A CRITERION IDENTIFYING WEAKLY-PERTURBING SYSTEMS, BASED ON MEASUREMENT OF ELASTIC SCATTERING

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A criterion is proposed for the identification of systems that perturb weakly the incident wave during scattering. It is based on a comparison of the elastic scattering differential cross sections measured at different incident-particle energies. The criterion is investigated for the case of scattering of neutrons by nuclei and some cases of scattering of fundamental particles. The existence of nuclei that perturb weakly the neutron wave is demonstrated. The Orear formula[1] for nucleon-nucleon scattering and some properties of the angular distribution of elastically scattered high-energy particles are interpreted as being a consequence of the weakness of perturbation of the incident wave.

STUDIES of the interaction and the structure of particles are frequently based on measurements of the scattering cross sections of these particles. In this case, it is of interest to find out the extent to which the scattering particle distorts the incident wave. Indeed, if it turns out in an analysis of elastic scattering that the distortion of incident wave is small during the interaction, then this makes it possible to describe clearly those physical conditions under which all other possible phenomena accompanying the particle scattering take place, and to present a more accurate interpretation of the measured characteristics of these processes.

Such a quantity as the total scattering cross section at a given energy cannot serve as a characteristic of the perturbation of the incident wave inside the system, since the increase of the volume of the interaction region, which distorts slightly the incident wave, leads to an appreciable increase of the total cross section, whereas the distortion can remain weak.

1. NONRELATIVISTIC CASE

1. If the distortion of the incident wave by the interaction is insignificant, it is possible to describe the scattering by means of Rayleigh-Gans-Born approximation[2,3]. The differential cross section for scattering through an angle ϑ in the c.m.s. can be represented in the form

$$\frac{d\sigma}{d\Omega}=\frac{m^2}{4\pi^2\hbar^4}\left|\int\exp\left\{i\frac{(\mathbf{p}_1-\mathbf{p}_2)\mathbf{r}}{\hbar}\right\}U(r)\,d\mathbf{r}\right|^2, \qquad (1)$$

where m is the reduced mass of the colliding particles, $\mathbf{p}_1$, and $\mathbf{p}_2$ are the c.m.s. momenta of the incident particles before and after the scattering, $|\mathbf{p}_1| = |\mathbf{p}_2| = |\mathbf{p}|$, $(\mathbf{p}_1-\mathbf{p}_2)^2 = p^2(1-\cos\vartheta)$, U(r) is the interaction potential, and $E = p^2/2m$ is the energy of the incident particle in the l.s.

It is seen from formula (1) that, regardless of the properties of the potential, the differential scattering cross section depends not on the two factors E and ϑ separately, but only on one quantity $x = \frac{1}{2}E(1-\cos\vartheta)$, i.e., the differential scattering cross section can be written in the form

$$d\sigma/d\Omega = F(\tfrac{1}{2}E(1-\cos\vartheta)). \qquad (2)$$

This property of the differential scattering cross section can be verified experimentally, and since it does not depend on the properties of the potential, it can serve as a criterion for the weakness of the perturbation of the incident wave during the scattering. Indeed, if the interaction perturbs the incident wave to such an extent that it is possible to use the classical formulas for the calculation of the small-angle scattering cross section, then it follows from the self-similarity condition (2) that the scattering cross section should have a Rutherford character.

For classical particles, the impact distance $\rho(\vartheta)$ is determined from the equation (see, for example,[4])

$$E\vartheta = -\rho\int_\rho^\infty \frac{dU}{dr}\frac{dr}{\sqrt{r^2-\rho^2}}.$$

It is seen from the last formula that ρ depends not on the energy and on the angle separately, but on the combination $y = E\vartheta$. Therefore the differential cross section also has the self-similarity property[1)] and can be represented, regardless of the properties of the potential, in the form

$$\frac{d\sigma}{d\Omega}=\left|\frac{1}{\vartheta}\rho(\vartheta)\frac{d\rho}{d\vartheta}\right|=\frac{E}{\vartheta}f(y).$$

If the condition (2) is satisfied in this case, then $F(x) = (E/\vartheta)f(y)$ or $4x^2F(x) = y^3f(y)$. The last equality, however, can be satisfied only if $F(x) = \mathrm{const}\cdot x^{-2}$, just as in scattering by a Coulomb potential.

Consequently, if the differential cross section does not have a form characteristic of Coulomb scattering and it depends only on $x = \frac{1}{2}E(1-\cos\vartheta)$, then it can be stated with a great degree of certainty that the incident wave is weakly disturbed by the interaction.

2. Let us consider several general consequences of the weakness of the perturbation. It follows from (2), in particular, that at all energies the differential cross section for scattering through zero angle should be constant and equal to F(0). A check on the satisfaction of this condition can serve as a criterion for the weakness of the perturbation.

1)This property can serve as an experimental criterion of the quasi-classical nature of the particle motion in the scattering.

The total elastic-scattering cross section $\sigma(E)$ can be calculated from the formula

$$\sigma(E)=\int \frac{d\sigma}{d\Omega}d\Omega=\frac{4\pi}{E}\int_0^E F(x)\,dx. \tag{3}$$

It is easy to see that formula (3) makes it possible (if the criterion for the weakness for the perturbation is satisfied) to calculate the function F(x) from the dependence of the total cross section on the energy

$$F(x)=\frac{1}{4\pi}\left(\frac{dE\sigma}{dE}\right)_{E=x}. \tag{4}$$

Thus, from the known dependence of the total elastic-scattering cross section on the energy at all values of energy smaller than E it is possible to reconstruct the differential scattering cross section at the energy E.

Comparison of the differential scattering cross sections calculated by means of formula (4) with the experimentally measured ones can also serve as a criterion for a verification of the weakness of the perturbation. This criterion is equivalent to (2).

3. In the measurement of the electron cross sections[5] it is customary to present the results of measurements of the dependence of the total and of the transport elastic-scattering cross sections on the energy. By definition, $\sigma_{tr} = \sigma\Delta p/p$, where Δp is the average momentum transfer. From the self-similarity condition (2) it follows that

$$\frac{d}{dE}\left(\frac{\Delta p}{p}\right)=\left(1-\frac{\Delta p}{p}\right)\frac{2}{E}+\left(2-\frac{\Delta p}{p}\right)\frac{1}{\sigma}\frac{d\sigma}{dE}. \tag{5}$$

The last equation admits of experimental verification if the total and transport elastic-scattering cross sections at different energies are known, and can be used as the already mentioned criterion.

4. The extensive experimental material reported in[6,7] makes it possible, by using the proposed criteria, to draw certain conclusions with respect to the distortion of a neutron wave propagating in atomic nuclei.

Since the neutron has a spin and the nuclei cannot be spherically symmetrical, it is useful to note that the differential elastic-scattering cross section averaged over all the asymmetry directions of the scattering center and the spins of both particles has, if the Born approximation is valid, the same fundamental property[2)], i.e., it is a function of only the combination $E(1-\cos\vartheta)$. Indeed, the scattering amplitude and cross section in a state with arbitrarily specified characteristics supplementing the description of the system, are functions of the quantity $E(1-\cos\vartheta)$ only. The averaging reduces in this case to a summation of a series of functions, each of which depends on $E(1-\cos\vartheta)$, so that the sum depends also on this variable. The same circumstance makes it possible to apply the foregoing analysis to the unlikely case when the nucleus is an amorphous mixture of protons and neutrons.

The reduction of the available experimental material, which pertains to measurement of angular distributions of elastically scattered neutrons, allows us to make a few remarks concerning the larger or smaller distortion of the neutron wave upon scattering by nuclei.

The data obtained by measuring the angular distribution of the scattered neutrons are not sufficient to reveal clear-cut regularities relating the atomic number of the scattering nucleus with the degree of distortion of the neutron wave. Therefore, the following statements are only a qualitative character. It has been observed that nuclei for which the criterion of the weakness of the wave distortion is satisfied occur more frequently among the heavy nuclei than among the light ones. Thus, for example, for nuclei with atomic weight smaller than 16 (up to oxygen inclusive), none of the differential cross sections presented by Gordeev, Kardashev, and Malyshev[7] reveal the regularities characteristic of the Born approximation, i.e., they all strongly distort the neutron wave (at any rate for neutron energies smaller than 14 MeV). Among the heavy nuclei, one encounters quite frequently those which distort the neutron wave weekly at neutron energies higher than 2–4 MeV. These include ${}_{13}Al^{27}$, ${}_{26}Fe^{56}$, and ${}_{83}Bi^{209}$; the differential cross sections for scattering by such nuclei as ${}_{29}Cu^{64}$, ${}_{48}Cd^{112}$ and ${}_{92}U^{238}$ exhibit regularities that are characteristic of the Born approximation even at neutron energies ~ 0.5 MeV. There exist several nuclei for which the Born approximation is valid at low energies (up to 3 MeV) and is violated when the energy is increased. The possible cause of this phenomenon will be analyzed later with uranium as an example.

Figure 1 shows several examples of F(x) plots reconstructed from the total cross section measurement data. These plots show the reduced experimental points[6] obtained by measuring the elastic scattering of neutrons by the nuclei ${}_{92}U^{238}$ and ${}_{29}Cu^{64}$ at different energies in the 0.3–2 MeV range. The good agreement

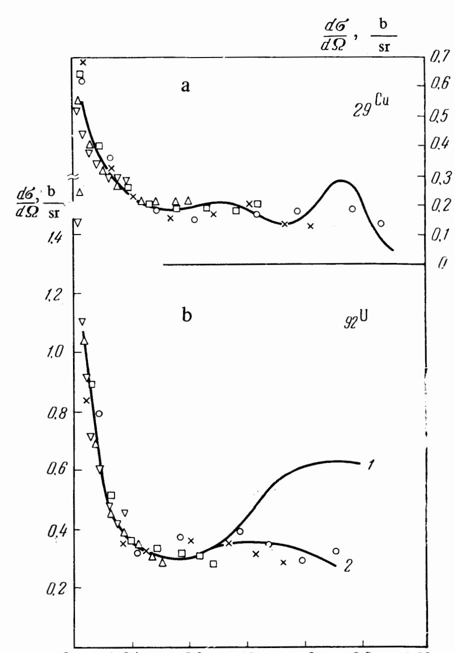


FIG. 1. Differential cross section, as a function of the parameter $x = \frac{1}{2}E_n(1-\cos\vartheta)$, for neutron scattering at different energies from copper and uranium nuclei. a–${}_{29}$Cu: ∇–E_n = 0.435 MeV, Δ–0.83 MeV, □–1.35 MeV, ×–1.75 MeV, ○–2.26 MeV. b–${}_{92}$U: ∇–E_n = 0.34 MeV, Δ–0.64 MeV, □–1.05 MeV, ×–1.54 MeV, ○–1.96 MeV.

[2)]The spin-orbit interaction, which depends on the velocity, leads to a modification of the criterion and is not considered in the present paper.

between the F(x) and the $(d(E\sigma)/dE)_{E=x}$ curves for copper (Fig. 1a) is quite evident. For uranium, the experimental data agree fairly well with the criterion at scattering angles $\vartheta \gtrsim 30°$. The values of $d(E\sigma)/dE$ (see Fig. 1b, curve 1) also coincide with the values of F(x) up to ~1 MeV. However, unlike the case of copper, the differential cross section for small-angle elastic scattering increases with energy at neutron energies $\gtrsim 1$ MeV, and for these energies the values of $d(E\sigma)/dE$ does not coincide with the corresponding value of F(x).

In our opinion, these discrepancies can be attributed to the fact that at energies $\gtrsim 1$ MeV the elastic and inelastic scattering cross sections become comparable. As is well known, inelastic scattering is accompanied by elastic diffraction scattering, the cross section of which is comparable with the inelastic-scattering cross section. Diffraction elastic scattering occurs essentially at small angles and increases the total elastic scattering cross section, thus causing a discrepancy between F(x) and $d(E\sigma)/dE$ at large neutron energies, and also causing an increase of the differential small-angle elastic scattering cross section. It is possible that this explains the already noted satisfaction of the criterion of weak distortion of the incident wave at low energies, and failure to satisfy this criterion with increasing energy.

If the foregoing considerations are valid, we can separate the diffraction elastic scattering from the total elastic scattering for systems that distort the incident wave weakly, and thus obtain the "true elastic scattering." To this end it is sufficient to reconstruct F(x) at small values of x, using the data on the differential elastic-scattering cross section in the energy region where the inelastic scattering cross section is much smaller than the elastic scattering cross section, and the diffraction scattering is negligible. From the obtained function it is possible to reconstruct the differential cross section of the "truly elastic scattering" through small angles at large energies. The large-angle elastic scattering cross section, as expected, is weakly distorted by the diffraction effects, so that it can be regarded as "true."

The reconstructed "truly elastic scattering" differential cross section can be used to calculate the total cross section. This procedure was used for the case of scattering of fast neutrons by uranium. We obtained a cross section σ_1 of the "truly elastic scattering" was used to construct the function $d(E\sigma)/dE$ a plot of which is shown in Fig. 1b (curve 2); this function agrees with the differential cross section data.

We note that the same reasoning, but in reverse sequence, can be used, starting from the data pertaining to elastic scattering, to determine the energy at which the inelastic processes begin to make a contribution comparable with the elastic scattering.

5. Once it is established that the wave is weakly perturbed by the scattering, the interpretation of the measurement data on the elastic scattering cross section can be continued, for in this case the value of the function F(x) makes it possible to determine (apart from the sign) the interaction potential of the colliding particles. Indeed, assuming spherical symmetry of the latter, we can rewrite (1) in the form [3]

$$[F(x)]^{1/2} = \frac{2\pi\hbar}{p\sin(\vartheta/2)}\int_0^\infty rU(r)\sin\left(\frac{2}{\hbar}p\sin\frac{\vartheta}{2}r\right)dr.$$

The last formula can be regarded as the sine-transform of the function rU(r), the inversion of which yield

$$U(r) = \frac{4}{\pi}\frac{1}{r}\int_0^\infty \sqrt{F(x)}\sin\left(\frac{2}{\hbar}\sqrt{2m}\,r\sqrt{x}\right)dx. \tag{6}$$

Using formula (6), we calculated the potential for the interaction of neutrons with the $_{29}Cu^{64}$ nucleus. To establish the values of F(x) we used the data represented in Fig. 1 for $x < 2.5$ MeV, as well as the differential scattering cross section data from [7] at E = 14 MeV and $2.5 \lesssim x < 14$ MeV. The result is shown in Fig. 2 (curve 1), which also shows for comparison the radial dependence of the proton density in the copper nucleus (curve 2)[8].

The error δU in the value of the potential, incurred by replacing the infinite upper limit in (6) by the finite limit $x_{max} \sim 14$ MeV, can be estimated from the formula

$$\delta U \sim \frac{dU}{dr}\frac{\hbar}{\sqrt{2m_n x_{max}}} = \frac{dU}{dr}\cdot 10^{13}\ \text{cm}.$$

The quantity

$$\frac{m}{2\pi\hbar^2}\int U(r)\,r\,dr,$$

which determines the possibility of using the Born approximation for the analysis of the differential scattering cross section, is $1/6 \ll 1$, which does not contradict the already established weakness of the distortion of the wave by the interaction.

Comparison of curves 1 and 2 of Fig. 2 shows that the interaction of the nucleons in the nucleus has an essentially nonlocal dependence on the nuclear density, since the same value of the nuclear density corresponds to different values of the potential.

2. RELATIVISTIC CASE

In the relativistic case, the foregoing analysis of the weakness of the perturbation of the incident wave by the scattering must be modified, first, because of the different connection (compared with the nonrelativistic case) between the energy and the momentum and second, because of the changes that the interaction introduces into the motion. These changes can be due to different causes:

(1) change of the particle mass in the interaction region;

(2) direct change of the energy and momentum of the particle (the mass remaining unchanged).

These two kinds of interaction differ in the transformation properties of the effective interaction potential. Namely, in the first case the interaction has scalar properties, and in the second vector properties.

We consider below the scattering of an incident particle in the c.m.s., for only in this coordinate system is it possible to analyze the elastic-scattering cross section on the basis of potential[3)] scattering.

We shall assume that the interaction potential in the c.m.s. is a real physical quantity, which depends in the rest system of the scattering particle only on the dis-

[3)] The fact that the Klein-Gordon equation with a certain effective potential that becomes local at high energies can be used to describe elastic scattering of relativistic particles was noted earlier in [9,10].

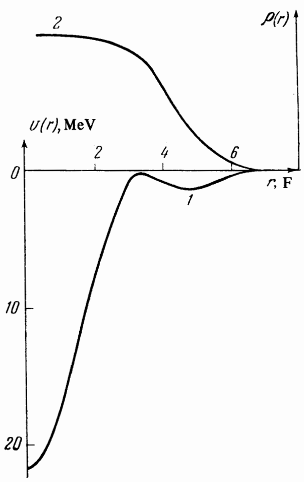


FIG. 2. Potential of the interaction between a neutron and a copper nucleus.

tance between particles. This quantity is scalar for the first type of the indicated interactions and vector for the second type. In the second case we shall assume that only the fourth component of the potential differs from zero in the rest system, something which is not always true and is not satisfied, for example, for electron scattering. It is easy to see that in this case the scattering properties depend on the particle to which the interaction field is ascribed. This remark does not pertain to particles having identical masses.

Case of Scalar Interaction

1. The wave function ψ satisfies in the c.m.s. the Klein-Gordon equation, in which the mass of the particle is distorted by the interaction φ and which can be written, with allowance for the relativistic transformation for the coordinates (Lorentz contraction) to the c.m.s. in the form

$$(E^2 - p^2)\psi = \left[\mu^2 - \varphi\left(\rho^2 + \frac{z^2}{1-\beta_c^2}\right)\right]\psi, \tag{7}$$

where μ is the mass of the incident particle, $E = i\partial/\partial t$, $p = -i\partial/\partial x$, β_c is the velocity of the c.m.s.; $\hbar = c = 1$; ρ and z are cylindrical coordinates with origin at the scattering particle and with z axis along the direction of motion of the incident particle.

In this case, for a weak interaction of the incident wave, the scattering amplitude is determined by the formula

$$f(E, \vartheta) = \frac{1}{4\pi}\int \exp\{i(\mathbf{k}_0 - \mathbf{k}_1)\mathbf{r}\}\, \varphi\left(x^2 + y^2 + \frac{z^2}{1-\beta_c^2}\right) dx\, dy\, dz,$$

where $\mathbf{k}_0$ and $\mathbf{k}_1$ are the wave vectors of the incident and scattered waves in the c.m.s., respectively ($|\mathbf{k}_0| = |\mathbf{k}_1| = k$).

We make the change of integration variable $\mathbf{r} \to \boldsymbol{\rho}$, where $\rho_x = x$, $\rho_y = y$, and $\rho_z = z/\sqrt{1-\beta_c^2}$. Then the scattering amplitude is written in the form

$$f = \frac{\sqrt{1-\beta_c^2}}{4\pi}\int \exp(i\varkappa\rho)\,\varphi(\rho^2)\, dV_\rho, \tag{8}$$

where

$$\varkappa_x = (\mathbf{k}_0 - \mathbf{k}_1)_x, \quad \varkappa_y = (\mathbf{k}_0 - \mathbf{k}_1)_y, \quad \varkappa_z = (\mathbf{k}_0 - \mathbf{k}_1)_z(1-\beta_c^2)^{1/2}.$$

We see that the integral depends only on the modulus of the vector $\boldsymbol{\kappa}$, which is equal to

$$\varkappa = k[\sin^2\vartheta + (1-\beta_c^2)(1-\cos\vartheta)]^{1/2}.$$

Thus, the differential elastic cross section can be written in the case of small distortion of the scattered wave in the form

$$d\sigma/d\Omega = (1-\beta_c^2)F(\varkappa^2). \tag{9}$$

The latter property can be used as a criterion which makes it possible to verify the weak distortion of the incident wave upon scattering.

2. Let us stop to discuss the dependence of κ on the scattering angle. When $\vartheta = 0$ we have $\kappa = 0$; from this it follows directly that the zero-angle differential elastic-scattering cross section is proportional to the factor $(1-\beta_c^2)$, that is, it is constant when $\beta_c \ll 1$ (as already noted above in the discussion of the nonrelativistic particles), and decreases with increasing energy like $1/\epsilon$ (ϵ—energy in the l.s.). When $\beta_c < 1/\sqrt{2}$, the quantity κ as a function of $x = \cos\vartheta$ decreases monotonically from a value $2m\beta_c$ at $x = -1$ to zero at $x = 1$. (Here m—mass of scattering particle.) If $\beta_c > 1/\sqrt{2}$, then $\kappa(x)$ has a maximum equal to $m/\sqrt{1-\beta_c^2}$ at $x = 1 - \beta_c^{-2}$; when $x = -1$, the value of κ is $2m\beta_c$ as before.

Such a dependence of κ on the scattering angle leads to certain experimentally-observable properties of the differential scattering cross section.

(a) when $\beta_c > 1/\sqrt{2}$, the function $\kappa(x)$ assumes certain values twice at $x = x_1$ and $x = x_2$, and the points at which this takes place are connected by the relation $x_1 + x_2 = -2(1-\beta_c^2)$. The same can be said with respect to the dependence of the differential scattering cross section on the angle at a given energy. Since we have $x_1 + x_2 \to 0$ when $\beta_c \to 1$, the points at which the differential scattering cross section has the same value become almost symmetrical with respect to the angle $\vartheta = \pi/2$. It should be noted that the latter circumstance does not always lead to an asymptotically (at large energies) symmetrical (with respect to the angle $\vartheta = \pi/2$) differential scattering cross section. Thus, at high energies the ratio of the cross sections for scattering through the angle $\vartheta = 0$ and through the angle $\vartheta = \pi$ equals $F(0)/F(4m^2)$, which can be different from unity.

(b) When $\beta_c < 1/\sqrt{2}$, the differential scattering cross section as a function of the scattering angle has an extremum at $\cos\vartheta = 1 - \beta_c^{-2}$, since $d\kappa/d\cos\vartheta = 0$ at this point, and consequently

$$\frac{d}{d\cos\vartheta}\left\{\frac{d\sigma}{d\Omega}\right\} = (1-\beta_c^2)\frac{dF}{d\varkappa}\frac{d\varkappa}{d\cos\vartheta} = 0.$$

If $F(\kappa) \to 0$ as $\kappa \to \infty$, then the extremum is a minimum and its position should tend at high energies to the angle $\vartheta = \pi/2$ like $\cos\vartheta = 1 - \beta_c^{-2}$.

3. Using formula (9), we can reconstruct the differential scattering cross section at an energy E from the known dependence of the total cross section on the energy at all energies. Indeed, from (9) we obtain for the total elastic-scattering cross section

$$\sigma(k) = \pi\frac{1-\beta_c^2}{k}\left\{\int_0^{4k^2(1-\beta_c^2)} \frac{F(x)\,dx}{\sqrt{k^2-\beta_c^2 x}} + \left(1 + \frac{k^2-m^2}{|k^2-m^2|}\right)\int_{4k^2(1-\beta_c^2)}^{k^2/\beta_c^2} \frac{F(x)\,dx}{\sqrt{k^2-\beta_c^2 x}}\right\} \tag{10}$$

This is an integral equation with respect to the function F(x). These are convenient numerical methods for solving such equations, such as Tikhonov's regularization method[11].

At large values of k, an approximate solution of (10) can be obtained analytically. To this end it is necessary to replace the upper limit of the first integral and the lower limit of the second one by their limiting values (as $k \to \infty$) $4m^2$. Solving the resultant Abel equation, we have

$$F(\varkappa^2) = \frac{1}{2\pi^2}\frac{\partial}{\partial(\varkappa^2)}\int\limits_{m^2}^{\varkappa^2}\frac{\sigma(z)\sqrt{z-m^2}\,\beta_c(z)\,dz}{[1-\beta_c^2(z)]\sqrt{\varkappa^2-z}}. \qquad (11)$$

Here $z = k^2 + m^2$ is the square of the energy of the incident particle in the c.m.s., and $\sigma(z)$ and $\beta_c(z)$ are the total elastic scattering cross section and the c.m.s. velocity.

Case of Vector Potential

4. In this case the interaction changes the characteristics of the particle motion without changing its mass. Assuming that in the system where the scattering particle is at rest the interaction vector is only a fourth component, which depends in spherically symmetrical fashion on the relative distance between the particles, we get on the c.m.s.

$$\left(E - \frac{\varphi}{\sqrt{1-\beta_c^2}}\right)^2\psi - \left(\mathbf{p} + \frac{\varphi\beta_c}{\sqrt{1-\beta_c^2}}\right)^2\psi = \mu^2\psi$$

(the notation is the same in formula (7)).

Assuming the interaction to be weak ($|\varphi| \ll E$), the effective interaction potential can be written in the form

$$\frac{2E\varphi}{\sqrt{1-\beta_c^2}} + \frac{\varphi^2(1-\beta_c^2)}{1-\beta_c^2} - \frac{p\varphi\beta_c}{\sqrt{1-\beta_c^2}} - \frac{\varphi p\beta_c}{\sqrt{1-\beta_c^2}}$$

In the construction of the matrix element we take into account the fact

$$\hat{\mathbf{p}}\psi_0 = \mathbf{k}_0\psi_0, \qquad \psi_0 = e^{ikz},$$

and in the case where the operator $\hat{p}$ acts on $\varphi\psi$, its action (with allowance for hermiticity) can be regarded as action on the scattered wave, so that it reduces to a multiplication by $\mathbf{k}_1$. Taking this into account, we obtain

$$f(E\vartheta) = (2E + k(1-\cos\vartheta))F_1(\varkappa),$$
$$d\sigma/d\Omega = [2E + k(1-\cos\vartheta)]^2F(\varkappa), \qquad (12)$$

(κ was defined above).

An analysis of formula (12) leads to the same results as in the case of scattering by a scalar potential, except that now it is necessary to consider not $d\sigma/d\Omega$, but $[2E + k(1-\cos\vartheta)]^{-2}d\sigma/d\Omega$. As to the integral equation for the function $F(\kappa)$, its structure changes significantly, so that numerical methods must be used for its solution.

5. The properties of the differential scattering cross section at large values of κ depend in the general case on the form of the potential. Their analysis is similar to the analysis of the Born approximation in the nonrelativistic case, as given in[3]. In particular, if the potential is an analytic function of r^2, then $F(\kappa)$ decreases exponentially at large values of κ and the argument of the exponential is determined by the imaginary part of the singularity of the potential r_0 closest to the real axis, i.e., $F(\kappa^2) \sim \exp\{-2\kappa \operatorname{Im} r_0\}$ (see[3], formula (127.10)).

If $F(\kappa)$ decreases sufficiently rapidly at large values of κ, so that the integral $\int_0^\infty F(x)dx$ converges, it is possible to draw certain conclusions concerning the behavior of the total scattering cross section at high energies.

In scattering by a scalar potential, the scattering cross section decreases to zero at large energies like $1/E^4$, namely:

$$\sigma(E)_{E\to\infty} \simeq \frac{1-\beta_c^2}{k^2}\left\{\int\limits_0^\infty F(x)\,dx + \int\limits_{4m^2}^\infty F(x)\,dx\right\}. \qquad (13)$$

In the case of a vector potential

$$\sigma(E)_{E\to\infty} \approx \left\{\frac{4(E+k)^2}{k^2}\int\limits_0^\infty F(x)\,dx + \frac{4E^2}{k^2}\int\limits_{4m^2}^\infty F(x)\,dx\right\}. \qquad (14)$$

It is important that the scattering cross section cannot drop to zero in the case of the vector potential.

The difference in the behavior of the total elastic-scattering cross sections at high energies can be used to determine the transformation properties of the interaction potential.

6. Let us consider, from the point of view presented above, certain experimental data pertaining to the scattering of elementary particles at ultrarelativistic energies.

At the present time we know the following experimentally established facts:

a) The total elastic-scattering cross section drops to a constant value for all particles[12].

b) The total elastic-scattering cross section, after subtracting the diffraction elastic scattering accompanying the inelastic scattering[13], drops to zero like $1/E^4$ for NN scattering.

c) The differential cross section for elastic scattering through an angle $\pi/2$ decreases much more rapidly than through an angle π, and (in the case of scattering of non-identical particles [4)]) a minimum whose position varies with the energy appears in the angular distribution[14].

d) The experimental data pertaining to the proton-proton scattering at energies E_p = 10—30 BeV are satisfactorily described by Orear's phenomenological formula[1]

$$\frac{d\sigma}{d\Omega} = \frac{A}{s}\exp(-ap_\perp), \qquad (15)$$

where A and a are constants, s is the square of the total energy in the c.m.s., and $p_\perp$ the momentum transfer in a direction perpendicular to the direction of the initial motion of the scattered particle. This formula is valid for scattering angles $\vartheta \sim 30 - 90°$.

e) In backward scattering there appears a maximum whose magnitude is much smaller than the differential forward scattering, and whose angular width is approximately equal to the angular width of the forward maximum[14].

f) In addition to these facts there are certain suggestions advanced by Lyubimov[13] with respect to the extension of Orear's formula to a larger angle interval and to πN scattering. These assumptions will be considered below.

[4)] In the case of scattering of identical particles, the scattering cross section is symmetrical with respect to the angle $\pi/2$, since it is impossible to distinguish between the scattering and scattered particles.

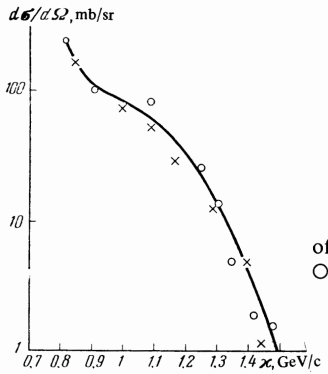
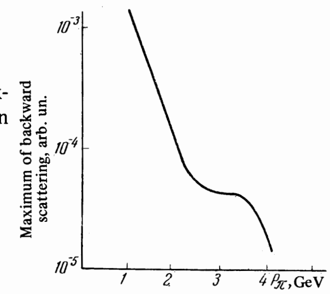


FIG. 3. Differential cross section of πN scattering as a function of κ. O—p_π = 4 BeV/c, X—p_π = 8 BeV/c.

FIG. 4. Dependence of the maximum of the backward scattering on the initial momentum of the incident particle.



We shall consider the foregoing facts and the possibility of explaining them under the assumption of weak perturbation of the incident wave and, what is more important from our point of view, the possibility of concluding from an analysis of these facts that the wave is weakly perturbed by the scattering.

A. In Lyubimov's paper[13], the first fact is connected with diffraction phenomena accompanying the inelastic scattering of the wave. The part of the differential cross section connected with these phenomena cannot give an idea of the weakness of the distortion of the incident wave by means of the criterion under consideration.

B. The decrease of the cross section of "truly elastic scattering" with increasing energy offers evidence that if the wave is weakly distorted by the scattering, then the interaction has a scalar character and $\int_0^\infty F(x)dx$ converges so that $F(x) \to 0$ as $x \to \infty$.

C. When the investigated criterion is satisfied, a minimum should occur in the angular distribution at an energy larger than a certain value. The incident-particle l.s. kinetic energy T_0 at which the minimum occurs is equal to the

$$T_0 = (m - \mu) + \sqrt{2(m^2 + \mu^2)},$$

where m and μ are respectively the masses of the scattering and incident particle. In the case of πN scattering T_0 = 2.24 BeV, and in the case of pn scattering T_0 = 2 BeV. The experimental data on the position of the minimum in the scattering of pions by protons are given in the paper of Ter-Martirosyan[14] and reduce to the following: $\cos\vartheta_{min} = -0.95$ when T = 2.3 BeV and $\cos\vartheta_{min} = -(0.4-0.6)$ when T = 4 BV.

The theoretically calculated values of $\cos\vartheta_{min}$ are equal respectively to $-(1-0.8)$ and -0.54.

D. Orear's formula, which agrees well with the experimental data on pp scattering, is a direct confirmation of the weakness of the perturbation of the incident wave in scattering from a scalar potential. Indeed, when $|\sin^2\vartheta| \gg (1-\beta_c^2)(1-\cos\vartheta)$ it is sufficient to put $F(\kappa^2) = A(2m)^{-2}e^{-a\kappa}$ to verify the identity of the formulas (17) and (9) at high energies.

E. The differential cross sections of elastic πN scattering through an angle smaller than 90° satisfy the criterion of weakness of the perturbation. This circumstance is demonstrated in Fig. 3, which shows the suitably reduced data on the differential scattering cross section at p_π = 4 and 8 BeV/c, as given in[14].

F. The existence of a backward minimum in the angular distribution of the scattered particles, its width and the ratio of its magnitude to the forward maximum were discussed in Sec. 2b. Assuming that the perturbation of the wave during scattering is small, and that Orear's formula (more accurately, the exponential dependence of the differential cross section on the value of κ) is valid for the π^--meson—neutron system, we can calculate the value of the backward scattering maximum. A plot of this quantity against the initial incident-particle momentum is shown in Fig. 4.

However, recent experimental data (communication from V. A. Lyubimov) did not improve the agreement between the experimental data[15] and the "theoretical" curve shown in Fig. 4. The new data reveal a fine structure in the dependence of the backward peak on the pion momentum, which is attributed to resonance effects[16]. In addition, the coefficient A obtained from data on scattering through angles smaller than $\pi/2$, which satisfy the criterion, is smaller by approximately one order of magnitude in the case of $\pi^\pm$p scattering than the values obtained from the analysis of the behavior of the backward peak. Finally, data on the charge-exchange cross section[17] which should satisfy the criterion in the case when the interaction is weak, do not satisfy the criterion. The last three circumstances suggest the possibility of an accidental satisfaction of the criterion at angles smaller than $\pi/2$ for πN scattering, and do not make it possible to conclude that the pion wave is weakly perturbed by the scattering.

7. Lyubimov[13] discussed the aforementioned data and the possibility of generalizing Orear's formula in order to predict certain properties of the differential πN elastic scattering cross section in the region in large angles (a similar question can also be raised with respect to the large-angle proton-neutron scattering). In particular, it was proposed to describe large-angles scattering by means of the formula $d\sigma/d\Omega = A(E)\exp(-ap_\perp)$, where $p_\perp = k\sin\vartheta$. The symmetry of the differential scattering cross section, which follows from this formula, was pointed out.

It is easy to see that if the proposed hypothesis concerning the weakness of the perturbation of the incident wave is valid, then Orear's formula should be generalized in a somewhat different fashion, and consequently the symmetry proposed in[13] should not take place, as discussed in detail in section 2b.

8. The potential corresponding to the function $F(\kappa^2)$ and obtained from Orear's formula can be easily reconstructed, and its value for pp scattering is

$$\varphi(r) = \alpha\left[r^2 + \left(\frac{a}{2}\right)^2\right]^{-2}, \quad \alpha = \frac{a}{2\pi m}\left(\frac{A}{\pi}\right)^2 = 3.7\,(\text{mb})^2(\text{BeV})^2.$$

We note in conclusion that the analysis of the relativistic case, unlike the unrelativistic case, is based on a hypothesis which is not verified experimentally, namely the hypothesis that a potential independent of the scattered particles actually exists. This hypothesis is in some respects doubtful since, first, it is difficult to perceive physically such a field on the basis of the notions concerning exchange of virtual particles and, second, the theory is not formulated symmetrically with respect to the colliding particles (with the exception of the case of particles with equal masses), and one of the particles takes on a preferred role in the collision. This circumstance undoubtedly makes the proposed criterion weaker, since this criterion should be based only on experimental data.

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