NONLINEAR GENERATION OF HIGHER HARMONICS OF THE ELECTRON-CYCLOTRON FREQUENCY IN A CURRENT-CARRYING PLASMA[1)]

M. A. LIVSHITZ and V. N. TSYTOVICH

P. N. Lebedev Physics Institute, Academy of Sciences U.S.S.R.

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We consider the nonlinear generation of electron-cyclotron harmonics as a result of the excitation of a two-stream instability in a plasma. The nonlinear growth rates are computed for the generation of the ordinary, extraordinary, and plasma electron-cyclotron waves for scattering of ion-acoustic waves by the electron stream. It is shown that Compton scattering predominates when the ordinary waves are generated whereas the predominant scattering process in the generation of extraordinary and plasma waves is nonlinear scattering. The interpretation of experiments on the generation of electron-cyclotron frequencies in a current-carrying plasma on the basis of the nonlinear excitation theory is discussed.

1. INTRODUCTION

A NUMBER of experiments[1-3] have exhibited strong generation of electromagnetic waves at frequencies that are multiples of the electron-cyclotron frequency $\omega_{He} = eH/mc$. Thus, wave generation is observed with characteristic frequencies $\nu\omega_{He}$ (ν is an integer) up to values of ν of the order of 10–12. Especially strong intensities of the radiation are observed at $\nu = 2$. The intensity of the radiation in these experiments[1-3] is many orders of magnitude beyond the thermal level. The effect has been called anomalous emission. However, a detailed working theory of this effect is not yet available. The authors of [1-3] have proposed one possible explanation based on the production of maser effects in the plasma at frequencies $\nu\omega_{He}$. In the language of present-day plasma theory, this means that an instability arises in the plasma at the indicated frequencies.

It is well known that a plasma can support the propagation of collective motions at frequencies close to $\nu\omega_{He}$, specifically, the so-called cyclotron frequencies. These waves propagate almost perpendicularly to the external magnetic field. Thus, it is natural to suggest that cyclotron waves have been excited under the experimental conditions described in [1-3]. This point of view has also been proposed by the authors of the work mentioned above. It should be noted that there are at least two possible mechanisms for the electron-cyclotron instability. The first is a linear instability, which can arise if the electron distribution in the plasma is not an equilibrium distribution (non-Maxwellian). The second mechanism is associated with a nonlinear instability that can arise if the plasma supports sufficiently strong oscillations of another kind (for example, ion-acoustic waves).

Only the first of the possibilities has been discussed in [1-3] and there was no detailed explanation of what mechanism in the plasma could give rise to a nonequilibrium electron distribution, nor a mechanism by which such a distribution could be maintained (for a time of the order of the generation time). It should be noted that the development of an instability that leads to the generation of electron-cyclotron waves must (because of the effect of the waves themselves on the electrons) modify the initial electron distribution in such a way that the instability is quenched (quasi-linear effect). The time required to quench the linear instability is usually of the order of γ^{-1} where γ is the linear growth rate, which must be higher than the frequency of collisions of electrons with neutral atoms ν_e (for the conditions in [1-3] $\nu_e \sim 10^8\ \text{sec}^{-1}$). Thus, in a time of the order of ν_e^{-1} the nonequilibrium electron distribution would be quenched by binary collisions.

For any reasonable estimates the time in question is extremely small and may be taken as much smaller than the generation time ($\sim 10^{-4}$ sec). Hence, if one is to explain the observed effects in terms of linear phenomena it must be assumed that the nonequilibrium features of the electron distribution are maintained by some extremely efficient mechanism. In other words, the use of the linear instability theory requires the introduction of a number of rather extreme assumptions.[2)]

In the present work we develop a theory for a nonlinear instability, that is to say, we consider the second of the possibilities mentioned above. The use of a nonlinear mechanism for the instability is quite reasonable and does not require the introduction of additional assumptions. The basic feature in the present analysis, from the present point of view, derives from the experimental result that the generation is observed in the presence of current flow in the plasma. It should be noted that the experiments in [1-3] made use of the inert gases Ne, Ar, Kr, and Xe, which are characterized by high ion masses. This means that the velocity of the ion-acoustic wave $v_s = \sqrt{T_e/m_i}$ is rather small. It is well known[5,6] that if the electron drift velocity with respect to the ions u exceeds v_s, the ion-acoustic instability develops when $T_e \gg T_i$. Using the experimen-

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[2)]In this sense, a plasma carrying a current is basically different from that used in [4] where the method of generation of the electron-cyclotron frequencies was based on the injection of an electron beam in a magnetic field, in which case the nonequilibrium distribution is produced beforehand by an external agency.

tal results[1-3] we find that u is larger than v_S and that in a number of cases it reaches values of the order

$$v_{Te} = \sqrt{m_i / m_e}\, v_s \sim 10^2 v_s.$$

Thus, the conditions in [1-3] allow the excitation of the ion-acoustic wave. A stabilizing feature, that hinders the development of this instability, is represented by binary collisions of ions with neutrals.[7] An estimate of the role of collisions can be made at frequencies $\omega \sim \omega_{0i}$ for which the growth rate for the ion-acoustic instability is a maximum. Assuming that $u > v_S$, the condition that the collisions be negligible can be written in the form

$$u > v_{Te}\frac{\nu_e}{\omega_{0e}}\sqrt{\frac{T_i}{T_e}},$$

where ω_{0e} is the electron plasma frequency, $\omega_{0e} = \sqrt{4\pi n_0 e^2/m_e}$, v_{Te} is the electron thermal velocity, $v_{Te} = \sqrt{T_e/m_e}$. Assuming that $\nu_e/\omega_{0e} \sim 10^{-2} - 5\times 10^{-2}$ in [1-3] we find that this condition is satisfied.

The development of the ion-acoustic instability is accompanied by the generation of intense waves at frequencies smaller than or equal to ω_{0i}. We note that electron cyclotron waves cannot be excited directly in the two-stream instability (the excitation of ion-cyclotron waves is considered in [8, 9]). This result follows because only the electrons participate in the electron-cyclotron oscillations so that the motion of ions with respect to the electrons is not important. We note, however, that there are possible nonlinear mechanisms for the excitation of electron-cyclotron waves and the present work is devoted specifically to the analysis of these mechanisms. Excitation of this kind is possible under induced scattering of ion-acoustic waves on a stream of electrons with resulting conversion to electron-cyclotron waves.

A specific feature of the conversion of waves under induced scattering on electrons is the possibility of an increase in the wave frequency associated with the existence of the electron drift. Under the conditions in [1-3] $\omega_{He} > \omega_{0i} \gtrsim \omega_S$. From energy conservation considerations, for scattering we have the relation

$$\omega_s - k_{1z}v_z - \omega_{\nu_0} = \nu\omega_{He},$$

where ω_S is the frequency of the ion-acoustic wave, $\omega_{\nu_0} \approx \nu_0\omega_{He}$ is the frequency of the electron-cyclotron wave, ν_0 is a positive integer, ν is any integer, and k_{1Z} is the component of the wave vector for the ion-acoustic wave.

The quantity v_Z (the electron velocity in the direction of the magnetic field) is of the order of the drift velocity u. For the ion-acoustic waves, the quantity $\omega_S(\mathbf{k}_1) - k_{1Z}v_Z$ in the conservation relation that governs the scattering can reach rather high values:

$$\omega_{0e}\sqrt{\frac{T_e}{T_i}}\frac{u}{v_{Te}} \gg \omega_s \approx \omega_{0e}\sqrt{\frac{m_e}{m_i}}.$$

This means that the frequency of the scattered wave can be large in the coordinate system in which the electron stream is at rest. If the scattering in this coordinate system occurs without frequency change ($\nu = 0$) the frequency of the scattered wave will be large, that is to say, it is possible to excite high harmonics of the electron-cyclotron frequency. In actuality, because of the fact that the allowed frequency change in scattering must be a multiple of ω_{He} ($\nu \neq 0$) the situation is actually more complicated and requires an analysis of the relative roles of processes that lead to different frequency changes in scattering.

A detailed analysis carried out below shows that it is precisely the waves with large k_1 that correspond to the excitation of higher harmonics of the gyrofrequency. In this respect the ion-acoustic waves are favored since they have the highest value of k_1 of all possible waves. Furthermore, the growth rate for the excitation of the ion-acoustic wave in the presence of a current flow in the plasma is larger than the growth rate for other waves.[10] It is precisely for these reasons that below we consider the nonlinear conversion of ion-acoustic waves.

It should be noted that other nonlinear mechanisms for the excitation of electron-cyclotron waves are essentially negligible. The effect of scattering on ions only leads to a reduction in the frequency of the wave; the same result follows in decay processes; in order for an "addition interaction" (up-conversion) to result in the frequency ω_{He} it is necessary to have simultaneous (or multiple) collisions of a large number of ion-acoustic plasmons, the probability of which is very small. The nonlinear growth rates obtained below for the electron-cyclotron waves are much higher in magnitude and can be used for other purposes than the interpretation of the results in [1-3]. In particular, below we discuss the question of a radiative-dissipation, ion-acoustic turbulence and the related question of efficiency of turbulent heating of a plasma.[11]

2. FORMULATION OF THE PROBLEM

We assume that the electron velocity distribution is a Maxwellian with a superimposed drift

$$f(\mathbf{v}) = \frac{n_0}{(2\pi)^{3/2}v_{Te}^3}\exp\left\{-\frac{(\mathbf{v}-\mathbf{u})^2}{2v_{Te}^2}\right\}, \quad \int f(\mathbf{v})\,d\mathbf{v} = n_0, \tag{1}$$

where $u \ll v_{Te}$. The direction of the external magnetic field $\mathbf{H}_0$ is taken to be along the z-axis. The wave vector for the ion-acoustic waves $\mathbf{k}_1 \parallel \mathbf{H}_0$ and the direction of the wave vector $\mathbf{k}$ for the scattered cyclotron wave is taken to be the x-axis.

It is well known[12-14] that the nonlinear growth rate for the excitation of a wave σ as a consequence of scattering of a wave σ' on a particle α can be computed if one knows the scattering cross section $W^{\sigma\sigma'\alpha}(\mathbf{p}, \mathbf{k}_\sigma, \mathbf{k}_{\sigma'})$ for the wave σ' with momentum $\mathbf{k}_1$ on a particle α with momentum $\mathbf{p}$ with the conversion to a wave σ with momentum $\mathbf{k}$. The expression for the scattering probability is[13]

$$W_\nu^{\sigma\sigma'\alpha}(\mathbf{p},\mathbf{k},\mathbf{k}_1) = 2\omega_1^2(\mathbf{k}_1)\,\delta[\omega(\mathbf{k}) - \omega_1(\mathbf{k}_1) - (k_z - k_{1z})v_z - \nu\omega_{He}]$$
$$\times\left|\frac{\partial}{\partial\omega}\omega^2\varepsilon^\sigma\right|^{-1}_{\omega=\omega(\mathbf{k})}\left|\frac{\partial}{\partial\omega}\omega^2\varepsilon^{\sigma'}\right|^{-1}_{\omega=\omega_1(\mathbf{k}_1)}|\Lambda_{p_\alpha}^{\sigma\sigma'}(\mathbf{k},\omega(\mathbf{k}),\mathbf{k}_1,\omega_1(\mathbf{k}_1))|^2; \tag{2}$$
$$\varepsilon^\sigma(\omega,\mathbf{k}) = a_i^*(\mathbf{k})\,\varepsilon_{ij}(\omega,\mathbf{k})\,a_j(\mathbf{k}) + \omega^{-2}(\mathbf{k}\mathbf{a}(\mathbf{k}))\,(\mathbf{k}\mathbf{a}^*(\mathbf{k}))\,c^2,$$

where

$$\Lambda_{p_\alpha}^{\sigma\sigma'} = a_i(\mathbf{k})\Lambda_{ij}(\mathbf{k},\omega(\mathbf{k}),\mathbf{k}_1,\omega_1(\mathbf{k}_1))\,a_j(\mathbf{k}_1).$$

The notation is that of [13]. The scattering cross section can be found if we write an expression for the nonlinear current $\mathbf{j}(\mathbf{k}, \omega)$, which is proportional to the first power of the electric field in the wave:

$$j_i(\mathbf{k}, \omega) = \int \Lambda_{ij}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1) E_j(\mathbf{k}_1, \omega_1) d\mathbf{k}_1 d\omega_1.$$

It should be noted that two physically different scattering mechanisms are possible: Thomson and nonlinear. The first, which is characterized by the quantity $\Lambda_{ij}^{(1)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1)$ is associated with the oscillation of the charge in the field of the wave, while the second $\Lambda_{ij}^{(2)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1)$ is associated with radiation on the plasma inhomogeneities produced by the ion-acoustic waves.[12] The components of the tensor $\Lambda_{ij}^{(2)}$ are expressed in terms of the components of the tensor $S_{ijs}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}_2, \omega_2)$ which arises in the second approximation in the expansion of the nonlinear plasma current $\mathbf{j}(\mathbf{k}, \omega)$ in the external field amplitude (cf. [12-13]):

$$\Lambda_{ij}^{(2)} = [S_{ijs}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k} - \mathbf{k}_1, \omega - \omega_1) + S_{isj}(\mathbf{k}, \omega, \mathbf{k} - \mathbf{k}_1, \omega - \omega_1, \mathbf{k}_1, \omega_1)] E_s^Q(\mathbf{k} - \mathbf{k}_1, \omega - \omega_1), \quad (3)$$

in which case the field $E_s^Q(\mathbf{k} - \mathbf{k}_1, \omega - \omega_1)$ is determined by means of Maxwell's equations with the current produced by a charge gyrating in the magnetic field along a helical line, neglecting the perturbation of its trajectory by the wave field.

The expression for the rate-of-change of the photons $N_{\mathbf{k}}^{\sigma}$ is

$$\frac{\partial N_{\mathbf{k}}^{\sigma}}{\partial t} = \sum_{\nu} \int W_{\nu}^{\sigma\sigma' e}(\mathbf{p}_e, \mathbf{k}, \mathbf{k}_1) \times N_{\mathbf{k}}^{\sigma} N_{\mathbf{k}_1}^{\sigma'} \left[(k_z - k_{1z}) \frac{\partial f}{\partial p_z} + \frac{\nu \varepsilon \omega_{He}}{p_{\perp} c^2} \frac{\partial f}{\partial p_{\perp}} \right] d\mathbf{p}\, d\mathbf{k}_1. \quad (4)$$

Here, we are interested in the excitation of the electron-cyclotron waves. The index σ in (4) corresponds to a wave $\omega_{\nu_0} \approx \nu_0 \omega_{He}$ while the index σ' corresponds to the ion-acoustic wave ω_s. The relation in (4) determines directly the desired nonlinear growth rate if we use the distribution function (1) in (4).

The problem is now reduced to the calculation of the probability $W_{\nu}^{\sigma\sigma' e}$. We first summarize the basic properties of the electron-cyclotron waves.

The dispersion relation breaks up into two parts when $\theta = \pi/2$ where θ is the angle between the vectors $\mathbf{k}$ and $\mathbf{H}$:

$$n^2 - \varepsilon_{33}(\omega, \mathbf{k}) = 0, \quad \varepsilon_{11}(\omega, \mathbf{k}) n^2 - \varepsilon_{11}(\omega, \mathbf{k}) \varepsilon_{22}(\omega, \mathbf{k}) - \varepsilon_{12}^2(\omega, \mathbf{k}) = 0,$$
$$n^2 = k^2 c^2 / \omega^2.$$

The first relation characterizes the so-called ordinary wave, while the second characterizes the extraordinary wave. The components of the dielectric tensor $\varepsilon_{ij}(\omega, \mathbf{k})$ are given in [15]. These expressions can be used when $|(\omega - l\omega_{He})/\omega_{He}| \gg v_{Te}^2/c^2$ (l in an integer). When $n^2 \gg \varepsilon_{22} + \varepsilon_{12}^2/\varepsilon_{11}$ the second relation yields the dispersion equation for the plasma wave $\varepsilon_{11}(\omega, \mathbf{k}) = 0$.

The ordinary waves are characterized by polarization vectors $\mathbf{a}(\mathbf{k}) = \{0, 0, 1\}$. In accordance with (2) the quantity $\varepsilon^{\sigma}(\omega, \mathbf{k})$ for these waves is given by $\varepsilon^{\sigma}(\omega, \mathbf{k}) = \varepsilon_{33}(\omega, \mathbf{k})$. Calculation of the quantity $\partial\omega^2\varepsilon^{\sigma}/\partial\omega|_{\omega=\omega(\mathbf{k})}$ that appears in (2) yields

$$\frac{\partial}{\partial \omega} \omega^2 \varepsilon^{\sigma} \Big|_{\omega=\omega(\mathbf{k})} = \sum_{\nu} \omega_{0e}^2 A_{\nu}(\mu) \frac{\nu \omega_{He}}{(\omega(\mathbf{k}) - \nu\omega_{He})^2}, \quad (5)$$

where

$$\mu = k^2 v_{Te}^2 / \omega_{He}^2, \quad A_{\nu}(\mu) = e^{-\mu} I_{\nu}(\mu),$$

and I_{ν} is a modified Bessel function. The actual form of the dependence $\omega(\mathbf{k})$ for the ordinary electron-cyclotron waves is given in [15]. We shall not give these relations here because, as will be shown below, the growth rate for nonlinear generation of ordinary cyclotron waves (under the assumption $\omega \approx \nu_0 \omega_{He}$) does not depend on small deviations of $\omega(\mathbf{k})$ from $\nu_0 \omega_{He}$.

The extraordinary waves are characterized by a polarization vector $\mathbf{a}(\mathbf{k}) = \{a_x, a_y, 0\}$.[16] Using Maxwell's equations and the normalization condition $\mathbf{a} \cdot \mathbf{a}^* = 1$ we find

$$a_y(\mathbf{k}) = \left(1 + \left|\frac{\varepsilon_{12}(\mathbf{k})}{\varepsilon_{11}(\mathbf{k})}\right|^2\right)^{-1/2}, \quad a_x(\mathbf{k}) = -\frac{\varepsilon_{12}(\mathbf{k})}{\varepsilon_{11}(\mathbf{k})} \left(1 + \left|\frac{\varepsilon_{12}(\mathbf{k})}{\varepsilon_{11}(\mathbf{k})}\right|^2\right)^{-1/2},$$

where, for example, $\varepsilon_{11}(\mathbf{k}) = \varepsilon_{11}(\mathbf{k}, \omega(\mathbf{k}))$ and $\omega(\mathbf{k})$ is the solution of the equation for the extraordinary waves.

We have investigated the extraordinary-wave branch, which becomes the plasma wave when $\kappa \ll \mu \ll 1$. The solution of the dispersion equation for this case yields (we take account of the resonance term and the terms characterized by $\nu = \pm 1$)

$$\frac{\omega(\mathbf{k}) - \nu_0 \omega_{He}}{\omega_{He}} = -\frac{\nu_0(\nu_0 - 1)[\mu(\nu_0 + 1) + 2\kappa\nu_0] I_{\nu_0}(\mu)}{\mu(\mu + \kappa)},$$
$$\kappa = \frac{\omega_{0e}^2}{\omega_{He}^2} \frac{v_{Te}^2}{c^2} = \frac{\mu}{n^2} \frac{\omega_{0e}^2}{\nu_0^2 \omega_{He}^2} = \frac{4\pi n_0 T_e}{H^2}, \quad (6)$$

where ν_0 is an integer. In accordance with (2), the quantity $\varepsilon^{\sigma}(\omega, \mathbf{k})$ for the extraordinary waves is

$$\varepsilon^{\sigma}(\omega, \mathbf{k}) = [\varepsilon_{11}(\omega, \mathbf{k}) (|\varepsilon_{12}(\mathbf{k})|^2 + |\varepsilon_{11}(\mathbf{k})|^2) + 2\varepsilon_{11}(\mathbf{k}) \varepsilon_{12}(\mathbf{k}) \varepsilon_{12}(\omega, \mathbf{k})] \frac{|a_y(\mathbf{k})|^2}{|\varepsilon_{11}(\mathbf{k})|^2} + \frac{k^2 c^2}{\omega^2} |a_x(\mathbf{k})|^2. \quad (7)$$

The quantity $\partial\omega^2\varepsilon^{\sigma}/\partial\omega|_{\omega = \omega(\mathbf{k})}$ that appears in (2) is given by

$$\frac{\partial}{\partial \omega} \omega^2 \varepsilon^{\sigma}(\omega, \mathbf{k}) \Big|_{\omega=\omega(\mathbf{k})} = \frac{\omega_{0e}^6}{\omega_{He}^5} \frac{\mu + \kappa}{\nu_0(\nu_0 - 1)^2 [\mu(\nu_0 + 1) + 2\kappa\nu_0](\nu_0 + 1)^2} \times \left[2 + \left(1 - \frac{2\nu_0 I_{\nu_0}}{\mu}\right) \frac{\mu(\mu + \kappa)}{\mu(\nu_0 + 1) + 2\kappa\nu_0} \frac{1}{I_{\nu_0}(\mu)}\right] \frac{|a_y(\mathbf{k})|^2}{|\varepsilon_{11}(\mathbf{k})|^2}. \quad (8)$$

The plasma waves are characterized by a polarization vector that has an x component $\mathbf{a}(\mathbf{k}) = \{1, 0, 0\}$. The quantity $\varepsilon^{\sigma}(\omega, \mathbf{k})$ for the plasma wave is given by $\varepsilon^{\sigma}(\omega, \mathbf{k}) = \varepsilon_{11}(\omega, \mathbf{k}) + k^2c^2/\omega^2$. The quantity $\partial\omega^2\varepsilon^{\sigma}/\partial\omega|_{\omega = \omega(\mathbf{k})}$ that appears in (2) is of the form

$$\frac{\partial}{\partial \omega} \omega^2 \varepsilon^{\sigma} \Big|_{\omega=\omega(\mathbf{k})} = \sum_{\nu} \frac{\omega_{0e}^2}{(\omega(\mathbf{k}) - \nu\omega_{He})^2} \nu\omega_{He} \frac{\nu^2 A_{\nu}(\mu)}{\mu}. \quad (9)$$

We have not given the actual form of the function $\omega(\mathbf{k})$ for the plasma wave[15] because in the calculation of the growth rate γ one need not use the explicit form of $\omega(\mathbf{k})$.

We can now consider the nonlinear effects in the generation of all three cyclotron waves.

3. NONLINEAR EXCITATION RATES FOR ELECTRON-CYCLOTRON WAVES

In order to avoid complicated expressions, we shall only give the results of the calculation of the components S_{ijs}; using the formulas of the preceding section, and knowing S_{ijs} it is not difficult to write the general expressions for the nonlinear excitation rates. The actual expressions we give here, however, will only be those for limiting cases of interest.

A. Ordinary waves. It follows from (2) that the quantities $\Lambda_{33}^{(1)}$ and $\Lambda_{33}^{(2)}$ are important in the scattering cross section. In order to compute $\Lambda_{33}^{(2)}$, in accordance with (3) we must compute the tensor components S_{331}, S_{313}, S_{333} and S_{333}. It will be shown below that the components S_{332} and S_{323} are not required. Calculation yields

$$S_{331}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1)$$
$$= \frac{\omega_{0e}^2 e}{m_e}\left(1+\frac{k_1 u}{\omega-\omega_1}\right)\frac{1}{2(2\pi)^{3/2} v_{Te}^3}\frac{1}{kk_1}\sum_m \frac{m\omega_{He}}{\omega - m\omega_{He}} A_m(\mu) I_+(\beta_m),$$
$$S_{313}(\mathbf{k}, \omega, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1, \mathbf{k}_1, \omega_1) = \frac{\omega_{0e}^2 e}{m_e}\frac{1}{2(2\pi)^{3/2} v_{Te}^3}\frac{1}{kk_1}\frac{1}{\omega-\omega_1}$$
$$\times\left[k_1 v_{Te}\gamma_0 I_+(\gamma_0) + \omega\left(\frac{u}{v_{Te}}+\gamma_0\right) I_+'(\gamma_0)\right]\sum_m \frac{m\omega_{He}}{\omega - m\omega_{He}} A_m(\mu),$$

$$S_{333}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1) = \frac{\omega_{0e}^2 e}{m_e}\frac{1}{2(2\pi)^{3/2} v_{Te}^3}\frac{1}{k_1}\sum_m \frac{1}{\omega - m\omega_{He}} A_m(\mu)$$
$$\times\left\{I_+(\beta_m)\left(\beta_m v_{Te} + u\frac{m\omega_{He}}{\omega-\omega_1}\right) - v_{Te}\sqrt{2\pi}\right\}, \tag{10}$$
$$S_{333}(\mathbf{k}, \omega, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1, \mathbf{k}_1, \omega_1) = \frac{\omega_{0e}^2 e}{m_e}\frac{1}{2(2\pi)^{3/2} v_{Te}^3}\frac{1}{k_1}\sum_m \frac{1}{\omega - m\omega_{He}} A_m(\mu)$$
$$\times\left\{v_{Te} I_+'(\gamma_0) + \frac{m\omega_{He}}{\omega-\omega_1}\left[v_{Te}\gamma_0 I_+(\gamma_0) + v_{Te} I_+'(\gamma_0)\left(\gamma_0 + \frac{u}{v}\right)^2\right]\right\}.$$

Here, we have used the notation

$$I_+(\beta_m) = \int_{-\infty}^{\infty} e^{-x^2/2}\frac{dx}{\beta_m - x}, \quad \beta_m = \frac{\omega-\omega_1+k_1 u - m\omega_{He}}{-k_1 v_{Te}}$$
$$\gamma_l = \frac{\omega_1 - k_1 u - l\omega_{He}}{k_1 v_{Te}}.$$

The function $I_+(t)$ is related to the probability integral of complex argument $W(z)$.[17, 18] In the calculation we have made use of the identity

$$W'(z) = \frac{2i}{\sqrt{\pi}} - 2zW(z). \tag{11}$$

In the region of high refractive index, as an approximation we can assume scattering occurs only through the virtual longitudinal wave. In this case, the inverse Maxwell operator is

$$\Pi_{iq}(\mathbf{k}_2, \omega_2) = -4\pi i\frac{k_{2i}k_{2q}}{\omega_2 \mathbf{k}_2^2 \varepsilon^l(\mathbf{k}_2, \omega_2)}, \quad \mathbf{k}_2 = \mathbf{k}-\mathbf{k}_1, \quad \omega_2 = \omega - \omega_1. \tag{12}$$

By virtue of our choice of coordinate axes above, it is then clear that S_{323} and S_{332} are not required.

We are interested in the scattered ordinary wave at frequencies close to $\nu_0\omega_{He}$; using the asymptotic expressions for the function $I_+(t)$ for small values of the argument we find

$$\Lambda_{p_e}^{(2)\sigma\sigma'} = \frac{1}{(2\pi)^3}\sum_\nu \delta(\omega-\omega_1+k_1 v_z - \nu\omega_{He}) B_{\nu_0}(\mathbf{k}, \mathbf{k}_1) J_\nu\left(\frac{kv_\perp}{\omega_{He}}\right), \tag{13}$$

where

$$B_{\nu_0}(\mathbf{k}, \mathbf{k}_1) = \frac{4\pi i e}{(\mathbf{k}-\mathbf{k}_1)^2 \varepsilon^l(\mathbf{k}-\mathbf{k}_1, \omega-\omega_1)}$$
$$\times\{[S_{133}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1) + S_{133}(\mathbf{k}, \omega, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1, \mathbf{k}_1, \omega_1)]k_1$$
$$-[S_{131}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1) + S_{113}(\mathbf{k}, \omega, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1, \mathbf{k}_1, \omega_1)]k\}$$
$$= -\frac{\omega_{0e}^2}{(\mathbf{k}-\mathbf{k}_1)^2 \varepsilon^l(\mathbf{k}-\mathbf{k}_1, \omega-\omega_1)}\frac{e^2}{m_e}\sqrt{\frac{\pi}{2}}\frac{\nu_0\omega_{He}}{\omega-\nu_0\omega_{He}}\frac{A_{\nu_0}(\mu)}{k_1 v_{Te}^3}. \tag{14}$$

The probability of the Compton scattering is proportional to

$$\Lambda_{p_e}^{(1)\sigma\sigma'} = \sum_\nu \frac{e^2}{n_e}\frac{1}{(2\pi)^3}\delta(\omega-\omega_1+k_1 v_z - \nu\omega_{He}) J_\nu\left(\frac{kv_\perp}{\omega_{He}}\right)\frac{i}{\omega-\nu\omega_{He}}. \tag{15}$$

It is evident from (9) and (11) that the Compton scattering is larger in a dense plasma.[3)] Substituting (11) in (1) and then in (4) we have $\partial N_{\mathbf{k}}/\partial t = \gamma N_{\mathbf{k}}$ where

$$\gamma = \frac{1}{4(2\pi)^{5/2}}\int N_{\mathbf{k}_1} d\mathbf{k}_1 \frac{1}{n_0 T_e}\frac{k_1 u - \nu_0\omega_{He}}{k_1 v_{Te}}\frac{\omega_{0e}^2}{\nu_0\omega_{He}}\frac{\omega_1^3(\mathbf{k}_1)}{\omega_{0i}^2}. \tag{16}$$

Thus, in accordance with (16) the excitation rate for the ordinary wave is independent of the spectral form of $\omega(\mathbf{k})$ in the range of $\mathbf{k}$ that is of interest.

B. Extraordinary waves. In order to compute the scattering cross section we must calculate $\Lambda_{13}^{(1)}$, $\Lambda_{13}^{(2)}$, $\Lambda_{23}^{(1)}$ and $\Lambda_{23}^{(2)}$. The tensor components $\Lambda_{13}^{(1)}$ and $\Lambda_{23}^{(1)}$ correspond to Compton scattering and vanish in the nonrelativistic approximation. The tensor components S_{ijs} needed for computing $\Lambda_{13}^{(2)}$ and $\Lambda_{23}^{(2)}$ are of the form

$$S_{131}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1) = 0,$$
$$S_{113}(\mathbf{k}, \omega, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1, \mathbf{k}_1, \omega_1)$$
$$= \frac{\omega_{0e}^2 e}{m_e}\frac{1}{2(2\pi)^{3/2}}\frac{1}{k^2 k_1 v_{Te}^4}\frac{\omega}{\omega-\omega_1} I_+'(\gamma_0)\sum_m \frac{m^2\omega_{He}^2}{\omega - m\omega_{He}} A_m(\mu),$$
$$S_{133}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1) = 0,$$
$$S_{133}(\mathbf{k}, \omega, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1, \mathbf{k}_1, \omega_1)$$
$$= \frac{\omega_{0e}^2 e}{m_e}\frac{1}{2(2\pi)^{3/2}}\frac{1}{kk_1^2 v_{Te}^4}\frac{\omega_1}{\omega-\omega_1} I_+'(\gamma_0)\sum_m \frac{m^2\omega_{He}^2}{\omega - m\omega_{He}} A_m(\mu),$$
$$S_{231}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}-\mathbf{k}_1\, \omega-\omega_1) = 0,$$
$$S_{213}(\mathbf{k}, \omega, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1, \mathbf{k}_1, \omega_1)$$
$$= \frac{\omega_{0e}^2 e}{im_e}\frac{1}{2(2\pi)^{3/2}}\frac{1}{k_1 v_{Te}^2}\frac{\omega}{\omega-\omega_1}\frac{I_+'(\gamma_0)}{\omega_{He}}\sum_m \frac{m\omega_{He}}{\omega - m\omega_{He}} A_m'(\mu),$$
$$S_{233}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1) = 0,$$
$$S_{233}(\mathbf{k}, \omega, \mathbf{k}-\mathbf{k}_1, \omega-\omega_1, \mathbf{k}_1, \omega_1) \tag{17}$$
$$= \frac{\omega_{0e}^2 e}{im_e}\frac{1}{2(2\pi)^{3/2}}\frac{k}{k_1^2 v_{Te}^2}\frac{\omega_1}{\omega-\omega_1}\frac{I_+'(\gamma_0)}{\omega_{He}}\sum_m \frac{m\omega_{He}}{\omega - m\omega_{He}} A_m'(\mu).$$

The probability of nonlinear scattering through the virtual longitudinal wave is proportional to

$$\Lambda_{p_e}^{(2)\sigma\sigma'} = a_x^*(\mathbf{k})\Lambda_{13}^{(2)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1) a_{1z}(\mathbf{k}_1) + a_y^*(\mathbf{k})\Lambda_{23}^{(2)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1) a_{1z}(\mathbf{k}_1),$$

where

$$\Lambda_{13}^{(2)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1) = \frac{1}{(2\pi)^3}\sum_{\nu m}\delta(\omega-\omega_1+k_1 v_z - \nu\omega_{He}) C_m(\mathbf{k}, \mathbf{k}_1) J_\nu\left(\frac{kv_\perp}{\omega_{He}}\right) \tag{18}$$

$$C_m(\mathbf{k}, \mathbf{k}_1) = -\frac{i\omega_{0e}^2}{(\mathbf{k}-\mathbf{k}_1)^2 \varepsilon^l(\mathbf{k}-\mathbf{k}_1, \omega-\omega_1)}\frac{e^2}{m_e}\frac{\nu^2\omega_{He}^2}{\omega-\nu_0\omega_{He}}\frac{1}{kk_1 v_{Te}^4} A_m(\mu), \tag{19}$$

$$\Lambda_{23}^{(2)}(\mathbf{k}, \omega, \mathbf{k}_1, \omega_1) = \frac{1}{(2\pi)^3}\sum_{\nu m}\delta(\omega-\omega_1+k_1 v_z - \nu\omega_{He}) D_m(\mathbf{k}, \mathbf{k}_1) J_\nu\left(\frac{kv_\perp}{\omega_{He}}\right), \tag{20}$$

$$D_m(\mathbf{k}, \mathbf{k}_1) = -\frac{\omega_{0e}^2}{(\mathbf{k}-\mathbf{k}_1)^2 \varepsilon^l(\mathbf{k}-\mathbf{k}_1, \omega-\omega_1)}\frac{e^2}{m_e}\frac{\nu_0\omega_{He}}{\omega-\nu_0\omega_{He}}\frac{k}{k_1 v_{Te}^2} A_m'(\mu)\frac{1}{\omega_{He}}, \tag{21}$$

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3)If the plasma is not dense we cannot use the expression for $\Sigma_{ij}(\omega, \mathbf{k})$ as a consequence of the limitation $|\omega - l\omega_{He}/\omega_{He}| \gg v_{Te}^2/c^2$ (CM. [15]).

Thus,

$$\Lambda_{p_e}^{(2)\sigma\sigma'} = \frac{1}{(2\pi)^3}\sum_{mv}\delta(\omega-\omega_1+k_1v_z-\nu\omega_{He})J_\nu\left(\frac{kv_\perp}{\omega_{He}}\right)$$
$$\times\{a_x^*(\mathbf{k})C_m(\mathbf{k},\mathbf{k}_1)a_{1z}(\mathbf{k}_1)+a_y^*(\mathbf{k})D_m(\mathbf{k},\mathbf{k}_1)a_{1z}(\mathbf{k}_1)\}$$
$$=\frac{1}{(2\pi)^3}\sum_{mv}\delta(\omega-\omega_1+k_1v_z-\nu\omega_{He})J_\nu\left(\frac{kv_\perp}{\omega_{He}}\right)\left(-\frac{e^2}{m_e}\right)$$
$$\times\frac{\omega_{0e}^2}{(\mathbf{k}-\mathbf{k}_1)^2\varepsilon^l(\mathbf{k}-\mathbf{k}_1,\omega-\omega_1)}\frac{m}{\omega-m\omega_{He}}\frac{k}{k_1v_{Te}^2}\frac{A_m'(\mu)}{\varepsilon_{11}(\mathbf{k})}a_y^*(\mathbf{k}). \quad (22)$$

Considering the propagation of a wave with frequency $\omega \approx \nu_0\omega_{He}$, we only consider the resonance term in the summation over m. Substituting (22) in (2) and in (4) we have

$$\gamma(\mathbf{k})=\frac{1}{4(2\pi)^{5/2}}\int\frac{N_{\mathbf{k}_1}d\mathbf{k}_1}{n_0T_e}\frac{k_1u-\nu_0\omega_{He}}{(k_1v_{Te})^7}$$
$$\times\nu_0^2A_{\nu_0}'^2(\mu)\,\omega_{0e}^2\omega_{He}^5\mu P(\mu,\nu_0,\varkappa)\,\omega_{0i}\left[1+\frac{\omega_{0e}^2}{k_1^2v_{Te}^2}\left(1+\frac{T_e}{T_i}\right)\right]^{-2}$$

and for

$$k_1\gg\frac{1}{\lambda_{De}},\quad k_1>k,\quad \lambda_{De}=\frac{v_{Te}}{\omega_{0e}}; \quad (23)$$

$$\gamma(\mathbf{k})=\frac{1}{4(2\pi)^{5/2}}\int\frac{N_{\mathbf{k}_1}d\mathbf{k}_1}{n_0T_e}\frac{k_1u-\nu_0\omega_{He}}{(k_1v_{Te})^5}$$
$$\times\nu_0^2A_{\nu_0}'^2(\mu)\,\omega_{He}^5\mu P(\mu,\nu_0,\varkappa)\,\omega_{0i}\left(2+\frac{T_e}{T_i}\right)^{-2} \quad (24)$$

while for $k_1\sim 1/\lambda_{De}$, $k_1 > k$

$$\gamma(\mathbf{k})=\frac{1}{4(2\pi)^{5/2}}\int\frac{N_{\mathbf{k}_1}d\mathbf{k}_1}{n_0T_e}\frac{k_1u-\nu_0\omega_{He}}{(k_1v_{Te})^3}\frac{\omega_{He}^5}{\omega_{0e}^2}$$
$$\times\nu_0^2A_{\nu_0}'^2(\mu)\,\mu P(\mu,\nu_0,\varkappa)\frac{\omega_1^3(\mathbf{k}_1)}{\omega_{0i}^2}\left(1+\frac{T_e}{T_i}\right)^{-2}, \quad (25)$$

and when $k_1\ll 1/\lambda_{De}$, $k_1>k$;

$$P(\mu,\nu_0,\varkappa)=\frac{\mu^2(\mu+\varkappa)(\nu_0+1)^2}{\nu_0I_{\nu_0}^2(\mu)[\mu(\nu_0+1)+2\varkappa\nu_0]}$$
$$\times\left[2+\left(1-\frac{2\nu_0I_{\nu_0}(\mu)}{\mu}\right)\frac{\mu(\mu+\varkappa)}{\mu(\nu_0+1)+2\varkappa\nu_0}\frac{1}{I_{\nu_0}(\mu)}\right]^{-1}. \quad (26)$$

C. Plasma waves. As in the case of the extraordinary waves, in the nonrelativistic approximation the plasma wave is subject only to nonlinear scattering $\Lambda_{p_e}^{\sigma\sigma'}=\Lambda_{p_e}^{(2)\sigma\sigma'}=\Lambda_{13}^{(2)}(\mathbf{k},\omega,\mathbf{k}_1,\omega_1)$. Using (18) and (19), substituting (19) in (2) and (4) we can compute scattering through the virtual longitudinal wave:

a) For a low density plasma (by virtue of the necessity for satisfying $k_1u-\nu_0\omega_{He}>0$ excitation is possible only when $k_1\gg 1/\lambda_{De}$)

$$\gamma(\mathbf{k})=\frac{1}{4(2\pi)^{5/2}}\int\frac{N_{\mathbf{k}_1}d\mathbf{k}_1}{n_0T_e}\frac{\omega_{0e}^6\nu_0A_{\nu_0}(\mu)\,\omega_{He}(k_1u-\nu_0\omega_{He})}{(k_1v_{Te})^7}$$
$$\times\omega_{0i}\left[1+\frac{\omega_{0e}^2}{k_1^2v_{Te}^2}\left(1+\frac{T_e}{T_i}\right)\right]^{-2}, \quad (27)$$

b) For a dense plasma for $k_1\gg 1/\lambda_{De}$, $k_1>k$ (27); when $k_1\sim 1/\lambda_{De}$, $k_1>k$

$$\gamma(\mathbf{k})=\frac{1}{4(2\pi)^{5/2}}\int\frac{N_{\mathbf{k}_1}d\mathbf{k}_1}{n_0T_e}\frac{\nu_0A_{\nu_0}(\mu)(k_1u-\nu_0\omega_{He})}{\omega_{0e}}\omega_{He}\omega_{0i}\left(2+\frac{T_e}{T_i}\right)^{-2}; \quad (28)$$

and when $k_1\ll 1/\lambda_{De}$, $k_1>k$

$$\gamma(\mathbf{k})=\frac{1}{4(2\pi)^{5/2}}\int\frac{N_{\mathbf{k}_1}d\mathbf{k}_1}{n_0T_e}\frac{\omega_{0e}^2\nu_0A_{\nu_0}(\mu)(k_1u-\nu_0\omega_{He})\,\omega_{He}\omega_1^3(\mathbf{k}_1)}{(k_1v_{Te})^3\omega_{0i}^2(1+T_e/T_i)^2}. \quad (29)$$

The expressions for a dense plasma apply when $\mu\ll 1$. When $\mu\gtrsim 1$ the frequency of the wave that propagates across $\mathbf{H}_0$ is not close to $\nu_0\omega_{He}$.

4. DISCUSSION OF RESULTS

1. The nonlinear excitation rates obtained above can be used to estimate the maximum values of the harmonics that can be excited by nonlinear effects. Specifically, the excitation is positive if

$$\nu_0<k_{1max}u/\omega_{He}\sim\frac{\omega_{0e}}{\omega_{He}}\sqrt{\frac{T_e}{T_i}}\frac{u}{v_{Te}}.$$

It should be noted that in the s-waves there must be waves directed in the opposite direction to the electron stream. These can be formed as a result of induced scattering of ion-acoustic waves on ions or by other nonlinear mechanisms that tend to randomize the oscillations. In particular, in scattering on ions the characteristic time to reach isotropy[19] is

$$\frac{1}{\tau}\sim\omega_{0i}\frac{W^s}{n_0T_e}\frac{T_i}{T_e}.$$

Furthermore, isotropy can be produced by various decay processes. In the presence of collisions with charged particles, effective excitation is possible only within a relatively narrow range $\Delta\mathbf{k}_1$ near $k_1\sim 1/\lambda_{De}$; in accordance with[6, 20] the drift velocity is of the order of or somewhat larger than $v_{Te}\sqrt{m_e/m_i}$.

The order-of-magnitude of the energy W^s of the ion-acoustic waves can be estimated from simple considerations of energy balance

$$\frac{dW^s}{dt}=\mathbf{Ej}-\nu_{coll}W^s,\quad j\approx en_0v_{Te}\sqrt{\frac{m_e}{m_i}},$$

where ν_{coll} is the effective frequency for collisions with neutrals and charged particles. When $t\gg 1/\nu_{coll}$

$$W^s\approx\frac{eEn_0v_{Te}}{\nu_{coll}}\sqrt{\frac{m_e}{m_i}}.$$

Using this value of W^s we can estimate the time required to produce isotropy. For example, with $E\sim 10$ V/cm this time is much smaller than the characteristic time required for nonlinear excitation of the cyclotron waves. Account should be taken of the fact that if the instability occurs only when $k_1\sim 1/\lambda_{De}$ the maximum emitted frequency is determined by the relation

$$\nu_0<\nu_{0\,max}\approx\omega_{0e}u/\omega_{He}v_{Te}.$$

Estimating the values of u and v_{Te} from the results of [1-3], we can estimate $\nu_{0\,max}\sim 10$. The experiments [1-3] exhibit intense emission at the second harmonic so that we estimate the excitation rates for $\nu_0=2$, $\mu\ll 1$:

$$\gamma_0^{(2)}(\mathbf{k})\approx\frac{W^s}{n_0T_e}\frac{\sqrt{2\pi}}{8}\frac{\omega_{0e}}{\omega_{He}}\left(\omega_{0e}\frac{u}{v_{Te}}-2\omega_{He}\right),$$
$$\gamma_{\text{II}}^{(2)}(\mathbf{k})\approx\frac{W^s}{n_0T_e}\mu^2\sqrt{2\pi}\left(\omega_{0e}\frac{u}{v_{Te}}-2\omega_{He}\right)\frac{\omega_{He}}{16\omega_{0e}}\left(2+\frac{T_e}{T_i}\right)^{-2},$$
$$\gamma_{\text{II}}^{(2)}(\mathbf{k})\approx\frac{W^s}{n_0T_e}\frac{9\mu^2\sqrt{2\pi}}{4(2+T_e/T_i)^2}\left(\frac{\omega_{He}}{\omega_{0e}}\right)^5\left(\omega_{0e}\frac{u}{v_{Te}}-2\omega_{He}\right).$$

It follows from these estimates that the ordinary wave has the largest excitation value. When $k_1 \sim 1/\lambda_{De}$ excitation occurs only in a dense plasma $\omega_{0e} \gg \omega_{He}$. As ν_0 increases the excitation rates for the ordinary and plasma waves fall whereas that of the extraordinary wave increases (when $\mu < 1$). This result holds when

$$v_0 < v_{0\,max} \sim \omega_{0e} u / \omega_{He} v_{Te}.$$

2. It should be noted that the energy of a plasma and extraordinary cyclotron waves can be emitted from the plasma in two ways. First, by virtue of the spectral excitation the wavelength of the cyclotron waves can be comparable with the system dimensions, in which case dipole radiation is possible. Another method for the emission of energy of the cyclotron waves from the plasma is the nonlinear conversion of cyclotron waves into transverse waves with subsequent emission from the plasma.

3. It is of interest that in the experiments reported in [1-3] the drift velocity u frequently exceeds v_s. According to the theory (cf. [6, 20]), which is developed without taking account of the radiative dissipation of the ion-acoustic turbulence which, in turn, is associated with emission of electromagnetic waves at the cyclotron frequency, the drift velocity must be of order v_s. We now estimate the possible role of radiative dissipation. If the excitation rate for the cyclotron waves is larger than the nonlinear rate for spectral transfer of ion-acoustic waves the latter need not be taken into account and the basic process is the excitation of cyclotron waves. Under these conditions we can assume that the ion-acoustic waves are concentrated in the region of the excitation maximum, i.e., $k_1 \sim 1/\lambda_{De}$.

Radiative dissipation can become important when the nonlinear excitation of the cyclotron waves is larger than or of the order of the linear growth rate for excitation of ion-acoustic waves. A comparison of the growth rates leads to an equality that can be satisfied under the conditions reported in [1-3]. It should also be noted that the growth rate for nonlinear conversion of acoustic waves into a longitudinal wave with frequency $\omega = \omega_{He} \cos\theta$, as indicated by direct calculation (calculations analogous to those given above), is (with $\theta \sim 0$, $k \sim \omega_{He}/v_{Te}$)

$$\gamma(\mathbf{k}) = \int \frac{N_{\mathbf{k}_1} d\mathbf{k}_1}{n_0 T_e} \frac{1}{4(2\pi)^{5/2}} \frac{\omega_1^3(\mathbf{k}_1)}{\omega_{0i}^2} \frac{\omega_{0e}^2}{\omega_{He}} \frac{k_1 u - \omega_{He}}{k_1 v_{Te}} \approx \frac{W^s}{n_0 T_e} \frac{\omega_{0e}^2}{\omega_{He}} \left(\frac{u}{v_{Te}} - \frac{\omega_{He}}{\omega_{0e}} \right), \quad (30)$$

That is to say, this is of the same order as the growth rate in (16).

Under conditions such that nonlinear generation can compete with linear excitation, as one observes alternate excitation and quenching of the oscillations.[21] One expects that in the present case similar effects will arise. The fact that the experimental observations reveal phenomena of this kind may be taken as further support for some of the ideas developed here.

The stopping of the oscillations can reduce the efficiency for the excitation of ion-acoustic waves and, consequently, can increase the value of the mean velocity u. The radiative dissipation associated with ion-acoustic turbulence can also be important on experiments in concerning turbulent heating of plasma,[11] since the heating efficiency can be reduced and spurious losses can be increased.

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