{imp}})}, \qquad (17)$$

where

$$\rho_{\text{imp}}=\frac{nU^2mp_0}{2\pi^2T_c},\quad \rho_d=\left(\frac{c_t}{c_l}\right)^4\frac{Db^2\varepsilon_f}{9\pi T_c},\quad \eta=\frac{7\zeta(3)\varepsilon_f}{12(\pi T_c)^2}$$

If the metal is very impure (ρ_{imp}, $\rho_d\gg 1$) the tensor χ_{ik} will be proportional to the conductivity tensor:

$$\chi_{ik}=\frac{2\pi^3T_cm}{7\zeta(3)Ne^2}\sigma_{ik}. \qquad (18)$$

The principal values of the conductivity tensor are obtained in Appendix C.

From Eq. (16) we find the critical field

$$H_{c2}=\varkappa_0\sqrt{2}H_{cm}/[\chi_\perp(\chi_\perp\cos^2\theta+\chi_\parallel\sin^2\theta)]^{1/2}, \qquad (19)$$

($\chi_\perp\leq\chi_\parallel$), θ the angle between the axis of the dislocation and the direction of the field, and κ_0 the value of the parameter κ for a pure metal.

In conclusion we note that we can average the tensor χ_{ik} over all possible directions of the axis of the dislocations (see Appendix C). As a result we get $\chi_{ik}=\delta_{ik}\times({}^{2}/_{3}\chi_\parallel+{}^{1}/_{3}\chi_\perp)$. In that case the critical field increases, while remaining isotropic.

COMPARISON WITH EXPERIMENTS

To compare the results obtained with experiments it is necessary to eliminate from them the quantities Db^2 and ω_0, which are not measured directly experimentally. This can be done by using Eq. (C.2) for the residual resistivity due to impurities and dislocations.

As a result we find a simple parametric connection:

$$T_c-T_{c0}={}^{5}/_{4}kt\rho(0),$$

$$\rho_\perp(t)=\rho(0)\left[1-\frac{3}{2}t+\frac{3}{4}\frac{t^2}{\sqrt{1+t}}\ln\left(\frac{\sqrt{1+t}+1}{\sqrt{1+t}-1}\right)\right]^{-1}, \qquad (20)$$

k is the asymptotic slope of the $\Delta T(\rho)$ curve for large t, $\rho_\perp(t)$ is the resistivity at right angles to the axis of the dislocation,

$$k=\frac{2Ne_f e^2 s}{5\pi^2 F}W\left(\frac{\omega_0}{2\pi T_{c0}}\right),\quad t=\frac{\pi^2 Db^2\hbar^2}{3\rho(0)p_0e^2}\left(\frac{c_t}{c_l}\right)^4,$$

N is the electron density, F the energy per unit length of edge dislocation; the remaining quantities were defined in (13).

The critical field of strongly contaminated metals can be expressed directly in terms of the conductivity:

$$H_{c2}=H_{c2}(0)\sigma(0)/\{\sigma_\perp(t)[\cos^2\theta\sigma_\perp(t)+\sin^2\theta\sigma(0)]\}^{1/2}, \quad (21)$$

$H_{c2}(0)$ is the critical field when there are no dislocations.

The expression (20) for the shift in the transition temperature agrees qualitatively with experiments on superconducting rhenium.[4] The quantity $k = 6.9/R^\circ K$ (R is the resistivity at room temperature). There is no quantitative agreement due to the coarseness of the model and also because we take for T_c some average value of the temperature in the transition region while in Eq. (20) T_c is the temperature where resistivity first occurs.

The anisotropy of critical fields has been observed in many experiments.[5,6] The non-linear dependence of the critical field on the resistivity was observed in ref. [4]. One can obtain qualitative agreement with Eq. (21) for $\theta = \pi/2$, but there is no information whatever about the value of θ.

The author expresses his deep gratitude to M. N. Mikheeva for manifold and useful discussions about the state of experiments and also to Professor B. T. Geĭlikman for a discussion of the results of this paper.

APPENDIX A

To determine the field at close distances to a vibrating dislocation ($\rho \ll 1/p_z \sim T_c/\Theta p_0 \sim 10^{-6}$: the static zone) it is sufficient to solve the equilibrium equation of the dislocation line:

$$\Delta\mathbf{u}+\frac{1}{1-2\sigma}\nabla\operatorname{div}\mathbf{u}=[\boldsymbol{\tau}\mathbf{b}]\delta(\boldsymbol{\rho}) \quad (A.1)$$

(the symbols are all taken from the book [15]).

Let the Burgers vector be along the x axis and let the equation of the dislocation line have the form (x = ξ(z, t), y = 0). We write u in the form

$$\mathbf{u}=\mathbf{u}^0(x-\xi(z,t),y)+\mathbf{w}(\mathbf{r},t), \quad (A.2)$$

$\mathbf{u}^0$ is the solution of Eq. (A.1) for a straight-line edge dislocation.[15]

Substituting (A.2) into (A.1) and restricting ourselves to terms linear in ξ we have

$$\begin{aligned}\Delta w_x+\frac{1}{1-2\sigma}\frac{\partial}{\partial x}\operatorname{div}\mathbf{w}&=\frac{\partial^2\xi}{\partial z^2}\frac{\partial u_x^{(0)}}{\partial x},\\ \Delta w_y+\frac{1}{1-2\sigma}\frac{\partial}{\partial y}\operatorname{div}\mathbf{w}&=\frac{\partial^2\xi}{\partial z^2}\frac{\partial u_y^{(0)}}{\partial x},\\ \Delta w_z+\frac{1}{1-2\sigma}\frac{\partial}{\partial z}\operatorname{div}\mathbf{w}&=\frac{1}{1-2\sigma}\frac{\partial\xi}{\partial z}\frac{\partial}{\partial x}\operatorname{div}\mathbf{u}^0.\end{aligned} \quad (A.3)$$

The quantity w is a one-valued function so that we can solve the set (A.3) by Fourier's method:

$$(\operatorname{div}\mathbf{w})_{\mathbf{p}}=\frac{\xi(p_z)p_z^2}{p_\perp^2+p_z^2}\left(\frac{\partial}{\partial x}\operatorname{div}\mathbf{u}^0\right)_{p_\perp}.$$

Hence we find the quantity $(\operatorname{div}\mathbf{u})_{\mathbf{p}}$ for any direction of the Burgers vector:

$$(\operatorname{div}\mathbf{u})_{\mathbf{p}}=(\operatorname{div}\mathbf{u}^0)_{\mathbf{p}}-\frac{\xi(p_z)[(\mathbf{b}\nabla)\operatorname{div}\mathbf{u}^0]_{p_\perp}p_\perp^2}{b(p_\perp^2+p_z^2)}. \quad (A.4)$$

We write the quantity $\operatorname{div}\mathbf{u}^0$ in "invariant" form

$$\operatorname{div}\mathbf{u}^0=\left(\frac{c_t}{c_l}\right)^2\frac{e_{\alpha\beta}r_\alpha b_\beta}{\pi r^2},$$

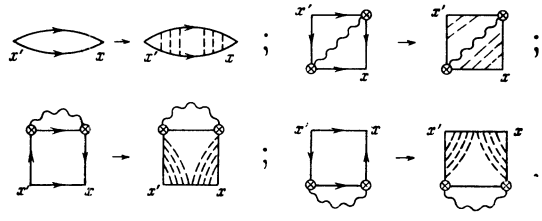
whence we find at once the Fourier components

$$\begin{aligned}(\operatorname{div}\mathbf{u}^0)_{\mathbf{p}}&=4\pi i\delta(p_z)\frac{e_{\alpha\beta}p_\beta b_\alpha}{p_\perp^2}\left(\frac{c_t}{c_l}\right)^2,\\ [(\mathbf{b}\nabla)\operatorname{div}\mathbf{u}^0]_{\mathbf{p}\perp}&=\left(\frac{c_t}{c_l}\right)^2\frac{2p_\alpha b_\alpha e_{\beta\gamma}b_\gamma p_\beta}{p_\perp^2}.\end{aligned} \quad (A.5)$$

Substituting the operator values of $\xi(p_z)$ from (4) into (A.4) we get the results (5) to (7).

APPENDIX B

We show that static defects do not change the shift in the transition temperature. To do this we average each term of Eq. (12) where we have put $x_1 = x_2 = x$. The averaging is done as follows:



Dashed lines depict static D-functions. Static and dynamic lines do not intersect. From this it is clear that all diagrams can be expressed through the average of the product of two G-functions joining in one point.

If the interaction with defects is given in the form (15) one can easily solve the equation for the average of two G-functions (see Gor'kov's paper[14]). As a result the product G(p)G(−p) acquires the factor

$$\eta(\theta)=1+\frac{1}{2\tau_{\text{imp}}|\omega|}+\frac{I}{2|\omega|\sin^2\theta}, \quad (B.1)$$

where

$$I=\left(\frac{c_t}{c_l}\right)^4\frac{Db^2e_f}{9\pi}\int_0^{2\pi}\frac{d\varphi}{(1-\cos\varphi+\delta^2)} \quad (B.2)$$

(δ is the screening parameter). Since each G-function contains the frequency with the same factor, the quantity $\eta(\theta)$ disappears from the answer after integration near the Fermi surface.

APPENDIX C

The calculation of the resistivity due to point impurities and static defects is a verbatim repeat of the corresponding calculations for a system of isotropic scatterers[11] with the only difference that now the reciprocal of the free flight time depends on the angle between the axis of the dislocation and the direction of the electron momentum (θ). We have

$$\frac{1}{\tau}=\frac{1}{\tau_{\text{imp}}}+\frac{I}{\sin^2\theta},\quad \frac{1}{\tau_{tr}}=\frac{1}{\tau_{\text{imp}}}+\frac{1}{\tau_d\sin^2\theta}. \quad (C.1)$$

The quantity I is defined in (B.2)

$$\frac{1}{\tau_d} = \frac{2Db^2\varepsilon_f}{9}\left(\frac{c_t}{c_l}\right)^4.$$

We must substitute the quantity (C.1) in the usual equation for $\Pi_{x,y}(p, \omega_+, \omega_-)$, which determines the conductivity in a direction at right angles to the axis of the dislocations (see [11]). The conductivity along the axis of the dislocations is determined by the point defects alone. After integrating over the direction of the vector p we find the principal values of the conductivity tensor

$$\sigma_{xx} = \sigma_{yy} = \sigma_\perp = \sigma_\parallel \cdot \frac{3}{2}\int_0^1 \frac{(1-x^2)\,dx}{1+t/(1-x^2)}$$

$$= \sigma_\parallel\left[1 - \frac{3}{2}t + \frac{3t^2}{4\sqrt{1+t}}\ln\left(\frac{\sqrt{1+t}+1}{\sqrt{1+t}-1}\right)\right],$$

$$t = \left(\frac{c_t}{c_l}\right)^4 \frac{\pi^2 D b^2 \sigma_\parallel}{3p_0 e^2}, \qquad \sigma_\parallel = \frac{Ne^2\tau_{imp}}{m}; \tag{C.2}$$

N is the electron density and D the two-dimensional density of dislocations.

[1] R. W. Shaw and D. E. Mapother, Phys. Rev. 118, 1474 (1960).

[2] V. I. Khotkevich and V. R. Golik, Zh. Eksp. Teor. Fiz. **20**, 427 (1950).

[3] J. J. Hauser and E. Buehler, Phys. Rev. **125**, 142 (1962).

[4] N. E. Alekseevskiĭ, M. N. Mikheeva, and N. A. Tulina, Zh. Eksp. Teor. Fiz. **52**, 875 (1967) [Sov. Phys.-JETP 25, 575 (1967)].

[5] J. Lowell, Solid State Comm. **5**, 323 (1967).

[6] T. G. Berlincourt, R. R. Hake, D. H. Leslie, Phys. Rev. Lett. **6**, 671 (1961).

[7] Ya. A. Iosilevskiĭ, Zh. Eksp. Teor. Fiz. **51**, 201 (1966) [Sov. Phys.-JETP **24**, 136 (1967)].

[8] A. Seeger, Phil. Mag. **1**, 651 (1956).

[9] G. Leibfried, in Dislokatsii i mekhanicheskie svoĭstva kristallov (Dislocations and Mechanical Properties of Crystals) IIL, 1960, p. 344.

[10] S. C. Hunter and F. R. N. Nabarro, Proc. Roy. Soc. **A220**, 542 (1953).

[11] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Methods of the Quantum Theory of Fields in Statistical Physics) Fizmatgiz, 1962, Sec. 35, 39 [English translation published by Pergamon Press, 1965].

[12] G. M. Èliashberg, Zh. Eksp. Teor. Fiz. **39**, 1437 (1960) [Sov. Phys.-JETP **12**, 1000 (1961)].

[13] Proceedings of a Course in the Theory of Defects in Crystals and Radiation Damage, Vol. III, Tbilisi, 1966, p. 169.

[14] L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **37**, 1407 (1959) [Sov. Phys.-JETP **10**, 998 (1960)].

[15] L. D. Landau and E. M. Lifshitz, Teoriya uprugosti (Theory of Elasticity) Fizmatgiz, 1965 [English translation to be published by Pergamon Press].

Translated by D. ter Haar
169