TOTAL BREMSSTRAHLUNG SPECTRUM OF ULTRARELATIVISTIC ELECTRONS

V. N. BAĬER, V. S. FADIN and V. A. KHOZE

Submitted July 7, 1967

Zh. Eksp. Teor. Fiz. 53, 2194–2202 (December, 1967)

Bremsstrahlung accompanying the collisions of ultrarelativistic electrons and positrons has been investigated, including the hard end of the spectrum. Bremsstrahlung cross sections have been obtained valid for all photon energies. It has been found that at the end of the spectrum the bremsstrahlung cross section for electron-positron collisions is appreciably greater than the corresponding cross section for electron-electron collisions.

1. Recently the process of emission of bremsstrahlung photons in electron-electron and electron-positron collisions has been intensively investigated in experiments on colliding beams[1,2]. The investigation of this process is of considerable interest since it can be utilized for monitoring collisions of the beams, and also from the point of view of studying radiation corrections. In a paper by the present authors[3] (cf., also[4]) the spectrum and the angular distribution of the emitted photons have been obtained which are valid for the whole range of frequencies with the exception of the end of the spectrum[1)]

$$E - \omega \gg m, \tag{1}$$

while at the end of the spectrum the photon energy is given by

$$\omega_{max} = E - m^2/E. \tag{2}$$

In this paper we consider the hard end of the spectrum of the emitted photons and we obtain general expressions describing the whole spectrum of bremsstrahlung from ultrarelativistic electrons. In this whole paper we shall systematically expand all quantities in powers of m^2/E^2 and retain only the leading terms of the expansion. As will be seen below the end of the emission spectrum has interesting specific properties, in particular, the bremsstrahlung cross sections for electron-electron and electron-positron collisions, which coincide when condition (1) is satisfied, differ appreciably at the end of the spectrum. The importance of the role played by recoil and the occurrence of interference lead to an appreciable qualitative difference between the case under consideration and the end of the bremsstrahlung spectrum for an electron in a Coulomb field.

2. The process of bremsstrahlung in electron-electron (electron-positron) collisions is described in the e^6-approximation of perturbation theory by eight diagrams (Fig. 1), where each diagram corresponds to Feynman diagrams with the emission of a photon before and after a collision. In the case of electron-positron collisions it is necessary to make the substitution

$$p_2 \to -p_1^+, \qquad p_4 \to -p_2^+. \tag{3}$$

In[3] it was shown that when condition (2) is satisfied with an accuracy up to terms of order m^2/E^2 it is necessary to consider only the contributions of the squares of the matrix elements of the diagrams a, b, a′, b′ for electron-electron collisions, and of diagrams a, b for electron-positron collisions. With the indicated accuracy it is possible to neglect all the interference terms. For the scattering diagrams in both processes this circumstance is associated with the fact that, firstly, an essential contribution is made only by small scattering angles, and, secondly, all the particles are ultrarelativistic and radiate into narrow cones of angle m/E; as a result the interference between diagrams associated with the overlapping of cones turns out to be $\sim m^2/E^2$. For the same reason the interference between direct and exchange diagrams gives no contribution in the case of electron-electron collisions. The contribution of the annihilation diagram and the interference between annihilation diagrams and scattering diagrams for electron-positron collisions drops out because of the large value of the transferred momentum ($\sim E^2$) in the annihilation diagram.

At the end of the spectrum the situation is changed in an essential manner. For photon energy

$$\omega = 2E(E-m)/(2E-m)$$

one of the final particles having emitted a hard photon can come to rest. At the very end of the spectrum for $\omega = \omega_{max}$ (2) both final particles are moving along the same straight line in the direction opposite to the direction of the photon momentum. As a result of this the choice of terms giving a contribution with the indicated degree of accuracy ($\sim m^2/E^2$) becomes different. It is true that the interference between the contributions of diagrams in which different initial particles radiate ((a, b), (a′, b), (a, b′), (a′, b′) for electron-electron collisions and (a, b), (a, b′), (b, b′), for electron-positron collisions), gives no contribution for the same combination of reasons. The interference between contributions of diagrams (a′, b′) for electron-positron collisions is exactly equal to zero[5]. No contribution is made likewise by the square of the contribution of the

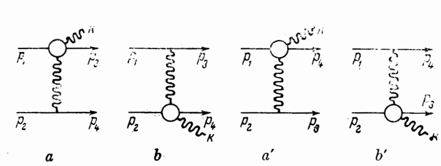

FIG. 1.

[1)]We consider bremsstrahlung in the c.m.s. for the initial particles and we utilize the notation of the preceding article[3].

annihilation diagram in which the final particles radiate due to the large value of the transferred momentum. But the square of the contribution of the annihilation diagram in which the initial particles radiate is by no means small because at the end of the spectrum the transferred momentum becomes of order m^2 (cf.[6]); also the interference contributions of the diagrams do not disappear in which the same initial particle radiates ((a, a′), (b, b′) for electron-electron and (a, a′), (b, a′) for electron-positron collisions). More exact estimates of the order of the neglected terms are given with the aid of the Schwarz inequality in Sec. 7.

3. We now obtain the contribution of the square of the matrix element of diagram a which is the same for both processes under consideration. The exact expression for the contribution of diagram a was obtained in[7] (formula (40)), and in the case when condition (1) is fulfilled the approximate expression with an accuracy up to terms $\sim m^2/E^2$ is given in[3] (formula (1)).

We now write out the terms giving the contribution to the spectrum of emitted photons with an accuracy up to terms $\sim m^2/E^2$ which is valid for all photon energies including also the end of the spectrum:

$$d\sigma_s = \frac{r_0^2\alpha}{\pi\nu^3}\int \frac{d\varkappa_1 d\varkappa_2 d\varkappa_3 d\Delta^2}{\Delta^4\sqrt{S}}\left\{-\frac{2\nu^2}{\varkappa_3^2}+\frac{1}{\varkappa_1\varkappa_3}\left[4\nu^2\left(1-\frac{\varkappa_2}{\nu}\right) + \Delta^2\nu^2\left[\left(1-\frac{\varkappa_2}{\nu}\right)^2+1\right]\right] - \frac{2\nu^2}{\varkappa_1^2}\left(1-\frac{\varkappa_2}{\nu}\right)^2 - \frac{\varkappa_1}{\varkappa_3}-\frac{\varkappa_3}{\varkappa_1} - \frac{\Delta^2}{\varkappa_1}\left(\nu-\frac{\varkappa_3}{2}\right)\right\}, \tag{4}$$

where

$$\nu = -(p_1p_2),\ \varkappa_i = -(kp_i),\ \Delta = p_2 - p_4,\ \varkappa = \varkappa_1+\varkappa_2 = \varkappa_3+\varkappa_4,$$
$$S = -[Q_i\varkappa_i^2 - 2P_i\varkappa_i + R_i], \qquad i = 1, 2, 3, 4. \tag{5}$$

and Q_3, P_3, R_3 coincide respectively with Q, P, R of[3] (formula (5)), while Q_1, P_1, R_1 are equal respectively to the quantities Q, P, R of[7] (formulas (33), (34)), if each of the latter is multiplied by $((\nu + 1)/\nu)^2$, and also the following equalities hold $Q_1 = Q_2$, $Q_3 = Q_4$. Here and in future we utilize the metric $(ab) = \mathbf{ab} - a_0b_0$ and $\hbar = c = m = 1$. The last term in the curly brackets in (4) gives a contribution only at the end of the spectrum.

The calculation of the spectrum of emitted photons taking into account its end is, generally speaking, quite complicated. But the situation can be considerably simplified if we choose for each term the appropriate order of integration. The ranges of integration when it is carried out in the following order: first over κ_1, then over Δ^2, then over κ_3 (κ_1, Δ^2, κ_3), are given in[7], and for the sequence (κ_3, Δ^2, κ_1) follow easily from formulas (7)–(9) of[3]. However, we note that in order to obtain the differential cross section with respect to the angle of emission of the photon we can utilize only the sequence (κ_3, Δ^2, κ_1).

In finding the contributions of the squares of the matrix elements of the diagrams the first integration is carried out in analogy to[3,7]:

$$I_i^{(n)} = \int \frac{\varkappa_i^n\, d\varkappa_i}{\sqrt{S}}. \tag{6}$$

Here we have

$$I_i^{(1)} = \frac{\pi P_i}{Q_i^{3/2}}, \quad I_i^{(0)} = \frac{\pi}{Q_i^{1/2}}, \quad I_i^{(-1)} = \frac{\pi}{R_i^{1/2}}, \quad I_i^{(-2)} = \frac{\pi P_i}{R_i^{3/2}}. \tag{7}$$

We consider the region of integration over the variables Δ^2, κ_1. Utilizing formulas (8), (9) of[3] we have

$$\Delta^2_{\substack{max\\min}} = \rho + \varkappa_1 \pm \nu Q_3^{1/2}\sqrt{\eta},$$
$$\frac{\varkappa}{2}\left(1-\frac{p}{E}\right) \leqslant \varkappa_1 \leqslant \frac{\varkappa}{2}\left(1+\frac{p}{E}\right), \tag{8}$$

where

$$\rho = \nu - \varkappa - 1 = 2E^2(1-\xi-1/E^2),$$
$$\lambda = \rho + 2 = \nu - \varkappa + 1 = 2E^2(1-\xi),$$
$$\eta = \rho/\lambda, \quad \xi = \omega/E, \quad p = \sqrt{E^2-1}, \tag{9}$$

the quantity η is characteristic for describing the behaviour of the end of the spectrum. At the end point of the spectrum $\rho(\omega_{max}) = 0$ (cf., (2)). The region of integration is shown in Fig. 2. The width of the region over Δ^2 for a given κ_1 is proportional to $\sqrt{\eta}$ and at the end point of the spectrum reduces to zero, so that the region degenerates into a segment of the straight line $\kappa_1 = \Delta^2$.

The calculation of the bremsstrahlung spectrum is simplified considerably if we carry it out in such a sequence that the terms with P_i do not arise in the numerator (formula (7)). As a result of this we obtain

$$d\sigma_s = r_0^2\alpha\frac{d\xi}{\xi}\left\{2(1-\xi)\left[1-\xi+\frac{1}{1-\xi}-\frac{2}{3}\right](L-\sqrt{\eta}) + \sqrt{\eta}\left[1-\frac{L^2}{2\rho}-\frac{1}{\rho^2}\left(\frac{L}{2}-\frac{\sqrt{\rho\lambda}E}{p}\right)^2\right]\right\} \tag{10}$$

where

$$L = 2\ln\left(\frac{1+E\sqrt{\eta}/p}{1-E\sqrt{\eta}/p}\right). \tag{11}$$

When condition (1) is satisfied the contribution is given only by the first term ($\eta \to 1$), obtained in[3] (formula (16)). At the end of the spectrum both terms are essential. Near the end point of the spectrum ($\eta, \rho \ll 1$) the inverse powers of ρ entering into the second term in formula (10) cancel so that the cross section assumes the form

$$d\sigma_s = 3r_0^2\alpha\frac{d\xi}{\xi}\sqrt{\eta}, \quad \eta \ll 1, \tag{12}$$

and at the end point of the spectrum tends to zero like $\sqrt{\eta}$. We note that the quantity $\sqrt{\eta}$ is proportional to the momentum of the final particle in the c.m.s. for the final particles.

The contribution of diagrams b to the spectrum is, naturally, equal to the contribution of diagrams a, it corresponds to radiation along the particle 2. The exchange diagrams make the same contributions to the

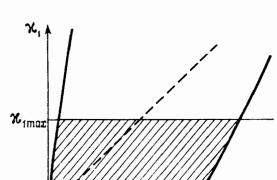

FIG. 2.

spectrum as the direct ones, but in virtue of the identical nature of the particles the total contribution should be divided by 2, so that the total contribution to the spectrum of the squares of the matrix elements of the diagrams is $2d\sigma_S$.

4. We now go on to consider the interference terms for the case of electron-electron collisions. We have already noted that at the end point both final particles move in the same direction with the same momenta. Therefore, the direct and the exchange diagrams in the case of the emission of a given particle give the same contributions and the interference between them is significant. From here it also follows that at the end point of the spectrum in virtue of the Pauli principle the final electrons cannot be in the same spin states. Thus, we must consider the interference of the contributions of the diagrams a, a′ and b, b′; the contributions of both interference terms to the spectrum are the same, and the total contribution, as has already been noted, should be divided by 2. Thus, it is sufficient to find the contribution of the interference of diagrams a and a′. We now write out the terms which give a contribution to the spectrum:

$$d\sigma_{ei} = -\frac{r_0^2\alpha}{\pi\nu^3}\int\frac{d\varkappa_1\,d\varkappa_2\,d\varkappa_3\,d\Delta^2}{\sqrt{S}\,\Delta^2\Delta'^2}\cdot\left\{-\frac{\nu}{\varkappa_1}+\frac{\nu\varkappa}{\varkappa_3}+\frac{\nu\varkappa}{\varkappa_4}-\frac{\Delta^2\varkappa^2}{2\varkappa_1\varkappa_3}-\frac{\Delta'^2\varkappa^2}{2\varkappa_1\varkappa_4}\right\}, \tag{13}$$

where

$$\Delta' = p_2 - p_3, \quad \Delta'^2 = -\Delta^2 + 2(\rho+\varkappa_1). \tag{14}$$

Carrying out the integration in the appropriate sequence (cf., the preceding section) we obtain the contribution of the interference to the spectrum in the case of an electron-electron collision

$$d\sigma_{ei} = r_0^2\alpha\frac{d\xi}{\xi}\left\{-\frac{L}{\rho}\ln(1+2\rho)+\sqrt{\eta}\left[\frac{L^2}{\rho}+\frac{2}{\rho^2}\left(\frac{L^2}{4}\right.\right.\right.$$
$$\left.\left.\left.-\frac{(1+E/p)L\sqrt{\lambda\rho}}{2(\sqrt{2\lambda}-1)}+2\sqrt{2\lambda}\frac{E}{p}\ln(\sqrt{2\lambda}-1)\right)\right]+2\sqrt{2}\,g_1-2g_2\right\}. \tag{15}$$

Here

$$g_1 = \frac{1}{\rho^{3/2}}\left[-2\ln(\sqrt{2\lambda}-1)+\frac{L_1\sqrt{2\rho}}{\sqrt{2\lambda}-1}-\ln r^{1/2}\operatorname{arctg} r^{1/2}\right.$$
$$\left.+\ln r^{3/2}\operatorname{arctg} r^{3/2}+\int_{r^{3/2}}^{r^{1/2}}\frac{\operatorname{arctg} y}{y}\,dy\right],$$

$$g_2 = \frac{1}{\rho}\left[2F(r)-F(r(1+2\rho))-F\left(\frac{r}{1+2\rho}\right)+\frac{\ln^2(1+2\rho)}{2}\right], \tag{16}$$

where

$$r = \frac{1-\sqrt{\eta}}{1+\sqrt{\eta}}, \quad L_1 = -\ln r, \quad F(x) = \int_0^x\frac{\ln(1-y)}{y}\,dy. \tag{17}$$

The integral appearing in g_1 cannot be evaluated in terms of elementary functions, but it can be represented in the form of a rapidly converging series convenient for carrying out calculations:

$$\int\frac{\operatorname{arctg} y}{y}\,dy = \sum_{n=0}^{\infty}(-1)^n\frac{y^{2n+1}}{(2n+1)^2}. \tag{18}$$

For $\rho \ll 1$ we have

$$g_1 = {}^5\!/\!_8\sqrt{\rho}+\ldots, \quad g_2 = 2\sqrt{2\rho}+\ldots, \tag{19}$$

for $1 \ll \rho \ll \nu$

$$g_1 = \frac{1}{2\sqrt{2}}\frac{\ln 2\rho+2}{\rho^2}+\ldots, \quad g_2 = \frac{1}{\rho}\left(\frac{\ln^2 2\rho}{2}+\frac{\pi^2}{6}\right)+\ldots. \tag{20}$$

The whole interference contribution for $\rho \ll 1$ has the form

$$d\sigma_{ei} = -4r_0^2\alpha\frac{d\xi}{\xi}\sqrt{\eta}, \quad \eta \ll 1. \tag{21}$$

The complete bremsstrahlung spectrum in the case of electron-electron collisions is given by

$$d\sigma_e = 2d\sigma_s + d\sigma_{ei} \tag{22}$$

and is given by formulas (10) and (15). For $\rho \ll 1$ the bremsstrahlung spectrum is given by formulas (12) and (21).

5. In the preceding sections we have utilized a very rational method of obtaining the bremsstrahlung spectrum. A much more awkward approach is the one involving the calculation of the differential cross section with respect to the angles of emission of the photon. But this cross section has an independent value for the analysis of the distribution of the emitted photons, and, therefore, we shall quote it here for the case of electron-electron collisions. With the adopted degree of accuracy we have

$$d\sigma_s = \frac{r_0^2\alpha}{2}\frac{d\varkappa}{\varkappa}\frac{d\varkappa_1}{\nu^3}\left\{\frac{\varkappa(\nu-\varkappa)}{\varkappa_1^2}\left[\left(\varkappa-3\nu+\frac{8\varkappa}{\varkappa_1}-\frac{4\varkappa^2}{\nu\varkappa_1^2}\right)\sqrt{\eta}\right.\right.$$
$$\left.+\left(2\nu+\frac{\varkappa^2}{\nu-\varkappa}+\frac{2\varkappa}{\varkappa_1}\left(1-\frac{\varkappa}{\nu\varkappa_1}\right)\right)L\right]-\frac{\varkappa\nu\mathcal{L}_1}{\varkappa_1\sqrt{Q_3}}$$
$$-\frac{\varkappa\sqrt{\eta}}{2\nu Q_3}\left[\nu\left(1+\frac{\lambda}{\varkappa_1}\right)+\frac{2\nu(\nu-\varkappa)\lambda}{\varkappa_1^2}\right]$$
$$\left.+\frac{\mathcal{L}_1\varkappa}{2\varkappa_1 Q_3^{3/2}\nu^2}[\lambda(-\varkappa+\varkappa_1\nu)+\varkappa_1^2\nu]+(\varkappa_1\leftrightarrow\varkappa_2)\right\}, \tag{23}$$

where

$$\mathcal{L}_1 = \frac{\rho+\varkappa_1+\nu\sqrt{Q_3\eta}}{\rho+\varkappa_1-\nu\sqrt{Q_3\eta}}, \tag{24}$$

$$d\sigma_{ei} = -\frac{r_0^2\alpha\,d\varkappa\,d\varkappa_1}{2\nu^3}\left\{\frac{1}{\rho+\varkappa_1}\left[-\frac{\nu\mathcal{L}_1}{\varkappa_1 Q_3^{1/2}}+\frac{\nu\varkappa}{\varkappa_1}L\right.\right.$$
$$\left.\left.+\frac{\varkappa}{\sqrt{c}}\mathcal{L}_2\left(\nu-\frac{\varkappa(\rho+\varkappa_1)}{\varkappa_1}\right)\right]+(\varkappa_1\leftrightarrow\varkappa_2)\right\}, \tag{25}$$

where

$$\mathcal{L}_2 = \ln\frac{\varkappa_2(\lambda\rho-\varkappa)+\varkappa^2+\nu\sqrt{c\lambda\rho}}{\varkappa_2(\lambda\rho-\varkappa)+\varkappa^2-\nu\sqrt{c\lambda\rho}}$$
$$c = \frac{\varkappa_2}{\nu^2}[\varkappa_2(\lambda\rho-2\varkappa)+2\varkappa^2]. \tag{26}$$

The total differential cross section with respect to the angles and frequencies is obtained on substituting the quantities found above into formula (22).

6. We now go on to the case of bremsstrahlung accompanying an electron-positron collision. In this case the cross section contains the squares of the contributions of the scattering diagrams (the same as in the case of electron-electron scattering), the square of the contribution of the annihilation diagram in which the initial particle is radiating, and the interference terms between the diagrams indicated above. The square of the contribution of the annihilation diagram has been

found in[5] (formula (2.25)); taking into account the ultrarelativistic approximation adopted by us it has the form

$$d\sigma_a = \frac{2r_0^2\alpha}{\lambda}\frac{d\xi}{\xi}\left(1-\frac{\eta}{3}\right)\sqrt{\eta}\left(\ln(2E)-\frac{1}{2}\right). \tag{27}$$

We note that from the law of conservation of charge parity a pair of particles in this diagram at the very end of the spectrum is created in the triplet state.

We now write out the interference terms giving a contribution to the spectrum:

$$d\sigma_{pi} = \frac{r_0^2\alpha}{2\pi\nu^3}\frac{1}{\lambda}\int\frac{d\varkappa_1\, d\varkappa_2\, d\varkappa_3\, d\Delta^2}{\Delta^2\sqrt{S}}\Big\{\varkappa_1+\nu-\Delta^2 + \frac{1}{\varkappa_1}\Big[\varkappa_3(2\lambda-1)-\varkappa(\nu-\varkappa)-\frac{\lambda\varkappa_3^2}{\nu}+\frac{\Delta^2}{2}(2\varkappa-\varkappa_3-\Delta^2)\Big] + \frac{1}{\varkappa_2}\Big[-2\nu^2-\nu\varkappa_4-\varkappa_4^2+\frac{\Delta^2}{2}(8\nu-3\varkappa_3-2\Delta^2)\Big] + \frac{\nu}{\varkappa_3}\Big(-\nu-\varkappa_1+\frac{3\Delta^2}{2}\Big)+\frac{\Delta^2}{2\varkappa_1\varkappa_3}\Big[-\varkappa^2+\Delta^2\Big(\nu-\frac{\Delta^2}{2}\Big)\Big] + \frac{1}{\varkappa_2\varkappa_3}\Big[2\nu^3-3\nu^2\Delta^2+\frac{\Delta^4}{2}\Big(3\nu-\frac{\Delta^2}{2}\Big)\Big]\Big\}. \tag{28}$$

The integration of this expression is carried out in the same way as in the case of electron-electron collisions. As a result of this we obtain

$$d\sigma_{pi} = \frac{r_0^2\alpha}{\lambda}\frac{d\xi}{\xi}\Big\{\frac{L}{8}\Big[5+\eta+2\ln(2\lambda)-\frac{1+\eta}{\sqrt{\eta}}L\Big] - \frac{\sqrt{\eta}}{2}+F\Big(\frac{1+\sqrt{\eta}}{2}\Big)-F\Big(\frac{1-\sqrt{\eta}}{2}\Big)\Big\}, \tag{29}$$

where the quantity L is determined by formula (11), while the Spence function F(x) is defined by formula (17). For $\rho \ll 1$ we have

$$d\sigma_{pi} = 0 + r_0^2\alpha\frac{d\xi}{\xi}O\Big(\eta^{3/2},\ \frac{\eta^{1/2}}{\nu}\Big). \tag{30}$$

The complete expression which applies to the whole bremsstrahlung spectrum in the case of an electron-positron collision has the form

$$d\sigma_p = 2d\sigma_s + d\sigma_a + 2d\sigma_{pi} \tag{31}$$

and is given by formulas (10), (27), and (29). Near the end of the spectrum one must correspondingly substitute into (31) the expressions (12) and (27).

7. We now make a few remarks with respect to the method of selecting the leading terms with the aid of the Schwarz inequality. To make the selection it is necessary to pick out the leading terms in the exact squares of the matrix elements of the scattering diagrams. The rigorous analysis that has been carried out shows that such a leading term, with an accuracy up to a logarithmic factor in the differential cross section with respect to the angle and the frequency, is given, just as in[3], by

$$r_0^2\alpha\frac{d\varkappa\, d\varkappa_1}{\varkappa_1^2\nu}. \tag{32}$$

It turns out that an estimate made with the aid of the Schwarz inequality for the interference of the scattering diagrams for both processes does not depend on the photon frequency and has the same form as formula (20) in[3]; in a similar manner one can demonstrate the disappearance of the contributions of the interference of diagrams (a′, b′), (a, b′), (a′, b) for the case of electron-electron collisions. For an estimate of the contributions of the interference of the diagram b′ (emission from final particles) and of the diagrams a, b for electron-positron collisions we utilize formula (2.35) of[5]. The leading term in the differential cross section for the emission by final particles has the form

$$d\sigma \sim \frac{r_0^2\alpha}{\nu}\frac{d\varkappa_1\, d\varkappa}{\varkappa^2}, \tag{33}$$

so that it is of order $1/\nu$. But then the contribution of the interference of the diagrams indicated above does not exceed

$$\frac{1}{\nu}\frac{d\varkappa}{\varkappa}\frac{d\varkappa_1}{\varkappa_1} \sim \frac{1}{\nu}, \tag{34}$$

and, consequently, is neglected as a quantity of order $1/\nu$ and not of order $1/\sqrt{\nu}$, as had been assumed by Altarelli and Bucella[4]).

8. We now discuss the results obtained above. We note first of all that at the end of the spectrum the square of the contribution of the annihilation diagram (formula (27)) contains the "large" logarithm $\ln(2\nu)$, while at the same time all the remaining contributions to the cross section of both processes do not contain it. Therefore, at the end of the spectrum the bremsstrahlung cross section for an electron-positron collision exceeds appreciably the bremsstrahlung cross section for an electron-electron collision[2)]. This circumstance is related to the fact that the integration over the angles of emission of the photon in diagram (a′) in the case of electron-positron collisions is carried out for a constant momentum transfer, i.e., the contribution comes from the whole region of variation of the angles at which photons are emitted. At the same time for scattering diagrams for $\rho \sim 1$ the transfer increases with an increase in κ_1 (in the limit the domain of integration degenerates into the straight line $\Delta^2 = \kappa_1$). This is related to the fact that for $\rho \sim 1$ both final particles move in the same direction opposite to the direction of emission of the photon, and with increasing κ_1 (deviation of the direction of emission of the photon from $\mathbf{p}_1$) the angle between the vectors $\mathbf{p}_2$ and $\mathbf{p}_4$ increases, and this leads to an increase in Δ^2.

These considerations also indicate that the angular distribution of photons in both cases has peaks in the direction of motion of the initial particles for any arbitrary photon energy including the end of the spectrum, but at the end of the spectrum the angular distribution of the bremsstrahlung photons in the case of electron-positron collisions is considerably broader than in the case of electron-electron collisions.

The shape of the ends of the spectrum for a specific energy E = 2000 is given in Fig. 3. Here there are shown: the bremsstrahlung cross section evaluated in[3] (curve 1),

$$d\sigma_c = 4r_0^2\alpha\frac{d\xi}{\xi}(1-\xi)\Big[\frac{1}{1-\xi}+1-\xi-\frac{2}{3}\Big]\Big(2\ln\frac{4E^2(1-\xi)}{\xi}-1\Big); \tag{35}$$

the bremsstrahlung cross section $d\sigma_e$ (22) (curve 2); the bremsstrahlung cross section $d\sigma_p$ (31) (curve 3).

The first of these cross sections has the same shape both for electron-electron and electron-positron colli-

2)The characteristic feature of the end of the spectrum of emission by an initial particle in the case of annihilation has been pointed out in[6].

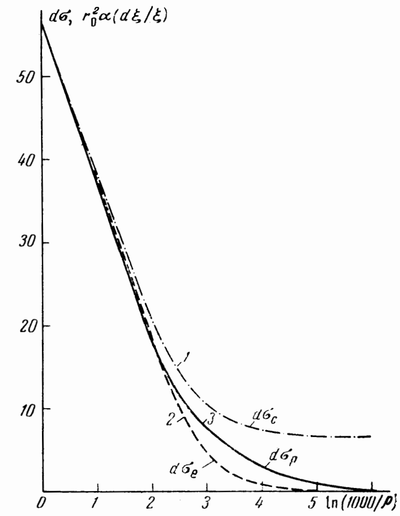

FIG. 3.

sions in the c.m.s. and l.s. and for the bremsstrahlung radiation in a Coulomb field (with an accuracy up to the relativistic recalculation of the energy in the argument of the logarithm). This fact is associated with the circumstance that the main contribution is given by small transferred Δ^2 and small scattering angles. At the end of the spectrum the situation is essentially altered. Firstly, as a result of recoil a kinematic difference appears in the case of bremsstrahlung accompanying a collision of electrons or accompanying scattering in a Coulomb field (for the limiting frequency $E - \omega = 1/E$ in the former case and $E - \omega = 1$ in the latter case); secondly, interference and emission accompanying annihilation (for electron-positron collisions) becomes essential. Therefore, $d\sigma_s$ (10) does not coincide with the ultrarelativistic limit of the formula for the bremsstrahlung cross section in the case of a Coulomb field; moreover, $d\sigma_e$ (22) and $d\sigma_p$ (31) contain additional terms. In Fig. 3 it can be seen that in accordance with the assertions made above at the end of the spectrum the bremsstrahlung cross section for electron-positron collisions at this energy exceeds by a factor of several times the cross section for electron-electron collisions. We note that the latter characteristic feature is strengthened by the fact that in the second case $d\sigma_{ei}$ (15) changes its sign near the end of the spectrum and becomes negative.

We note in conclusion that the difference between the cross sections (22) or (31) and the approximate formula (35) essentially becomes noticeable only for $\rho \ll E$, i.e., $E - \omega \ll 1$. This range of frequencies makes a very small contribution to the cross section integrated over the frequency, since we have shown that at the end of the spectrum there are no local rises in the cross section. Therefore, approximating the whole spectrum by formulas of the type (35) is sufficiently good, provided only that we are not specially interested in the end of the spectrum where one should utilize the exact formulas obtained above.

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Translated by G. Volkoff
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