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ROLE OF OPTICAL PHONONS IN INTERBAND MAGNETOOPTICAL ABSORPTION OF SEMICONDUCTORS

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The one-particle Green's function is calculated and the spectrum is derived for an electron in a magnetic field at zero temperature by taking the interaction with polarization phonons into account. At magnetic field strengths corresponding to magnetophonon resonance, the spectrum consists of two branches separated by an energy gap defined by the coupling constant. The damping of the higher-energy branch of the spectrum is obtained and the cut-off point of the branch is determined. It is found that the damping of the lower branch of the spectrum is caused by other scattering mechanisms and by temperature corrections to the mechanism under consideration. The interband magnetooptical absorption coefficient in the magnetophonon resonance region is calculated. It is shown that in this region the absorption coefficient has two maxima, the distance between them being on the order of the distance between electron-spectrum branches in the conduction band. The shape of the peak corresponding to transfer of electrons from the valence band to the upper branch of the spectrum in the conduction band is obtained. The results of the theory agree with those of the experiments.

1. INTRODUCTION

A system of discrete electronic energy levels (Landau bands) is produced in a semiconductor placed in a magnetic field, provided the energy uncertainty of the electronic state is smaller than the cyclotron energy. The light absorption coefficient in such a system, connected with the transfer of the electrons from the valence band (v-band) to the conduction band (c-band) oscillates as a function of the frequency of the light wave (see, for example,[1]). If the states of the electron in the c-band and of the hole in the v-band can be regarded as stationary, then the calculated absorption coefficient is singular at certain frequencies, this being connected with the infinite values of the density of the electron states in the magnetic field at the point $k_z = 0$ (k_z—projection of the wave vector of the electron on the direction of the magnetic field **H**). The singularities of the absorption coefficient are eliminated by taking into account the interaction of the electron with the various types of scatterers present in the crystal (phonons, defects, etc.).

Recently Johnson and Larsen[2] measured the interband magnetooptical absorption in InSb at low temperatures and observed that in a certain region of magnetic-field values the magnetooptic peak corresponding to transitions between the Landau bands with quantum numbers $n = 1$ splits in two. This interesting effect was attributed by them to the influence exerted on the electron spectrum in the c-band by the interaction with the optical phonons. Two terms of the electron-phonon system (the first—electron in a state with $n = 1$, the second—electron in a state with $n = 0$ and one optical phonon) intersect at a value of H at which $\Omega_c = \omega_0$ (Ω_c—cyclotron frequency in the c-band, ω_0—limiting frequency of the optical phonon), if the interaction between them is neglected. Turning on the interaction of the electron with the phonon realigns the spectrum, lifting the degeneracy at the point $\Omega_c = \omega_0$, and leading in turn to a splitting of the absorption peak[2].

Thus, the interaction with the scatterers not only broadens the absorption bands, but in some cases leads to an appreciable realignment of the magnetooptical spectrum, causing the appearance of new oscillations.

In this communication we present the results of the theory of the interband magnetooptic absorption spectrum with consistent account taken of the inelastic interaction between the electrons and the optical (polarization) lattice vibrations (see also[3]).

2. FORMULATION OF PROBLEM AND FUNDAMENTAL RELATIONS

We consider a cubic crystal in a constant homogeneous magnetic field directed along one of the symmetry axes. We assume that the c- and v-bands lie in the center of the Brillouin zone and that a direct dipole transition between them is allowed. The cause of the change of the electron spectrum will be assumed to be the interaction of the electron with the longitudinal optical phonons, and the interaction itself will be assumed to be weak, meaning that the dimensionless coupling constant α_0 is small compared with unity, as is the case in crystals with a small fraction of ionic bond. In the general case the form of the magnetooptical oscillations is determined by the scattering of the electrons in the c-band and of the holes in the v-band. However, the observed doubling of the peak was connected with an interaction between the electron and phonons precisely in the c-band, since it took place near $\Omega_c = \omega_0$. Since the electron and hole masses are usually quite different, the condition for the doubling of the hole spectrum ($\Omega_v = \omega_0$) will not be satisfied when $\Omega_c = \omega_0$. We can therefore neglect the interaction between the holes and the phonons. In addition, we confine ourselves to the case of zero temperature, when there are no optical phonons, and the interaction leads only to their emission.

The Hamiltonian of the interaction between the electron and the phonons in a magnetic field in the c-band is written in the form

$$\mathcal{H}=\sum_{\mathbf{q}}\sum_{\alpha\alpha'}[C_{\mathbf{q}}I_{\alpha\alpha'}(\mathbf{q})b_{\mathbf{q}}+\mathrm{h.c.}]a_{\alpha}^{+}a_{\alpha'}. \tag{1}$$

Here $a_{\alpha}^{+}(a_{\alpha'})$—operator of creation (annihilation) of an electron in a state $\alpha(\alpha')$, $b_{\mathbf{q}}^{+}(b_{\mathbf{q}})$—operator for creation (annihilation) of phonons with wave vector **q**, α—state of the conduction electron in the c-zone: $\alpha = (n_{\alpha}, k_{y\alpha}, k_{z\alpha})$, n_{α}—number of the Landau band, $k_{z\alpha}$—projection of the wave vector on the direction of the magnetic field **H**, $I_{\alpha\alpha'}(\mathbf{q})$ is the matrix element of the operator $\exp(i\mathbf{q}\cdot\mathbf{r})$, calculated using the wave functions of the electron in the magnetic field,

$$C_{\mathbf{q}}=-i\hbar\omega_0\left[\frac{4\pi\alpha_0 l_0^3}{V}\right]^{1/2}\frac{1}{l_0 q},\qquad l_0^2=\frac{\hbar}{2m_c\omega_0}; \tag{2}$$

α_0—dimensionless coupling constant of the Froehlich interaction Hamiltonian[4]

$$\alpha_0=\frac{e^2}{2\hbar\omega_0 l_0}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right); \tag{3}$$

$(-e)$—electron charge, m_c—effective mass of the electron in the c-band, ϵ_0 and ϵ_∞—static and high-frequency dielectric constants, and V—normalization volume.

The light absorption coefficient $K(\omega)$ (ω—frequency of light), neglecting the scattering of the holes, is carried out with the aid of the Kubo formula[5] in the form

$$K(\omega)=\frac{\pi}{cn_0}\frac{e^2|P|^2}{\hbar m_0^2V}\operatorname{Re}\int_0 dt\, e^{-i\omega t}\sum_{\alpha}\frac{e^{i\omega_{v\alpha}t}}{\omega_{cv}}\langle a_{c\alpha}(t)a_{c\alpha}(0)\rangle, \tag{4}$$

where P is the interband matrix element of the momentum and corresponds to a transition in the center of the Brillouin zone, c the velocity of light in vacuum, n_0 the refractive index, m_0 the mass of the free electron, $\omega_{cv} = \omega_{c\alpha} - \omega_{v\alpha}$, where $\omega_{c\alpha}$ and $\omega_{v\alpha}$ are the frequencies of the electron in the c- and v-bands. We shall assume henceforth a very simple band structure, the bands are nondegenerate, parabolic, and their extrema are located at the center of the Brillouin zone. Then

$$\omega_{c\alpha}=\Omega_c\left(n_\alpha+\frac{1}{2}\right)+\frac{\hbar k_z^2}{2m_c},\qquad \Omega_c=\frac{eH}{m_c c};$$
$$\omega_{v\alpha}=-\omega_g-\Omega_v\left(n_\alpha+\frac{1}{2}\right)-\frac{\hbar k_z^2}{2m_v},\qquad \Omega_v=\frac{eH}{m_v c}; \tag{5}$$

ω_g is the width of the forbidden band, m_v the effective mass of the hole, $a_{c\alpha}(t)$ the operator in the Heisenberg representation, and the symbol $\langle \ldots \rangle$ denotes averaging over the ground state of the system.

In the model under consideration, the ground state should be taken to mean a completely filled valence band and an empty conduction band. We shall therefore calculate all the quantites in the zeroth approximation in the electron density in the c-band. In this case

$$\langle a_{c\alpha}(t)a_{c\alpha}^{+}(0)\rangle = iG_r(\alpha, t),$$

where $G_r(\alpha, t)$ is the retarded single-particle Green's function of the electron in the c-band in a magnetic field in the (α, t) representation, and

$$K(\omega)=\frac{\pi}{cn_0}\frac{e^2|P|^2}{\hbar m_0^2V}\operatorname{Re}\sum_{\alpha}\frac{i}{\omega_{cv}}G_r(\alpha,\omega-\omega_{v\alpha}). \tag{6}$$

In the zero-density approximation, G_r coincides with the corresponding causal Green's function; its Fourier component $G_r(\alpha, \epsilon)$ in terms of the variables r and t can be expressed in terms of the mass operator $\Sigma(\alpha, \epsilon)$:

$$G_r(\alpha,\varepsilon)=[\varepsilon-\omega_{c\alpha}-\Sigma(\alpha,\varepsilon)+is]^{-1} \tag{7}$$

(s—adiabatic parameter, s → 0).

3. ANALYSIS OF THE PERTURBATION-THEORY SERIES FOR THE MASS OPERATOR

It is convenient to calculate the function $\Sigma(\alpha, \epsilon)$ by using the Feynman diagram technique. The rules for making up the diagrams are standard. In the first approximation in the coupling constant (diagram a of Fig. 1) we have

$$\Sigma_1=\sum_{q}\sum_{\alpha_1}\hbar^{-2}|C_{\mathbf{q}}|^2|I_{\alpha\alpha_1}(\mathbf{q})|^2(\varepsilon-\omega_0-\omega_{c\alpha_1}+is)^{-1}. \tag{8}$$

We are interested in the spectrum of the electron-phonon system for n = 1. Since the electron situated at the level n = 1 can only emit a phonon, the resonant terms of the sum over n_{α_1} in (8) will be only those corresponding to a transition between n = 1 and n = 0. The remaining terms of the series are nonresonant, depend little on ϵ, and can be eliminated by renormalization of ω_g. They will be disregarded from now on. Then (8) takes the form

$$\Sigma_1=\frac{\alpha_0\omega_0}{2\pi}\left(\frac{\omega_0}{\Omega_c}\right)^{1/2}\int_0^{\infty}du\int_{-\infty}^{+\infty}dx\,\frac{ue^{-u}}{(u+x^2)[\gamma+\lambda-(k-x)^2+is]}, \tag{9}$$

where

$$\gamma=\frac{\varepsilon-3\Omega_c/2}{\Omega_c},\qquad \lambda=\frac{\Omega_c-\omega_0}{\Omega_c},$$
$$k=\frac{k_z l_H}{\sqrt{2}},\qquad l_H=\left(\frac{c\hbar}{eH}\right)^{1/2}.$$

In calculating the integral in (9) we can neglect x^2 in the factor $(u + x^2)^{-1}$ with accuracy to terms of order $\alpha_0^{1/3}$, after which the integral can be readily calculated and

$$\Sigma_1=-i\omega_0\frac{\alpha_0}{2}\left(\frac{\omega_0}{\Omega_c}\right)^{1/2}(\gamma+\lambda+is)^{-1/2}. \tag{10}$$

In the next order of perturbation theory, two terms appear, Σ_{21} (diagram b of Fig. 1) and Σ_{22} (diagram c of Fig. 1). When calculated at the same accuracy as Σ_1, their values are

$$\Sigma_{21}=i\omega_0\left(\frac{\alpha_0}{2}\right)^2\left(\frac{\omega_0}{2\Omega_c}\right)^{1/2}(\gamma+\lambda+is)^{-3/2}, \tag{11}$$

$$\Sigma_{22}\cong\omega_0\left(\frac{\alpha_0}{2}\right)^2(\gamma+\lambda+is)^{-1}. \tag{12}$$

4. SPECTRUM OF ELECTRON IN THE CONDUCTION BAND WITH ALLOWANCE FOR THE ELECTRON-PHONON INTERACTION

To determine the spectrum of the electron in the c-band it is necessary to find the poles $G_r(\alpha, \epsilon)$, which

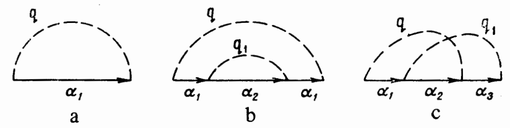

FIG. 1. Diagrams for mass operator $\Sigma(\alpha,\epsilon)$ of first-order (a) and second-order (b, c) perturbation theory.

should be located in the lower half-plane of the complex variable ϵ. We shall seek the poles of G_r by replacing the mass operator $\Sigma(\alpha, \epsilon)$ by the approximate value of Σ_1. Then the expression (7) reduces to the form

$$G_r(k,\gamma)=\left\{\gamma-k^2+\frac{i\eta}{(\gamma+\lambda)^{1/2}}\right\}^{-1},\quad \eta=\frac{\alpha_0}{2}\left(\frac{\omega_0}{\Omega_c}\right)^{3/2}. \tag{13}$$

The poles of $G_r(k, \gamma)$ are determined by the zeroes of the denominator in (13).

In the case $k_z = 0$, which is of importance in our problem, we must solve the equation

$$\gamma+i\eta/\sqrt{\gamma+\lambda}=0. \tag{14}$$

Let us determine first the roots of (14) in certain limiting cases. If $\lambda = 0$ (resonant case $\Omega_c = \omega_0$), then the roots of (14) will be

$$\gamma_1=\eta^{2/3}e^{-i\pi/3},\quad \gamma_2=-\eta^{2/3}. \tag{15}$$

When $\lambda > 0$ and $\lambda \gg \gamma$ or $\gamma \to -\lambda$, respectively,

$$\gamma_1=-\frac{i\eta}{\sqrt{\lambda}}+\frac{\eta^2}{2\lambda^2},\quad \gamma_2\cong-\lambda-\frac{\eta^2}{\lambda^2}. \tag{16}$$

On the other hand, if $\lambda < 0$ and $|\lambda| \gg \gamma$, then

$$\gamma_2\cong-\eta|\lambda|^{-1/2}. \tag{17}$$

In the last limiting case, (14) has only one root, whereas in each of the other limiting cases there are two roots. This means that one branch of the spectrum vanishes as λ shifts towards negative values. The point of vanishing of one branch of the spectrum, λ_0, is determined from the exact solution of (14) by reducing it to a third-degree equation

$$\gamma^3+\lambda\gamma^2+\eta^2=0, \tag{18}$$

the roots of which are

$$\gamma_1=-\frac{\lambda}{3}+R_1^{1/3}e^{i\pi/3}+R_2^{1/3}e^{-i\pi/3}, \tag{19}$$

$$\gamma_2=-\left(\frac{\lambda}{3}+R_1^{1/3}+R_2^{1/3}\right),$$

$$\gamma_3=-\frac{\lambda}{3}+R_1^{1/3}e^{-i\pi/3}+R_2^{1/3}e^{i\pi/3};$$

$$R_{1,2}=\left(\frac{\lambda}{3}\right)^3+\frac{\eta^2}{2}\mp\eta\left[\left(\frac{\lambda}{3}\right)^3+\left(\frac{\eta}{2}\right)^2\right]^{1/2}, \tag{20}$$

where $R_1^{1/3}$ and $R_2^{1/3}$ must be taken to mean the branch of the cubic root with the smallest phase. The root γ_2 is always real.

Substituting the roots of (18) into (14) we can verify that in the region where

$$(\lambda/3)^3+(\eta/2)^2>0,$$

the roots of (14) will be γ_1 (upper branch of the spectrum) and γ_2 (lower branch of the spectrum). On the other hand, if

$$(\lambda/3)^3+(\eta/2)^2=0, \tag{21}$$

then $R_1 = R_2$ and all the roots of (18) become real:

$$\gamma_2=-(\eta/2)^{2/3},\quad \gamma_1=\gamma_3=2(\eta/2)^{2/3}. \tag{22}$$

However, γ_1 and γ_3 of (22) satisfy not the equation (14), but its complex conjugate. Thus, at a magnetic field value such that

$$\lambda=\lambda_0=-3(\eta/2)^{2/3}, \tag{23}$$

the upper branch of the spectrum vanishes, and in weaker magnetic fields there remains one branch γ_2. The spectrum of the electron-phonon system as a function of the magnetic field is shown schematically in Fig. 2.

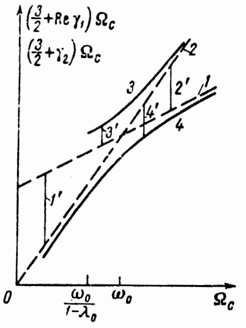

FIG. 2. Schematic spectrum of electron with allowance for interaction with optical phonons. 1, 2–initial states of electron-phonon system without interaction; 3,4–upper and lower branches of spectrum. The resonance point $\Omega_c = \omega_0$ and the point of termination of the upper branch of the spectrum $\Omega_c = \omega_0/(1 - \lambda_0)$ are shown. 1′, 2′, 3′, 4′ denote respectively.

The vanishing of the upper branch of the spectrum γ_1 at the point λ_0 should be taken to mean the vanishing of the connection between the states of the electron with n = 1 and n = 0. Indeed, when the magnetic field is decreased the cyclotron frequency Ω_c becomes so much smaller than ω_0 that when $k_z = 0$ the energy of the electron thrown by the light to the "bottom" of the level n = 1 is insufficient for a transition to the bottom of the level n = 0 with emission of an optical phonon.

The distance $\Delta(\lambda)$ between the branches of the spectrum at the point λ_0 (23) is equal to

$$\Delta(\lambda_0)=3(\eta/2)^{2/3}\Omega_c. \tag{24}$$

When $\lambda_0 = 0$ it is equal to

$$\Delta(0)={}^3/_2\eta^{2/3}\Omega_c \tag{25}$$

and is a minimum.

We note also that the lowest branch of the spectrum does not have an imaginary increment, i.e., it corresponds to the stationary state, as should be the case at T = 0 when there are no optical phonons.

Let us estimate the error in the determination of the poles of G_r, including the quantity Σ_{21} in $\Sigma(\alpha, \epsilon)$. In lieu of (14) we obtain the more complicated expression

$$\gamma+\frac{i\eta}{\sqrt{\gamma+\lambda}}-\frac{i\eta^2}{\sqrt{2}(\omega_0/\Omega_c)^{3/2}(\gamma+\lambda)^{3/2}}=0. \tag{26}$$

Denoting the correction to the roots γ_i of Eq. (14) by $\Delta\gamma_i$ and solving (26) approximately, assuming the term $\sim\eta^2$ to be small, we get $\Delta\gamma_i \cong \eta$ when $\lambda = 0$, and consequently the error in the determination of the spectrum is proportional to $\alpha_0^{1/3}$. Thus, the approximations made above in the mass operator are justified if the coupling constant is sufficiently small.

5. CALCULATION OF THE LIGHT ABSORPTION COEFFICIENT AND DISCUSSION OF RESULTS

Substituting in the formula for $K(\omega)$ the function $G_r(\alpha, \omega - \omega_{v\alpha})$ from (13), we obtain

$$K(\omega)=K_0\,\mathrm{Re}\,[iF(\Gamma)], \tag{27}$$

$$K_0=\frac{\pi}{cn_0}\frac{\sqrt{2}e^2|P|^2m_c}{(2\pi)^2\hbar^2\omega_g m_0^2 l_H},$$

where

$$F(\Gamma)=\int_{-\infty}^{\infty} dk\left\{\Gamma-\frac{m_c+m_v}{m_v}k^2+\frac{i\eta}{\sqrt{\Gamma+\lambda-m_ck^2/m_v}}\right\}^{-1}, \quad (28)$$
$$\Gamma=[\omega-\omega_g-{}^3/_2(\Omega_c+\Omega_v)]\,/\,\Omega_c.$$

The value of F(Γ) can be easily calculated in the limiting case when $m_c/m_v \ll 1$. (This takes place in many III-V compounds; for example in InSb we have $m_c/m_v \cong 0.05$.) In this case

$$F(\Gamma)=-i\pi(\Gamma+i\eta\,/\sqrt{\Gamma+\lambda})^{-1/2}. \quad (29)$$

Putting again λ = 0 and separating the real and imaginary parts in (29), we get

$$K(\omega)=K_0\begin{cases}0, & \Gamma\leqslant-\eta^{2/3},\\ \pi/\sqrt{f_1(\Gamma)(\Gamma+\eta^{2/3})}, & -\eta^{2/3}\leqslant\Gamma\leqslant0, \quad (30)\\ \pi f_2(\Gamma)\Big/\left\{\left(\Gamma-\frac{\eta^{2/3}}{2}\right)^2+\frac{3}{4}\eta^{4/3}\right\}^{1/2}, & \Gamma\geqslant0, \quad (31)\end{cases}$$

$$f_1(\Gamma)=\frac{\Gamma^2-\eta^{2/3}\Gamma+\eta^{4/3}}{\Gamma^2+\eta\sqrt{-\Gamma}},\quad f_2(\Gamma)=\left(\frac{\sqrt{\Gamma^4+\eta^2\Gamma}+\Gamma^2}{2(\Gamma+\eta^{2/3})}\right)^{1/2}. \quad (32)$$

It is seen from the foregoing formulas that the absorption coefficient has two maxima, at the points $\Gamma_1 = -\eta^{2/3}$ and $\Gamma_2 \cong \eta^{2/3}/2$ (the second maximum is shifted, somewhat towards larger Γ, owing to the monotonic function $f_2(\Gamma)$. The distance Δ_1 between maxima is

$$\Delta_1={}^3/_2\eta^{2/3}\Omega_c, \quad (33)$$

which coincides with the splitting of the electron spectrum at λ = 0 (see (25)). If we take into account the correction to the mass operator Σ, necessitated by Σ_{21}, then the function K(ω) changes at λ = 0 by the factor

$$1+\eta^2/2|\Gamma|(\eta-|\Gamma|^{3/2})\quad\text{(for formula (30)),}$$
$$1+\eta^3/\Gamma(\Gamma^3+\eta^2)\quad\text{(for formula (31)).}$$

It follows therefore that in the region $(-\eta \le \Gamma \le \eta)$ and near $(-\eta^{2/3})$ it is impossible to limit oneself to the first approximation Σ_1 in the calculation of the mass operator, but it is necessary to take into account the entire perturbation-theory series[1]. In the remaining regions of variation of Γ, and particularly in the region of the maximum of the second peak, these corrections are small like $\eta^{1/3}$.

Let us trace now the behavior of the absorption coefficient when λ approaches λ_0, corresponding to the point of vanishing of the upper branch of the spectrum. For arbitrary λ we get from (27) and (29)

$$K(\omega)\sim\left\{\left[\Gamma+\frac{\lambda}{3}-\frac{1}{2}(R_1^{1/3}+R_2^{1/3})\right]^2+\frac{3}{4}[R_1^{1/3}-R_2^{1/3}]^2\right\}^{-1/2}. \quad (34)$$

This formula is valid for Γ + λ > 0 (we leave out the complicated monotonic function similar to $f_2(\Gamma)$ in (31)). If $\lambda \to \lambda_0$, then $R_1 \to R_2$, and the maximum of the oscillation shifts towards larger Γ and becomes sharper. However, in addition, the region of values of Γ where formula (34) is valid also shifts to the right by a larger amount than the shift of the maximum of the oscillation, and therefore the peak vanishes when $\lambda = \lambda_0$.

The theory outlined above describes the main features of the effect observed by Johnson and Larsen[2], namely the doubling of the peak at a magnetic field such that $\Omega_c \cong \omega_0$, and the vanishing of the additional peak with decreasing magnetic field. The point of vanishing of the upper branch of the spectrum (23) is satisfactorily described by the formulas given above for the electron spectrum. If we use the values of the parameters of the theory

$$\hbar\omega_0=0{,}023\text{ eV},\ \alpha_0=0{,}02,\quad m_c=0{,}014\,m_0,$$

which are characteristic of InSb, then the value of H calculated from the condition (21) is ~ 2.9 × 10⁴ Oe, whereas in[2] the second maximum vanishes at H = 2.8 × 10⁴ Oe. The remaining quantities also agree with the data of Johnson and Larsen: the distance between peaks of the absorption coefficient at resonance ($\Omega_c = \omega_0$) is $\hbar\Delta \cong 1.6 \times 10^{-3}$ eV, and the width of the oscillation in the region of larger values of the optical frequencies is $\delta \approx 0.8 \times 10^{-3}$ eV. The values of the corresponding quantities in the experiment of[2] are 1.8×10^{-3} eV and 10^{-3} eV.

The form of the oscillations at the lower light frequencies is not determined by the aforementioned scattering mechanism at zero temperature. Indeed, if it is assumed that there are no optical phonons, then the lower state is only stationary; when $\Omega_c < \omega_0$, this state will be the level n = 1, and if $\Omega_c > \omega_0$, this will be the level n = 0 and a phonon. For the same reason, when the magnetic field is increased ($\Omega_c > \omega_0$), the vanishing of the peak located at smaller light frequencies is not described by the theory, since this peak diverges. Its form is determined by other scattering mechanisms, for example by the interaction with impurities, and also by the temperature corrections to the considered interaction mechanism.

Certain simplifying assumptions were made in the calculation, but allowance for them will apparently not introduce any appreciable changes in the results. The scattering of the holes makes no contribution to the braodening of the oscillations, since $m_v \gg m_c$, and the holes cannot emit optical phonons at all. Allowance for the non-parabolicity cannot change strongly the result, since it is sufficient, for the description of the effect, to take into account only two Landau levels (n = 0 and n = 1) and the form of the oscillation is determined by the small k_z. Allowance for the dispersion of the optical phonons is likewise of no significance, since the main effect is connected with multiple resonant transitions of the phonon with zero momentum between the electronic states with $k_z = 0$.

In conclusion we note that the contribution made by the aforementioned interaction mechanism to the form of the peak containing no divergences is described here exactly. It is therefore of interest to estimate the contribution made to the maximum of this peak by the interaction with acoustic phonons. For order-of-magnitude

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[1] When $m_c/m_v = 0$, direct use of (29) leads to a value K = 0 at the point Γ = −λ. In particular, when λ = 0 we have K = 0 at the point Γ = 0, as is seen also from (30) and (31). However, if account is taken of the finite value of the ratio m_c/m_v, then (27) and (28) yield at the point Γ = −λ a nonzero value of K. In the case of resonance (λ = 0) we have

$$K\left(\Gamma=0,\frac{m_c}{m_v}\right)=K_0\frac{\pi}{3}\eta^{-1/3}\left(\frac{m_c}{m_v}\right)^{1/6}. \quad \text{(a)}$$

On the other hand, the value of K near the maximum at $\Gamma = (1/2)\eta^{2/3}$ is

$$K\left(\Gamma=\frac{1}{2}\eta^{2/3},\ \frac{m_c}{m_v}=0\right)\approx K_0\frac{2\pi}{3}\eta^{-1/3}. \quad \text{(b)}$$

When $m_c/m_v = 0.05$, as is the case in InSb, the ratio of (b) to (a) is equal to ~ 3.3, i.e., the maximum is quite clearly pronounced. The case of an arbitrary ratio m_c/m_v will be considered in detail by the authors in a separate paper.

estimates we shall use the formula for the light absorption coefficient in the case of scattering of an electron by acoustic phonons, a formula which is valid in the quasiclassical case[6]. The overall absorption coefficient is

$$K(\omega) \sim \eta^{-1/3}(1 - \gamma_c / 2\Omega_c\eta^{2/3}), \tag{35}$$

$$\frac{\gamma_c}{\Omega_c\eta^{2/3}} \approx \left[\frac{E_c{}^2 T}{10\alpha_0(\hbar\omega_0)^2(\rho u_{\text{ac}}{}^3 l_H)^3} \right]^{2/3}, \tag{36}$$

where E_c is the constant of the deformation potential in the c-band, ρ is the crystal density, u_{ac} is the speed of sound, and T is the temperature in energy units.

Substituting in (36) the values of the parameters for InSb ($E_c = 4.5$ eV, $\rho = 5.8$ g/cm^3, $u_{ac} = 3.5 \times 10^5$ cm/sec) and also assuming a temperature of 30°K[2] and $l_H = 10^{-6}$ cm, we get

$$\gamma_c / \Omega_c\eta^{2/3} \approx 10^{-3},$$

i.e., the interaction with the acoustic phonons makes a negligibly small contribution to the form of the peaks at the given temperature. It follows from this also that in sufficiently pure crystals the form of the diverging peak will be determined by the temperature corrections to the considered broadening mechanism, and the same corrections will determine the form of the second peak at finite temperatures. A consideration of this question, however, is beyond the scope of the present article.

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