SOVIET PHYSICS JETP VOLUME 27, NUMBER 3 SEPTEMBER, 1968

THE VAN DER WAALS FORCES IN FERRODIELECTRICS

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Submitted October 10, 1967

Zh. Eksp. Teor. Fiz. 54, 994–1000 (March, 1968)

The molecular forces of interaction between two ferrodielectrics separated by a narrow gap are determined in the case $l \gg \hbar c/kT$. It is shown that allowance for the magnetic-moment fluctuations leads to the appearance of an additional attraction force that depends on the magnetic permeability of bodies, and that the electric and magnetic moment fluctuations make independent contributions of the same order to the general Van der Waals attraction force.

1. Long-wave fluctuations of the electromagnetic field in an absorbing medium are a source of long-range forces, having the same nature as Van-der-Waals forces of attraction between molecules at large distances. The long-range character of these forces leads to non-additivity of the free energy, as a result of which, for example, the free energy of a system of solids depends on the distances between them. The interaction forces between nonmagnetic macroscopic bodies, the surfaces of which are very close to one another, were first obtained by E. M. Lifshitz [1] (see also [2,3]). These forces have the character of attraction and are determined completely by the dispersion properties of the bodies.

It is obvious that the main contribution to the interaction between solids is made by electromagnetic fluctuations with frequencies which are at any rate not larger than ω_{abs} - the frequencies of the upper edge of the absorption spectrum of the given bodies, for when $\omega > \omega_{abs}$ the dielectric constants of the bodies $\epsilon(\omega) \to 1$. The electromagnetic fluctuations themselves either have the character of quantum fluctuations with wavelengths on the order of the distance l between the bodies (with frequency $\omega_q \sim c/l$), or are principally thermal fluctuations with frequencies on the order of $\omega_T = kT/\hbar$. Specifically, the character of the fluctuations is determined by which of the frequencies, ω_q or ω_T, is higher. For condensed bodies, the characteristic frequency of the thermal fluctuations ω_T is always much smaller than the frequency ω_{abs}, which lies, for example, in the optical band, and therefore the dependence of the attraction force on the distance is determined entirely by the location of the frequency ω_q relative to the frequencies ω_T and ω_{abs}.

As shown in [1], in the limiting case of "small" distances, when $\omega_q \gg \omega_{abs}$, the principal role is played by quantum fluctuations with frequencies $\omega \lesssim \omega_{abs}$, the attraction force is proportional to $F(l) \sim \hbar\omega_{abs}/l^3$, and does not depend on the temperature.

In the case of "large" distances and low temperatures, when $\omega_T \ll \omega_q \ll \omega_{abs}$, the contribution to the attraction force is made by quantum fluctuations of the electromagnetic field with frequencies $\omega \lesssim \omega_q$, and the attraction force $F(l) \sim \hbar\omega_q/l^3 \sim \hbar c/l^4$ does not depend on the temperature [1].

In the preceding two cases, the magnetic properties of the bodies did not play any role, since the magnetic permeability of non-ferromagnetic bodies differs little from unity, and that of ferromagnetic bodies differs from unity only at frequencies not exceeding the characteristic frequency of the ferromagnetic resonance $\omega_0 \sim 10^{10}$ Hz, which is much smaller than the frequencies ω_{abs} and ω_q[1)]. On the other hand, in the case of "large" distances and high temperatures, when $\omega_q \ll \omega_T \ll \omega_{abs}$, the main contribution to the attraction force is made by thermal fluctuations of the electromagnetic field in the gap between the surfaces of the solids, and, in perfect analogy to the fluctuations of density in a two-dimensional system, appreciable fluctuations of the electromagnetic field with extremely small frequencies occur in a narrow gap, so that the expression for the attraction force contains the static values of the dielectric constant and the magnetic permeability, which are quantities of the same order in the case of ferrodielectrics.

The present paper is devoted to a calculation of the Van-der-Waals force of interaction between two ferrodielectrics separated by a narrow gap, in the case when $\omega_q \ll \omega_T \ll \omega_{abs}$. Allowance for the fluctuations of the magnetic moment leads to the appearance of an additional attraction force, and the fluctuations of the electric and magnetic moments give independent contributions of equal order to the total attraction force.

2. We shall visualize the interacting photodielectrics in the form of two plane-parallel plates of sufficient thickness, separated by a narrow gap bounded by the planes $Z = 0$ and $Z = l$. In order to exclude the dipole interaction of the magnetized plates, we direct the magnetic moments of the plates parallel to the gap in the direction of the X axis.

The force of interaction between the plates, per unit surface area, is equal to the component σ_{ZZ} of the stress tensor of the fluctuating electromagnetic field.

Dzyaloshinskiĭ and Pitaevskiĭ [4] obtained an expression for σ_{ik} in terms of the temperature Green's functions $D_{ik}(\mathbf{r}_1, \mathbf{r}_2; \omega_n)$ of the electromagnetic field. In the vacuum that separates the two media, the expression for σ_{ik} is of the form

$$\sigma_{ik} = -\frac{T}{2\pi}\sum_{n=0}^{\infty}{}' \{D_{ik}^E(\mathbf{r}, \mathbf{r}; \omega_n) + D_{ik}^B(\mathbf{r}, \mathbf{r}; \omega_n) - {}^1/_2\, \delta_{ik}[D_{ll}^E(\mathbf{r}, \mathbf{r}; \omega_n) + D_{ll}^B(\mathbf{r}, \mathbf{r}; \omega_n)]\}. \quad (1)$$

1)The frequencies $\omega_q \gtrsim \omega_0$ correspond to distances $l \gtrsim 1$ cm. At such distances, the Van-der-Waals forces are vanishingly small, although formally the magnetic effects contribute in this case to the force.

The summation is carried out here over the values $\omega_n = 2\pi nT/\hbar$, and the term with $n = 0$ is taken with half-weight. The functions D_{ik}^E and D_{ik}^B are made up of components of the operators of the electric field intensity and the magnetic field induction, in exactly the same manner as D_{ik} is constructed from components of the vector potential of the field, and play the role of mean values of the products of the components of the corresponding operators:

$$D_{ik}^E(\mathbf{r}, \mathbf{r}'; \omega_n) = -\omega_n^2 D_{ik}(\mathbf{r}, \mathbf{r}'; \omega_n),$$

$$D_{ik}^B(\mathbf{r}, \mathbf{r}'; \omega_n) = \mathrm{rot}_{il}\,\mathrm{rot}_{km}' D_{lm}(\mathbf{r}, \mathbf{r}'; \omega_n). \tag{2}$$

The temperature Green's function $D_{ik}(\mathbf{r}, \mathbf{r}'; \omega_n)$ satisfies the equation

$$[\omega_n^2 \varepsilon_{il}(\mathbf{r}, i\omega_n) + \mathrm{rot}_{ij}\,\mu_{jm}^{-1}(\mathbf{r}, i\omega_n)\,\mathrm{rot}_{ml}] D_{lk}(\mathbf{r}, \mathbf{r}'; \omega_n) = -4\pi\delta_{ik}\delta(\mathbf{r} - \mathbf{r}'). \tag{3}$$

Here $\epsilon_{il}(\mathbf{r}, \omega)$ is the dielectric tensor and $\mu_{jm}^{-1}(\mathbf{r}, \omega)$ is the reciprocal magnetic-permeability tensor.

On the boundary between the two media, the components D_{ik} should satisfy the boundary conditions corresponding to the continuity of the tangential components of the electric and magnetic fields. Because of the fact that Eq. (3) does not take into account the spatial dispersion of the dielectric and magnetic permeabilities, the quantities $D_{ik}^E(\mathbf{r}, \mathbf{r}')$ and $D_{ik}^B(\mathbf{r}, \mathbf{r}')$ become infinite at $\mathbf{r} = \mathbf{r}'$, and therefore the infinite contribution of the short-wave photons must be cut off before these quantities are substituted in formula (1).

Thus, the force $F(l)$ acting on a unit surface is

$$F(l) = \frac{T}{8\pi}\sum_{n=0}^{\infty}{}' \{D_{XX}^E(l, l; \omega_n) + D_{YY}^E(l, l; \omega_n) - D_{ZZ}^E(l, l; \omega_n) + D_{XX}^B(l, l; \omega_n) + D_{YY}^B(l, l; \omega_n) - D_{ZZ}^B(l, l; \omega_n)\}. \tag{4}$$

Thus, the problem reduces to a solution of Eq. (3) with boundary conditions.

To simplify the calculations we first find the force $F(l)$ assuming that $\epsilon_{il}(\mathbf{r}, \omega) = \delta_{il}$ in all of space. The tensor $\mu_{ik}^{-1}(i\omega_n)$ in ferrodielectrics is equal to

$$\mu_{ik}^{-1}(i\omega_n) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta_1 & \eta_3 \\ 0 & -\eta_3 & \eta_2 \end{pmatrix}, \tag{5}$$

where

$$\eta_1 = \frac{\omega_0\omega_2 + \omega_n^2 + 2\omega_1\omega_n\lambda/gM_0}{\omega_r^2 + \omega_n^2 + \omega_n\gamma_r}, \quad \eta_3 = \frac{4\pi\omega_n g M_0}{\omega_r^2 + \omega_n^2 + \omega_n\gamma_r},$$

$$\eta_2 = \frac{\omega_1^2 + \omega_n^2 + 2\omega_0\omega_n\lambda/gM_0}{\omega_r^2 + \omega_n^2 + \omega_n\gamma_r}, \quad \omega_0 = gM_0\left(\beta + \frac{H_0^{(i)}}{M_0}\right),$$

$$\omega_1 = \omega_0 + 4\pi g M_0, \quad \omega_2 = \omega_0 + 8\pi g M_0, \quad \omega_r = \sqrt{\omega_1\omega_2},$$

$$\gamma_r = (\omega_1 + \omega_2)\lambda / gM_0,$$

here λ is the magnetic-moment relaxation constant, β the anisotropy constant, $H_0^{(i)}$ the constant magnetic field inside the magnet, directed along the easiest magnetization axis (the X axis), and ω_r and γ_r are the frequency and width of the homogeneous ferromagnetic resonance line.

By virtue of the homogeneity of the problem with respect to the variables X and Y, the function $D_{ik}(\mathbf{r}, \mathbf{r}')$ depends only on the difference $X - X'$ and $Y - Y'$. We carry out a Fourier transformation with respect to these variables:

$$D_{ik}(Z, Z', \boldsymbol{\rho}) = \frac{1}{(2\pi)^2}\int e^{i\mathbf{q}\boldsymbol{\rho}} D_{ik}(Z, Z', \mathbf{q})\,d\mathbf{q}.$$

The vectors $\mathbf{q}$ and $\boldsymbol{\rho}$ lie in the XY plane, $\boldsymbol{\rho} = \{X - X', Y - Y'\}$. We choose a new coordinate system x, y, z in which the y axis is along the vector $\mathbf{q}$ and the z axis along the old Z axis. Equations (3) for the Fourier components $D_{ik}(z, z', \mathbf{q}; \omega_n)$ take on in the new coordinate system the form

$$\left[\omega_n^2 + q^2\eta_2 - (\sin^2\alpha + \eta_1\cos^2\alpha)\frac{d^2}{dz^2}\right] D_{xk} - i\sin\alpha\left[q\eta_3\frac{d}{dz} + i\cos\alpha(1 - \eta_1)\frac{d^2}{dz^2}\right] D_{yk} - q\sin\alpha\left[q\eta_3 + i\cos\alpha(1 - \eta_1)\frac{d}{dz}\right] D_{zk} = -4\pi\delta(z - z')\delta_{xk},$$

$$iqD_{yk} + \frac{d}{dz}D_{zk} = -\frac{4\pi}{\omega_n^2}\left(iq\,\delta_{yk} + \delta_{zk}\frac{d}{dz}\right)\delta(z - z'),$$

$$-q\sin\alpha\left[q\eta_3 + i\cos\alpha(1 - \eta_1)\frac{d}{dz}\right] D_{xk} + iq(\cos^2\alpha + \eta_1\sin^2\alpha)\frac{d}{dz}D_{yk} + [\omega_n^2 + q^2(\cos^2\alpha + \eta_1\sin^2\alpha)] D_{zk} = -4\pi\delta(z - z')\delta_{zk}. \tag{6}$$

Here α is the angle between the axes X and x. Since we are interested only in the Green's function in the region of the gap, we can immediately confine ourselves to the case $0 < z' < l$. Then the functions D_{ik} in the regions $z \le 0$ and $z \ge l$ will be determined by Eqs. (6) without the right sides, and in the region $0 < z < l$ it is necessary to put in (6) $\eta_1 = \eta_2 = 1$, and $\eta_3 = 0$. The boundary conditions reduce in this case to the requirement of continuity of the functions D_{ik} and $d(\cos\alpha D_{yk} - \sin\alpha D_{xk})/dz$ at the points $z = 0$ and $z = l$.

3. The solution of Eqs. (6) with k = z, y, and z is carried out in perfect analogy, and we therefore consider only the case k = x.

In the region $0 < z < l$, Eqs. (6) take the form

$$\left(w^2 - \frac{d^2}{dz^2}\right) D_{xx} = -4\pi\delta(z - z'),$$

$$iqD_{yx} + \frac{d}{dz}D_{zx} = 0, \quad iq\frac{d}{dz}D_{yx} + w^2 D_{zx} = 0, \tag{7}$$

where $w^2 = \omega_n^2 + q^2$. From the system (7) we get

$$D_{xx}(z, z') = -\frac{2\pi}{w}e^{-w|z-z'|} + a_x(z')e^{wz} + b_x(z')e^{-wz},$$

$$D_{yx}(z, z') = a_y(z')e^{wz} + b_y(z')e^{-wz},$$

$$D_{zx}(z, z') = -\frac{iq}{w}a_y(z')e^{wz} + \frac{iq}{w}b_y(z')e^{-wz}, \tag{8}$$

here a_x, b_x, a_y, and b_y are unknown functions. In the regions $z \ge l$ and $z \le 0$ we shall seek the solution of the system of homogeneous equations (6) in the form $D_{ik} \sim e^{\kappa z}$. The following dispersion equation is obtained for the determination of κ:

$$\eta_1\varkappa^4 - [w_1^2 + w_2^2\eta_1 + q^2(\eta_1 - 1)(1 - \eta_2)\cos^2\alpha + q^2\eta_3^2\sin^2\alpha]\varkappa^2 + w_1^2 w_2^2 + q^4\eta_3^2\sin^2\alpha = 0, \tag{9}$$

where $w_1^2 = \omega_n^2 + q^2(\cos^2\alpha + \eta_1\sin^2\alpha)$ and $w_2^2 = \omega_n^2 + q^2\eta_2$. It can be shown that Eq. (9) has only real roots $\kappa_1 \ge \kappa_2 \ge 0$, $\kappa_3 = -\kappa_1$, and $\kappa_4 = -\kappa_2$. Simple calculations lead to the following result: in the region $z \le 0$, the solution is of the form

$$D_{xx}(z, z') = \alpha(z')d_1 e^{\varkappa_1 z} + \beta(z')d_2 e^{\varkappa_2 z},$$

$$D_{yx}(z, z') = \alpha(z')\frac{i\varkappa_1}{q}e^{\varkappa_1 z} + \beta(z')\frac{i\varkappa_2}{q}e^{\varkappa_2 z},$$

$$D_{zx}(z, z') = \alpha(z')e^{\varkappa_1 z} + \beta(z')e^{\varkappa_2 z}, \tag{10}$$

and in the region $z \geq l$ we obtain from (6)

$$D_{xx}(z, z') = A(z') d_3 e^{-\varkappa_1 z} + B(z') d_4 e^{-\varkappa_2 z},$$
$$D_{yx}(z, z') = -A(z') \frac{i\varkappa_1}{q} e^{-\varkappa_1 z} - B(z') \frac{i\varkappa_2}{q} e^{-\varkappa_2 z},$$
$$D_{zx}(z, z') = A(z') e^{-\varkappa_1 z} + B(z') e^{-\varkappa_2 z}, \qquad (11)$$

where

$$d_\xi = \frac{\varkappa_\xi^2 (\cos^2\alpha + \eta_1 \sin^2\alpha) - w_1^2}{q \sin\alpha [q\eta_3 - i(1-\eta_1)\varkappa_\xi \cos\alpha]}, \qquad \xi = 1, 2, 3, 4.$$

To determine the unknown functions a_x, b_x, a_y, b_y, α, β, A, and B, we shall use the boundary conditions for the Green's functions. From the condition for the continuity of the functions

$$D_{xx}, D_{yx}, D_{zx}, \quad \frac{d}{dz}(D_{yx}\cos\alpha - D_{xx}\sin\alpha)$$

at the points $z = 0$ and $z = l$ we obtain a system of eight equations:

$$\alpha d_1 + \beta d_2 = -2\pi e^{-wz'}/w + a_x + b_x,$$
$$\alpha i\varkappa_1/q + \beta i\varkappa_2/q = a_y + b_y, \quad \alpha + \beta = -iqa_y/w + iqb_y/w,$$
$$\alpha\rho_1\varkappa_1 + \beta\rho_2\varkappa_2 = 2\pi q e^{-wz'}\sin\alpha$$
$$+ (a_y - b_y) wq\cos\alpha - (a_x - b_x) wq \sin\alpha,$$
$$A d_3 e^{-\varkappa_1 l} + B d_4 e^{-\varkappa_2 l} = -2\pi e^{-w(l-z')}/w + a_x e^{wl} + b_x e^{-wl},$$
$$A\frac{i\varkappa_1}{q} e^{-\varkappa_1 l} + B\frac{i\varkappa_2}{q} e^{-\varkappa_2 l} + a_y e^{wl} + b_y e^{-wl} = 0,$$
$$A e^{-\varkappa_1 l} + B e^{-\varkappa_2 l} = -\frac{iq}{w} a_y e^{wl} + \frac{iq}{w} b_y e^{-wl},$$
$$A\rho_3\varkappa_1 e^{-\varkappa_1 l} + B\rho_4\varkappa_2 e^{-\varkappa_2 l} = 2\pi q\, e^{-w(l-z')}\sin\alpha$$
$$-(a_y\cos\alpha - a_x\sin\alpha) wq\, e^{wl} + (b_y\cos\alpha - b_x\sin\alpha) wq\, e^{-wl}, \qquad 12)$$

where

$$\rho_\xi = i\varkappa_\xi\cos\alpha - d_\xi q\sin\alpha.$$

Eliminating successively all the functions except α and β, we obtain the following equations:

$$\alpha F_{11} + \beta F_{12} = E_1(w) e^{-wz'} - E_1(-w) e^{wz'}$$
$$\alpha F_{21} + \beta F_{22} = E_2(w) e^{-wz'} - E_2(-w) e^{wz'}. \qquad (13)$$

The quantities contained here have the following structure:

$$F_{ik} = F_{ik}(w) - F_{ik}(-w),$$
$$F_{11}(w) = (\rho_1 - \rho_4)(\varkappa_1 + w)^2 e^{wl},$$
$$F_{12}(w) = (\rho_2 - \rho_4)(\varkappa_1 + w)(\varkappa_2 + w) e^{wl},$$
$$F_{21}(w) = (\rho_1 - \rho_3)(\varkappa_1 + w)(\varkappa_2 + w) e^{wl},$$
$$F_{22}(w) = (\rho_2 - \rho_3)(\varkappa_2 + w)^2 e^{wl},$$
$$E_1(w) = 4\pi q(\varkappa_1 + w) e^{lw}\sin\alpha, \quad E_2(w) = 4\pi q(\varkappa_2 + w) e^{wl}\sin\alpha.$$

The functions $\alpha(z')$ and $\beta(z')$, obtained from (13), must be substituted in the first four equations of the system (12). From these equations we get the unknown functions $a_x(z')$, $b_x(z')$, $a_y(z')$, and $b_y(z')$. Knowledge of these functions enables us to calculate by means of formulas (8) the Green's functions D_{xx}, D_{yx}, and D_{zz} in the region $0 \leq z \leq l$. The Green's functions (8) have the following structure:

$$D(z, z') = f_1(z - z') + f_2(z + z').$$

It can be shown that the second term makes no contribution to expression (4) for the force, so that we shall henceforth give only the expressions for the functions D^+, which depend only on $z - z'$.

In addition, we must subtract the infinite parts in the Green's functions D^E and D^B. This is equivalent to subtraction of terms of the type $-2\pi w^{-1} \exp[-w|z - z'|]$ in formula (8) for D_{xx}. As a result of such calculations we obtain the following expressions for the Green's functions:

$$D^+_{xx}(z - z', \mathbf{q}; \omega_n) = \frac{2\pi}{w} e^{w(z-z')}$$
$$+ \hat{S}_{12}\hat{S}_w \frac{(iw\cos\alpha - \rho_1)(\varkappa_1 + w)}{2qw\Delta\sin\alpha}[F_{22}E_1(w) - F_{12}E_2(w)] e^{w(z-z')},$$
$$D^+_{yx}(z - z', \mathbf{q}; \omega_n) = i\hat{S}_{12}\hat{S}_w \frac{\varkappa_1 + w}{2q\Delta}[F_{22}E_1(w) - F_{12}E_2(w)] e^{w(z-z')},$$
$$D^+_{zx}(z - z', \mathbf{q}; \omega_n) = \hat{S}_{12}\hat{S}_w \frac{\varkappa_1 + w}{2w\Delta}[F_{22}E_1(w) - F_{12}E_2(w)] e^{w(z-z')}, \qquad (14)$$

where $\Delta = F_{11}F_{22} - F_{12}F_{21}$. In formulas (14), to abbreviate the notation, we have introduced the symmetrization operators

$$\hat{S}_{12} f(\varkappa_1, \varkappa_2) = f(\varkappa_1, \varkappa_2) + f(\varkappa_2, \varkappa_1),$$
$$\hat{S}_w f(w) = f(w) + f(-w).$$

Analogous calculations enable us to find all the remaining components of the Green's functions, after which it remains only to go over to the initial system coordinates X, Y, Z and substitute the obtained values for D^E_{ik} and D^B_{ik} in formula (4) for the force. As a result we obtain the following expression:

$$F_\mu(l) = \frac{T}{2\pi^2}\int_0^{2\pi} d\alpha \int_0^\infty q\,dq \sum_{n=0}^{\infty}{}' w\hat{S}_{12}\left[\frac{F_{11}(w)F_{22} - F_{21}(w)F_{12}}{F_{11}(-w)F_{22} - F_{21}(-w)F_{12}} - 1\right]^{-1} \qquad (15)$$

In the case $\hbar c/l \ll kT$ of interest to us, formula (15) takes the form

$$F_\mu(l) = \frac{T}{32\pi^2 l^3}\int_0^{2\pi} d\alpha \int_0^\infty x^2\,dx$$
$$\times\left\{\left[\frac{\sqrt{\omega_0 + 4\pi g M_0\cos^2\alpha} + \sqrt{\omega_0\omega_1/\omega_2}}{\sqrt{\omega_0 + 4\pi g M_0\cos^2\alpha} - \sqrt{\omega_0\omega_1/\omega_2}}\right]^2 e^x - 1\right\}^{-1}. \qquad (16)$$

Calculation of the force in the case $\epsilon \neq 1$ are quite cumbersome, so that we present only the final result:

$$F(l) = F_\mu(l_1 + F_\varepsilon(l), \qquad (17)$$

where $F_\epsilon(l)$ is the force of attraction between nonmagnetic dielectrics[1]:

$$F_\varepsilon(l) = \frac{T}{16\pi l^3}\int_0^\infty x^2\,dx\left\{\left(\frac{\varepsilon_0 + 1}{\varepsilon - 1}\right)^2 e^x - 1\right\}^{-1}.$$

Expression (16) is essentially positive, therefore allowance for the fluctuations of the magnetic moment leads to the appearance of an additional attraction force, which comes into play at distances

$$l \gg \frac{\hbar c}{kT} \sim 10^{-4}\ \text{c.m.}$$

In conclusion the author is grateful to V. G. Bar'yakhtar and S. V. Peletminskiĭ for suggesting the topic and a discussion of the work.

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Translated by J. G. Adashko
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