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ABSORPTION OF ULTRASOUND IN DIELECTRIC CRYSTALS WITH HIGH IMPURITY CONCENTRATIONS

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Sound absorption is studied in crystals with a high concentration of impurities or point defects. It is shown that in the low-frequency region $\omega \ll \omega_0(T, c)$ (where c is the concentration of impurities and ω_0 is some characteristic frequency), the sound absorption is less than in the pure crystal and has a different temperature dependence. It is proportional to $T^{-1/3}$ for $T \ll T_D$ and to $T^{2/3}$ for $T \gg T_D$ (T_D is the Debye temperature). Between the low frequency region, where the absorption $\gamma \propto \omega^2$, and the high frequency region, where $\gamma \propto \omega$, there is an intermediate region where $\gamma \propto \omega^{7/4}$. The temperature and frequency limits of this region are found.

1. According to the phenomenological theory, the absorption of sound takes place as a result of the viscosity and thermal conductivity of the medium. The sound absorption due to viscosity in pure dielectrics has been studied by Akhiezer.[1] He considered sound as a classical field which adiabatically modulates the energy of the thermal vibrations of the lattice and consequently creates local departures of the phonon distribution function from the equilibrium value. The collisions of phonons with one another, due to anharmonic interactions, decrease these departures. In the relaxation process of the distribution function, the entropy of the crystal increases and an irreversible loss in the energy of the sound wave takes place. Such a phenomenological consideration is applicable if the changes in the sound field in space and time are sufficiently slow in comparison with the phonon path length and the frequency of the phonon relaxation. Both these conditions reduce to the requirement of low sound frequency $\omega \ll \Gamma_p(q_T)$, where $\Gamma_p(q_T)$ is the collision frequency of phonons with mean wave vector q_T. Akhiezer showed that in a pure crystal the inverse of the relaxation time of such sound due to viscosity is

$$\gamma \approx \frac{u(T)}{\rho s^2}\frac{\omega^2}{\Gamma_p(q_T)} \tag{1}$$

where u(T) is the thermal energy per unit volume, s the sound velocity, ρ the density of the crystal;

$$\begin{gathered} q_T \approx \frac{1}{a}, \quad \Gamma_p(q_T) \approx \frac{T_D}{\hbar}\frac{T}{Ms^2}, \quad \gamma \approx \frac{\hbar\omega^2}{T_D} \\ \text{if} \quad T \gg T_D; \\ q_T \approx \frac{T}{\hbar s}, \quad \Gamma_p(q_T) \approx \frac{T}{\hbar}\frac{T}{Ms^2}\left(\frac{T}{T_D}\right)^3, \quad \gamma \approx \frac{\hbar\omega^2}{T} \\ \text{if} \quad T \ll T_D; \end{gathered} \tag{2}$$

M and πa are the mass and dimension of the elementary cell of the crystal. (Here and below, the temperature is written in energy units.)

For transverse sound, the Akhiezer mechanism is the only source of sound absorption. For longitudinal waves, the heat exchange between the regions of compression and rarefaction leads to sound absorption, which is also proportional to ω^2. In germanium, silicon, and quartz it is much less than the absorption due to viscosity.[2]

The absorption of transverse sound of high frequency $\omega \gg \Gamma_p(q_T)$ was calculated by Landau and Rumer.[3] In this case the phenomenological method is inapplicable and it is necessary to use quantum perturbation theory. The absorption of the transverse sound of frequency $\omega(q_T) \gg \omega \gg \Gamma_p(q_T)$ is

$$\gamma \approx \begin{cases} \omega \dfrac{T}{Ms^2}\left(\dfrac{T}{T_D}\right)^3, & T \ll T_D \\ \omega \dfrac{T}{Ms^2}, & T \gg T_D \end{cases} \tag{3}$$

The same result has been obtained in a number of researches[4-6] for longitudinal sound at not very low temperatures and not very high frequencies.

Figure 1 shows the regions in the (ω, T) plane where there exists some sort of dependence of the absorption on these parameters for a pure crystal. Curves 1 and 2 are described by the equation $\omega = \omega(q_T)$ and $\omega = \Gamma_p(q_T)$. Above curve 2, $\gamma \propto \omega$, and below, $\gamma \propto \omega^2$.

2. In the present research we consider the sound absorption in a dielectric crystal with a high concentration of randomly located impurities or point defects. The impurities lead to an additional scattering of the phonons. The scattering of long wave phonons $q \ll 1/a$ can differ from the Rayleigh scattering and has singularities only in the presence of quasilocal states with frequencies much smaller than the Debye frequency. Such states appear if the impurity atom is much heavier than the regular atom or is much more weakly bound to its neighbors. We consider the case in which the masses and the force constants of the two components are of the same order, so that the low-frequency quasi-local levels are absent and the scattering of the long-wave phonons has a Rayleigh character:

$$\Gamma_i(q) = \varepsilon^2 c(1-c) s a^3 q^4, \tag{4}$$

where s is the sound velocity in the impure crystal and ε is a nondimensional parameter characterizing the size of the perturbation produced by the impurity. If, for definiteness, we speak of an impurity which differs from the regular atom only in mass, then

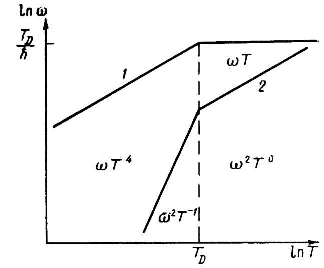

FIG. 1. Dependence of the sound absorption of frequency and temperature in a pure crystal. Equations of the curves: 1—$\omega = \omega(q_T)$; 2—$\omega = \Gamma_p(q_T)$. Above curve 2, the value of γ is determined in (3), below, in (2).

$\epsilon = |\Delta M/M|$. For $q \approx 1/a$, (4) is valid only in order of magnitude.

Doubt can arise as to the validity of (4) for $c \approx 1/2$ and $\epsilon \approx 1$. In order to clarify this problem, we use the rezults of Tatarskiĭ and Gertsenshteĭn,[7] who constructed a theory of scattering of waves in a medium with random inhomogeneities. An example of such scattering is the scattering of electromagnetic waves in a medium with inhomogeneities in the dielectric permittivity κ or sound waves in a medium with inhomogeneities in the density ρ and the elastic modulus K. It was shown in[7] that if the wavelength is much greater than the correlation length of the inhomogeneities l, then $\Gamma_i(q) = \sigma^2 s l^3 q^4$, where σ is the mean square of the relative fluctuations of κ, ρ, or K.

We apply this general theory to the case of sound propagation in an ideal solid solution of atoms with different masses M_1 and M_2. In this case, the correlation length for the density ρ is the lattice constant ($l = a$), and

$$\sigma^2 = \overline{(\rho - \bar{\rho})^2} / (\bar{\rho})^2 = \overline{(M - \bar{M})^2} / (\bar{M})^2 = |\Delta M / \bar{M}|^2 c(1-c),$$

and for any c and ϵ we get Eq. (4) for $\Gamma_i(q)$. Similar reasoning shows that (4) is valid also in the case in which the components of the solution differ in their force constants. According to (4), the long-wave phonons scatter weakly even at $\epsilon \approx 1$ and $c \approx 1/2$ ($\Gamma_i(q_i) \ll sq$), although the change in the averaged properties of the crystal (density, elastic moduli) can considerably alter their velocity and anharmonic interaction, because of the presence of the impurities.

The presence of impurities has a double effect on the sound absorption. In the first place, the impurities directly scatter the sound wave; in the second place, they scatter the phonons and thus change their contribution to the sound absorption. Using (4), it is easy to establish the fact that the direct effect of point defects on the absorption of the sound wave is vanishingly small. Even for an impurity concentration $c \approx 1/2$ and comparatively high frequency $\omega = 10^{10}$ sec^{-1} ($qa \approx 10^{-3}$), the damping length due to scattering on the impurities $s/\Gamma_i(q)$ is seen to be of the order of 10^4 cm. The second effect is substantial if the thermal phonons are scattered by impurities more often than by one another: $\Gamma_i(q_T) \gg \Gamma_p(q_T)$. Comparing (4) for $q = q_T$ and Eq. (2) for $\Gamma_p(q_T)$, we find that, to satisfy this inequality, sufficiently high concentrations of impurities are necessary, satisfying the condition $c(1-c) > T/\epsilon^2 M s^2$. We shall consider here just such concentrations, including $c \approx 1/2$.

It would appear that for a quantitative account of the effect of impurities on the sound absorption by phonons when $\omega \ll \Gamma_i(q_T)$ it suffices to replace $\Gamma_p(q_T)$ by $\Gamma_i(q_T)$ in Eq. (1), and to regard $\omega = \Gamma_i(q_T)$ as the criterion for the transition from the low-frequency case to the high-frequency one (from the quadratic frequency dependence of the absorption to the linear). However, such a course is an erroneous one. The fact is that if we systematically compute the sound absorption by the method of Akhiezer, assuming that the phonons are scattered only by the impurities, then γ is seen to be infinite because of the divergence of the integral over q as $q \to 0$. This divergence is associated with the rapid decay of $\Gamma_i(q)$ with decreasing q and is completely analogous to the divergence of the thermal conductivity, which arises for the same phonon scattering mechanism.

To obtain a finite sound absorption, it is important to take into account two circumstances. First, for subthermal phonons, $q \gg q_T$, it is necessary to consider their interaction with the thermal phonons. It is described by (3) and for sufficiently low q it can be comparable with or greater than the scattering from the impurities.[1] Second, the subthermal phonons q may not satisfy the criterion of applicability of the phenomenological theory of Akhiezer $\omega \ll \Gamma_i(q)$ in spite of the fact that $\omega \ll \Gamma_i(q_T)$, and use of the more general theory is necessary. A theory of the absorption of sound of frequency $\omega \ll \Gamma_i(q_T) \ll T/\hbar$ in solid solutions, taking these two factors into account, will be constructed below.

3. In computing the sound absorption, we shall make use of the Green's function technique, which is free of the limitations of the phenomenological theory of Akhiezer. We shall show that in sound absorption at a frequency $\omega \ll \Gamma_i(q_T)$ the principal role is played by the long-wave phonons $q_i \ll 1/a$. The exact Green's function of the long-wave phonon in a crystal with impurities is obtained in the already cited paper[7]:

$$D(q_1, \omega) = \frac{1}{\hbar} \frac{2\omega(q_1)}{\omega^2 - \omega^2(q_1) + 2i\omega(q_1)\Gamma_i(q_1)}, \tag{5}$$

where $\Gamma_i(q_i)$ is defined by (4). We shall represent by a thin line on the diagrams given below the temperature Green's function $D_0(q_1, i\omega_m)$ corresponding to (5). Since the long-wave phonons are weakly scattered in the solid solution ($\Gamma_i(q) \ll sq$) and therefore represent plane waves with great accuracy, the Hamiltonian of the triple anharmonic interaction of the long-wave phonons has the same form as in the pure crystal:

$$H_{\text{int}} = \sum_{\substack{\mathbf{q}\mathbf{q}_1\mathbf{q}_2 \\ jj_1j_2}} U_{\substack{\mathbf{q}\mathbf{q}_1\mathbf{q}_2 \\ jj_1j_2}} (a_{\mathbf{q}j} + \overset{+}{a}_{-\mathbf{q}j})(a_{\mathbf{q}_1j_1} + \overset{+}{a}_{-\mathbf{q}_1j_1})(a_{\mathbf{q}_2j_2} + \overset{+}{a}_{-\mathbf{q}_2j_2})\delta_{\mathbf{q}+\mathbf{q}_1+\mathbf{q}_2,0}, \tag{6}$$

where

$$U_{\substack{\mathbf{q}\mathbf{q}_1\mathbf{q}_2 \\ jj_1j_2}} = AV^{-1/2}\left(\frac{\hbar}{2\rho}\right)^{3/2}\left(\frac{qq_1q_2}{s_js_{j_1}s_{j_2}}\right)^{1/2} \tag{7}$$

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[1] In the case of the thermal conductivity in impure crystals, this circumstance was considered in [8], and led to excellent agreement with experiment.)

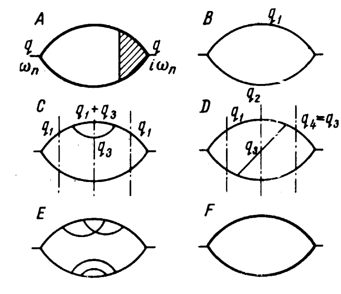

FIG. 2. Polarization operator $\Pi(q, i\omega_n)$.

(a^+_{qj} is the phonon creation operator, j the index of polarization,[2] and V the volume of the crystal). However, the sound velocities s, the density ρ, and the elastic modulus A that depends only on the angle, can differ in (7) from the same quantities for the pure crystal.

The absorption of sound of frequency ω and wave vector q is determined by the imaginary part of its polarization operator $\gamma = \hbar^{-1}\,\mathrm{Im}\,\Pi(q, \omega)$.[4,5] $\Pi(q, \omega)$ is the analytic continuation of the "temperature" polarization operator $\Pi(q, i\omega_n)$ corresponding to the diagram in Fig. 2A. The heavy lines in Fig. 2A denote the exact Green's functions of phonons with account of the anharmonic interaction of them with one another; the vertex part is shaded. If we do not take the anharmonic interaction of the thermal vibrations of the lattice into account (we shall find later the conditions for which this is possible), then the diagram corresponding to the polarization operator $\Pi(q, i\omega_n)$ has the form of Fig. 2B, i.e.,

$$\Pi(q, i\omega_n) = 18\frac{V}{(2\pi)^3}\sum_{j_1 j_2}\int d^3q_1 |U_{\substack{\mathbf{q}\mathbf{q}_1\mathbf{q}_2\\ jj_1j_2}}|^2 \times T\sum_{i\omega_m} D_0(q_1, i\omega_m) D_0(q_2, -i\omega_n - i\omega_m). \tag{8}$$

This expression describes the process of combination of a sound quantum q with the phonon $q_1 \gg q$, as a result of which the phonon q_2 appears. The factor 18 in (8) has a combinatorial origin and is explained in[10].

The Lehmann expansion of the Green's function D_0 in (8) with use of (5), summation, integration and analytic continuation, leads to the following result (similar calculations are given in detail in[5,6]):

$$\begin{aligned}\gamma_j &= \frac{9}{(2\pi)^3}\frac{\omega V}{\hbar^2}\sum_{j_1 j_2}\int d^3q_1 |U_{\substack{\mathbf{q}\mathbf{q}_1\mathbf{q}_2\\ jj_1j_2}}|^2 \\ &\times \frac{\partial N_1}{\partial\omega_1}\frac{2\Gamma_i(q_1)}{[\omega_{j_1}(q_1) + \omega - \omega_{j_2}(q_2)]^2 + 4\Gamma_i^2(q_1)} \\ &\approx \frac{\hbar\omega^2}{\rho^3}\sum_{j_1}\int d^3q_1 \frac{A^2 q_1^2}{s_j^2 s_{j_1}^2}\frac{\partial N_1}{\partial\omega_1}\frac{2\Gamma_i(q_1)}{\omega^2(s_{j_1}s_j^{-1}\cos\theta - 1)^2 + 4\Gamma_i^2(q_1)},\end{aligned} \tag{9}$$

where $N_1 = [\exp(\hbar\omega_{j_1}(q_1)/T) - 1]^{-1}$, and θ is the angle between q and q_1.

The "smeared" δ function

$$\Delta(\Omega) = 2\Gamma_i(q_1) / (\Omega^2 + 4\Gamma_i^2(q_1)),$$

takes into account the finiteness of the lifetime of the thermal phonon in the state of a plane wave with a definite wave vector. It replaces the present δ function of first-order perturbation theory, which neglects not only the anharmonic interaction of the phonons with each other but also their scattering from impurities. In (9), the components with $j_1 = j_2$ remain, inasmuch as Ω is minimal in this case.

We shall show that for low sound frequencies $\omega \ll \Gamma_i(q_T)$ the integrand of Eq. (9) $f(q_1)$ has a maximum near the value

$$q_1 = q' \equiv [\omega / \varepsilon^2 c(1 - c) s a^3]^{1/4} \ll q_T, \tag{10}$$

for which $\Gamma_i(q') = \omega$. Actually, for $q_1 < q_T$, we have $\partial N_1/\partial\omega_1 \propto 1/q_1^2$, and if $q_1 \gg q'$, then $\Delta = \frac{1}{2}\Gamma_i^{-1}(q_1) \propto 1/q_1^4$ and $f(q_1) \propto 1/q_1^2$. Integration over the angles in this case gives only a numerical factor of order unity. If now $q_1 \ll q'$, i.e., $\omega \gg \Gamma_i(q_1)$ and, moreover, the condition

$$\min\left|\frac{s_{j_1}}{s_j}\cos\theta - 1\right| \ll \frac{\Gamma_i(q_1)}{\omega} \tag{11}$$

is satisfied, then $\Delta(\cos\theta)$ has a sharp maximum $1/\Gamma_i(q_1)$ in the region of angles of order $\Gamma_i(q_1)/\omega$ and, consequently, integration over the angles makes a factor $1/\omega$ from Δ. Therefore, for $q_1 \ll q'$, the function $f(q_1) \propto q_1^2$. For those f_1 for which (11) is not satisfied, $f(q_1)$ falls off with decrease in q_1 even more rapidly. In all cases, the maximum value of $f(q_1)$ takes place for $q_1 \approx q'$ and consequently the subthermal phonons with $q \approx q' \ll q_T$ make the largest contribution to the sound absorption. This makes it easy to compute γ in order of magnitude:

$$\begin{aligned}\gamma &\approx \frac{A^2\omega^2\hbar}{\rho^3 s^4}\int_{q'}^{q_{max}} dq_1\, q_1^4 \frac{\partial N_1}{\partial\omega_1}\frac{1}{\Gamma_i(q_1)} \approx \frac{T\omega^2}{Ms^3\varepsilon^2 c(1-c)}\int_{q'}^{\infty}\frac{1}{q_1^2}dq_1 \\ &\approx \frac{T}{Ms^2}\frac{\omega}{[\varepsilon^2 c(1-c)]^{3/4}}\left(\frac{\hbar\omega}{T_D}\right)^{3/4}.\end{aligned} \tag{12}$$

Here we have used the estimate $A \approx \rho s^2$.

Up to the present time, we have not considered the anharmonic interaction of the phonons with one another. This is true only in the case when the anharmonic damping of the existing phonons q' is much smaller than their impurity damping $\Gamma_p(q') \ll \Gamma_i(q')$. In the opposite case, the considered zeroth approximation in the anharmonic interaction of the phonons is insufficient and, as we shall see below, summation of all the approximations of perturbation theory on this interaction is required.

In the lowest approximation, we obtain in place of Fig. 2B two diagrams—2C and 2D. It is easy to establish the fact that the diagram referring to the vertex part is $D \ll C$. The mean cross sections give in diagrams C and D denominators of the same order, while the limiting cross sections give

$$\begin{gathered}\left[\omega\left(\frac{s_{j_1}}{s_j}\cos\theta - 1\right) + 2i\Gamma_i(q_1)\right]^{-2}, \\ \left\{\left[\omega\left(\frac{s_{j_1}}{s_j}\cos\theta - 1\right) + 2i\Gamma_i(q_1)\right]\right. \\ \left.\times\left[\omega\left(\frac{s_{j_4}}{s_j}\cos(\mathbf{q}\mathbf{q}_4) - 1\right) + 2i\Gamma_i(q_1 + q_3)\right]\right\}^{-1}.\end{gathered} \tag{13}$$

respectively. Since $\Gamma_i(q_1)$ fall off sharply with decrease in q_1 in both diagrams, the $q_1 \approx q' \ll q_T$ are important. So far as q_3 is concerned, in diagram C it

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[2] Acoustic phonons are considered. The optical long-wave phonons are strongly scattered by the impurities ($\Gamma_i(q) \propto q$, and not q^4[9]) and do not make any appreciable contribution to the sound absorption, even for $T \gtrsim T_D$, when they are excited.

does not enter into the denominator that is written down and therefore the $q_3 \approx q_T$ are important in the integral over q_3, so that $\gamma_c \propto \Gamma_p(q')/\Gamma_i^2(q')$ ($\Gamma_p(q')$ results from the integration over q_3).

At the same time the quantity q_3 in diagram 2D enters into the analogous denominator and therefore $q_3 \approx q_1 \approx q'$ are important. This means that in D the subthermal phonon q_1 interacts only with the subthermal phonons which have a momentum not exceeding its own. In this case, its anharmonic damping, which is obtained as the result of integration over q_3 is proportional to $\Gamma_p(q')(q'/q_T)^3$ and therefore

$$\gamma_D/\gamma_C \approx (q'/q_T)^3 \ll 1.$$

It can be shown that for the same reason, the vertex part is also small in the succeeding approximations. Furthermore, in the calculation of $\Pi(q, i\omega_n)$, one can discard diagrams similar to the diagram 2E which contains the intersecting or mutually encompassing "supports," inasmuch as the corresponding expressions have a smaller number of small denominators of the type (13) than the diagrams with successive "supports," in this same approximation of perturbation theory. Thus the principal contribution to $\Pi(q, i\omega_n)$ is made by diagrams of the latter type, which can be represented by the loops of the heavy lines (Fig. 2F), while the point Green's function $D(q_1, i\omega_m)$ corresponding to these lines is a solution of the Dyson equation, represented graphically in Fig. 3:

$$D(q_1, i\omega_m) = \frac{D_0(q_1, i\omega_m)}{1 - \Pi(q_1, i\omega_m) D_0(q_1, i\omega_m)}. \tag{14}$$

The calculation of the diagram 2F, entirely analogous to the calculation of the diagram 2B, leads to the following general expression for the sound absorption, which takes into account both the scattering of the phonons from the impurities as well as their anharmonic interaction:

$$\gamma \approx \frac{\hbar\omega^2}{\rho^3} \sum_{j_1} \int d^3q_1 \frac{A^2 q_1^2}{s_j^2 s_{j_1}^2} \frac{\partial N_1}{\partial \omega_1} \times \frac{2(\Gamma_i(q_1) + \Gamma_p(q_1))}{\omega^2 (s_{j_1} s_j^{-1} \cos\theta - 1)^2 + 4(\Gamma_i(q_1) + \Gamma_p(q_1))^2}. \tag{15}$$

Equation (15) differs from (9) in the addition of the anharmonic damping $\Gamma_p(q_1) = \hbar^{-1}\,\mathrm{Im}\,\Pi(q_1, \omega(q_1))$ in the "smeared" δ function. For subthermal phonons, $q_1 \ll q_T$, which play a basic role in the sound absorption, $\Gamma_p(q_1)$ q_1 and is determined by Eq. (3).

Thus, in our problem, it appears possible to introduce rigorously the relaxation time of phonons due to the anharmonic interaction $\Gamma_p^{-1}(q_1)$. The physical reason for this "good luck" is that only the subthermal phonons are important for sound absorption in a solid solution; these represent a small part of the total number of phonons and therefore the small vertex part $\Pi(q, i\omega_n)$. i.e., the terms of small account in the kinetic equation for phonons corresponding to $\Pi(q, i\omega_n)$ kinetic equation for phonons.

We study the general expression (15). If $\Gamma_p(q') \ll \Gamma_i(q')$, then all the discussions touching on (9) remain in force and we get (12) for γ. If $\Gamma_p(q') \gg \Gamma_i(q')$, then the anharmonic damping is essential and $f(q_1)$ has a maximum not for q' but for q'', for which $\Gamma_p(q'') = \Gamma_i(q'')$; here $q' \ll q'' \ll q_T$. Actually, for $q' < q_i < q_T$,

$$\Delta = \frac{1}{2(\Gamma_i(q_1) + \Gamma_p(q_1))}, \quad f(q_1) \sim \frac{q_1^2}{\Gamma_i(q_1) + \Gamma_p(q_1)}$$

while for $q_1 > q''$, we have $f(q_1) \propto 1/q_1^2$ while for $q_1 < q''$, we have $f(q_1) \propto q_1^2$. Using (3) and (4), we get

$$q'' = \begin{cases} \dfrac{1}{a}\left[\dfrac{T}{Ms^2\varepsilon^2 c(1-c)}\right]^{1/3}, & T \gg T_D \\[2ex] \dfrac{1}{a}\dfrac{T}{T_D}\left[\dfrac{T}{Ms^2\varepsilon^2 c(1-c)}\right]^{1/3} & T \ll T_D \end{cases} \tag{16}$$

The existence of a maximum of $f(q_1)$ allows us to estimate (15) easily here:

$$\gamma \approx \frac{A^2\omega^2\hbar}{\rho^3 s^4} \int_{q''}^{\infty} q_1^4 \frac{\partial N_1}{\partial \omega_1} \frac{1}{\Gamma_i(q_1)}\, dq_1 \approx$$

$$\approx \begin{cases} \omega \dfrac{\hbar\omega}{T_D}\left[\dfrac{T}{Ms^2\varepsilon^2 c(1-c)}\right]^{2/3}, & T \gg T_D \\[2ex] \omega \dfrac{\hbar\omega}{T}\left[\dfrac{T}{Ms^2\varepsilon^2 c(1-c)}\right]^{2/3}, & T \ll T_D \end{cases} \tag{17}$$

Formulas (12) and (17) determine the dependence of the sound absorption in crystals with impurities on the frequency, temperature and concentration of impurities for $\omega \ll \Gamma_i(q_T)$. The first holds for $q' \gg q''$, the second for $q' \ll q''$.

For sufficiently low frequencies $q' \ll q''$ and in accord with (17), $\gamma \propto \omega^2$ in correspondence with the phenomenological theory. Comparison of (17) and (2) leads to the interesting conclusion that the low-frequency sound absorption in a solid solution is less than the absorption in the pure crystal in the ratio $[T/Ms^2\epsilon^2 c(1-c)]^{2/3}$. This factor materially changes both the absolute value and the temperature dependence of the absorption.

Upon increase in the sound frequency, the dependence of γ on the frequency and temperature is changed, when the characteristic frequency $\omega_0(T)$ is reached, which satisfies the condition $q'(\omega_0) = q''$. Using (10) and (16), we get

$$\omega_0(T) = \begin{cases} \dfrac{T_D}{\hbar}\left(\dfrac{T}{Ms^2}\right)^{4/3} \dfrac{1}{[\varepsilon^2 c(1-c)]^{1/3}} & T \gg T_D \\[2ex] \dfrac{T}{\hbar}\left(\dfrac{T}{Ms^2}\right)^{4/3} \left(\dfrac{T}{T_D}\right)^3 \dfrac{1}{[\varepsilon^2 c(1-c)]^{1/3}}, & T \ll T_D \end{cases}. \tag{18}$$

In the region of frequencies $\Gamma_i(q_T) \gg \omega \gg \omega_0(T)$, where $q' \gg q''$, γ is determined by Eq. (12). At still higher sound frequencies, $\omega \gg \Gamma_i(q_T)$, in crystals with impurities, as also in pure crystals, (3) is satisfied.

We note that the result (12) was obtained by Miller,[11] who, however, did not take into account the anharmonic interaction and therefore could not obtain the low frequency limit of applicability of (12) and of the expression (17) for the sound absorption in the case $\omega < \omega_0(T)$.

Figure (4) shows the regions of applicability of the results in the (ω, T) plane. Curves 1, 2, 3, 4 are described by $\omega = \omega(q_T)$, $\omega = \Gamma_p(q_T)$, $\omega = \Gamma_i(q_T)$ and $\omega = \omega_0(T)$. Curves 2, 3, 4 intersect at the point A for

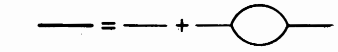

FIG. 3. Dyson equation for $D(q_1, i\omega_m)$.

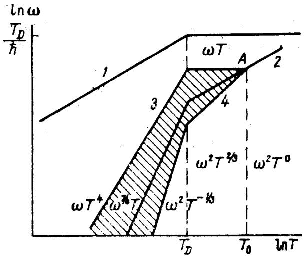

FIG. 4. Dependence of the sound absorption on frequency and temperature in a crystal with impurities. Equations of the curves: $1-\omega = \omega(q_T)$; $2-\omega = \Gamma_p(q_T)$; $3-\omega = \Gamma_i(q_T)$; $4-\omega = \omega_0(T)$. In the shaded region γ must be expressed by (17), under it, by (12), above it, by (3). For $T > T_0$, for the value of γ is determined by Eq. (2).

$T = T_0 = \epsilon^2c(1 - c)Ms^2$; for $T > T_0$, the presence of impurities has no effect on the sound absorption ($\Gamma_p(q_T) > \Gamma_i(q_T)$) and the expressions (2) obtained by Akhiezer are valid. In the shaded region, γ is determined by (12), and in the region lying under it, by (17). Equations (3) are satisfied above the curve 3. For a decrease in the concentration of impurities the point A shifts downward along curve 2, so that the shaded region and the region of applicability of (17) decrease and Fig. 4 gradually goes over into Fig. 1. For very small c the difference between the impure crystal and the pure one is appreciable only for very low frequencies and temperatures.

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200