POLARIZATION OF NUCLEAR SPINS BY A DIRECT CURRENT IN SEMICONDUCTORS

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Polarization of nuclear spins due to the passage of a strong current of hot electrons through a semiconductor located in strong crossed electric and magnetic field is considered. A set of kinetic equations, which are nonlinear with respect to the electric field, is solved for the interacting electrons and nuclei in the Fokker-Planck approximation. Expressions are obtained for the induced polarization of nuclei as a function of the Mach number of the electron current for various mechanisms of scattering of the momentum, energy and magnetization of the hot electrons.

1. Feher and Clark[1] have shown that the flow of a strong direct current in a semiconductor placed in crossed electric and magnetic fields can lead to an appreciable increase of the stationary magnetization of the nuclear spins. This phenomenon is caused by the fact that the mechanisms and the characteristic relaxation times of the momentum, energy, and magnetization of a flux of hot electrons are in general different, and this leads to a deviation of the spin degrees of freedom of the electrons from equilibrium with their orbital motion. The interaction of such a nonequilibrium distribution of the conduction electrons with the spins of the nuclei leads to a deviation of the nuclear magnetization from the thermodynamic-equilibrium value. This deviation depends on the scattering mechanisms of the momentum, energy, and magnetization of the hot electrons and on the nuclear-spin relaxation mechanisms.

A theory of this effect, as applied to the case of high conduction-electron density, when the decisive role in their kinetics is played by interelectron collisions, was constructed in[2]. In the case of low concentrations it is impossible in general, to use the effective values of the kinetic and spin temperatures, and the nonequilibrium distribution function of the hot electrons should be obtained from a kinetic equation which is nonlinear in the electric field and takes into account scattering process with and without conservation of the spin orientation.

We shall consider here an electron gas and strong crossed electric (0E0) and magnetic (00H) fields, interacting with acoustic phonons, impurities, and nuclear spins. The interelectron collisions are disregarded. The magnetic field is assumed to be strong in the sense that $\omega_0 > \tilde{\omega}$, where $\omega_0 = eH/mc$ is the cyclotron frequency and $\tilde{\omega}$ is the average momentum-scattering frequency of the electron flux. In this case the kinetic equation for the conducting electrons is

$$\dot{f}_{\nu\sigma} = \sum_{\nu'\sigma',\mathbf{q}} (1 - P_{\nu\sigma,\nu'\sigma'})[W^{\mathbf{q}}_{sph}(\nu'\sigma'|\nu\sigma) + W_{si}^{\mathbf{q}}(\nu'\sigma'|\nu\sigma) + W_{sd}^{\mathbf{q}}(\nu'\sigma'|\nu\sigma) + W_{sn}^{\mathbf{q}}(\nu'\sigma'|\nu\sigma)]; \tag{1}$$

$$W^{\mathbf{q}}_{sph}(\nu'\sigma'|\nu\sigma) = 2\pi\hbar^{-1}|U^{\mathbf{q}}_{sph}(\nu'\sigma'|\nu\sigma)|^2[(N_q+1)f_{\nu'\sigma'} - N_q f_{\nu\sigma}]\cdot \times \delta(\varepsilon_{\nu'\sigma'} - \varepsilon_{\nu\sigma} - \hbar\Omega_q + \hbar q_x V),$$

$$W_{si}^{\mathbf{q}}(\nu'\sigma'|\nu\sigma) = 2\pi\hbar^{-1}|U_{si}^{\mathbf{q}}(\nu'\sigma'|\nu\sigma)|^2 N_i[f_{\nu'\sigma'} - f_{\nu\sigma}]\delta(\varepsilon_{\nu'\sigma'} - \varepsilon_{\nu\sigma} + \hbar q_x V),$$

$$W_{sd}^{\mathbf{q}}(\nu'\sigma'|\nu\sigma) = 2\pi\hbar^{-1}|U_{sd}^{\mathbf{q}}(\nu'\sigma'|\nu\sigma)|^2(n_\sigma f_{\nu'\sigma'} - n_{\sigma'}f_{\nu\sigma})\delta(\varepsilon_{\nu'\sigma'} - \varepsilon_{\nu\sigma} - (\sigma'-\sigma)\Delta_d + \hbar q_x V),$$

$$W_{sn}^{\mathbf{q}}(\nu'\sigma'|\nu\sigma) = 2\pi\hbar^{-1}|U_{sn}^{\mathbf{q}}(\nu'\sigma'|\nu\sigma)|^2(N_\sigma f_{\nu'\sigma'} - N_{\sigma'}f_{\nu\sigma})\delta(\varepsilon_{\nu'\sigma'} - \varepsilon_{\nu\sigma} - (\sigma'-\sigma)\Delta_n + \hbar q_x V). \tag{2}$$

Here $f_{\nu\sigma}$, N_q, n_σ, and N_σ are respectively the distribution functions of the conduction electrons, photons, magnetic-impurity spins, and nuclear spins; N_i is the concentration of the nonmagnetic impurities; $P_{\alpha\beta}(\alpha, \beta) = (\beta, \alpha)$ and $U^q(\ldots|\ldots)$ are the matrix elements of the harmonic components of the scattering potentials with wave vector $\mathbf{q}$, the indices s, ph, i, d, and n pertaining respectively to the conduction electrons, phonons, nonmagnetic impurities, magnetic impurities, and nuclear spins; $\epsilon_{\nu\sigma} = \epsilon_\nu + \sigma\Delta_s$ are the eigenvalues of the kinetic-energy operator in the absence of an electric field; $V = cE/H$; $\sigma, \sigma' = \pm 1$ are the spin quantum numbers; $\Omega_q = sq$ is the phonon frequency, s is the speed of sound, and $2\Delta_{d,s,n} = g_{d,s,n}\mu_0 H$ are the Zeeman splittings for the spins of the magnetic impurities, conduction electrons, and nuclei (μ_0—Bohr magneton, g_i—spectroscopic splitting factors).

The kinetic equation for the populations of the nuclear Zeeman sublevels are written in the form

$$\dot{N}_\sigma = \sum_{\nu'\sigma',\nu\mathbf{q}} (1 - P_{\sigma\sigma'})W_{sn}^{\mathbf{q}}(\nu'\sigma'|\nu\sigma) - (N_\sigma - N_{0\sigma})\omega_{nl},$$

$$N_{0\sigma} = \frac{N_n \exp(-\sigma\Delta_n/T)}{2\,\mathrm{ch}(\Delta_n/T)}, \tag{3}$$

N_n is the concentration of the nuclear spins, ω_{nl} is the frequency of the relaxation of the magnetization of the nuclei as a result of their interaction with the lattice, and T is common temperature of the system prior to turning on the electric field.

2. To find the stationary solutions of the system (1) and (3), we represent the conduction-electron distribution function in the form

$$f_{\nu\sigma} = f(\varepsilon_{\nu\sigma}) - \sigma\Delta_s S(\varepsilon_{\nu\sigma})\partial f(\varepsilon_{\nu\sigma}) / \partial\varepsilon_{\nu\sigma}. \tag{4}$$

The function $S(\epsilon)$ corresponds to the saturation parameter of the electron-paramagnetic-resonance theory based on the concept of spin temperature, and describes the deviation of the effective magnetic field acting on the spins of the electron with energy ϵ from the value H. The expression for $S(\epsilon)$ is determined by the scattering mechanisms of the hot electrons in the semiconductor. The function $S(\epsilon)$ can be related with the fluctuation of the longitudinal magnetization $\delta M(\epsilon)$ of the electrons with energy ϵ:

$$S(\varepsilon)=\frac{\delta M(\varepsilon)}{\chi(\varepsilon)H},\quad \chi(\varepsilon)=-\left(\frac{g_s\mu_0}{2}\right)^2 g(\varepsilon)\frac{\partial f(\varepsilon)}{\partial\varepsilon}, \tag{5}$$

$g(\epsilon)$ is the state density of the conduction electrons. The functions $f(\epsilon)$ and $S(\epsilon)$ can be obtained from the system (1) and (3).

Multiplying (1) by $\delta(\epsilon_{\nu\sigma}-\epsilon)$ and $(1/2)\mu_0 g_s\sigma\delta\times(\epsilon_{\nu\sigma}-\epsilon)$, respectively, and summing over all the quantum numbers, we obtain in the stationary state

$$\sum_{\nu'\sigma',\nu\sigma,q}\left[1-\exp\left\{(\varepsilon_{\nu\sigma}-\varepsilon_{\nu'\sigma'})\frac{\partial}{\partial\varepsilon}\right\}\right]\sum_i W_i^q(\nu'\sigma'|\nu\sigma)\delta(\varepsilon_{\nu\sigma}-\varepsilon)=0, \tag{6}$$

$$\sum_{\nu'\sigma',\nu\sigma,q}\left[\sigma-\sigma'\exp\left\{(\varepsilon_{\nu\sigma}-\varepsilon_{\nu'\sigma'})\frac{\partial}{\partial\varepsilon}\right\}\right]\sum_i W_i^q(\nu'\sigma'|\nu\sigma)\delta(\varepsilon_{\nu\sigma}-\varepsilon)=0. \tag{7}$$

The index i numbers here the interaction mechanisms. In the case of small scattering inelasticity (which is valid for all mechanisms considered in the present paper) $\hbar\Omega_q/\bar{\epsilon}\ll 1$ and $\Delta_d, \Delta_n/\bar{\epsilon}\ll 1$, where $\bar{\epsilon}$ is the average energy of the conduction electrons; expanding the differential operators $\exp(\epsilon_{\nu\sigma}-\epsilon_{\nu'\sigma'})\partial/\partial\epsilon$ in powers of the operation $(\epsilon_{\nu\sigma}-\epsilon_{\nu'\sigma'})\partial/\partial\epsilon$, we obtain from (6) an equation for the differential energy balance of the conduction electrons

$$j(\varepsilon)E=P(\varepsilon),$$

$$j(\varepsilon)=j_{sph}(\varepsilon)+j_{si}(\varepsilon)+j_{sd}(\varepsilon)+j_{sn}(\varepsilon), \tag{8}$$

where

$$j_{sph}(\varepsilon)=e\alpha^2\sum_{\nu'\sigma',\nu\sigma,q}q_xW_{sph}^q(\nu'\sigma'|\nu\sigma)\delta(\varepsilon_{\nu\sigma}-\varepsilon)$$

$$\simeq-\frac{2\pi e^2\alpha^4}{\hbar}\sum_{\nu'\sigma',\nu\sigma,q}q_x^2|U_{sph}^q(\nu'\sigma'|\nu\sigma)|^2N_q\frac{\partial f(\varepsilon)}{\partial\varepsilon}\delta(\varepsilon_{\nu'\sigma'}-\varepsilon-\hbar\Omega_q+\hbar q_xV)\delta(\varepsilon_{\nu\sigma}-\varepsilon), \tag{9}$$

$$j_{si}(\varepsilon)=e\alpha^2\sum_{\nu'\sigma',\nu\sigma,q}q_xW_{si}^q(\nu'\sigma'|\nu\sigma)\delta(\varepsilon_{\nu\sigma}-\varepsilon)$$

$$\approx-\frac{2\pi e^2\alpha^4}{\hbar}\sum_{\nu'\sigma',\nu\sigma,q}q_x^2|U_{si}^q(\nu'\sigma'|\nu\sigma)|^2N_i\frac{\partial f(\varepsilon)}{\partial\varepsilon}\delta(\varepsilon_{\nu'\sigma'}-\varepsilon+\hbar q_xV)\delta(\varepsilon_{\nu\sigma}-\varepsilon), \tag{10}$$

$$j_{sd}(\varepsilon)=e\alpha^2\sum_{\nu'\sigma',\nu\sigma,q}q_xW_{sd}^q(\nu'\sigma'|\nu\sigma)\delta(\varepsilon_{\nu\sigma}-\varepsilon)$$

$$\approx-\frac{2\pi e^2\alpha^4}{\hbar}\sum_{\nu'\sigma',\nu\sigma,q}q_x^2|U_{sd}^q(\nu'\sigma'|\nu\sigma)|^2n_\sigma\frac{\partial f(\varepsilon)}{\partial\varepsilon}\delta(\varepsilon_{\nu'\sigma'}-\varepsilon-(\sigma'-\sigma)\Delta_d+\hbar q_xV)\delta(\varepsilon_{\nu\sigma}-\varepsilon); \tag{11}$$

$$j_{sn}(\varepsilon)=e\alpha^2\sum_{\nu'\sigma',\nu\sigma,q}q_xW_{sd}^q(\nu'\sigma'|\nu\sigma)\delta(\varepsilon_{\nu\sigma}-\varepsilon)$$

$$\approx-\frac{2\pi e^2\alpha^4}{\hbar}\sum_{\nu'\sigma',\nu\sigma,q}q_x^2|U_{sd}^q(\nu'\sigma'|\nu\sigma)|^2N_\sigma\frac{\partial f(\varepsilon)}{\partial\varepsilon}\delta(\varepsilon_{\nu'\sigma'}-\varepsilon-(\sigma'-\sigma)\Delta_n+\hbar q_xV)\delta(\varepsilon_{\nu\sigma}-\varepsilon) \tag{12}$$

are the densities of the dissipative conduction-electrons currents due to different scattering mechanisms, the total current being

$$j=\int_0^\infty d\varepsilon j(\varepsilon),\quad \alpha^2=\frac{c\hbar}{eH}.$$

$P(\epsilon)$ is the energy given up by the conduction electrons with energy ϵ per unit time in the inelastic scattering by phonons ($P_{sph}(\epsilon)$), magnetic impurities ($P_{sd}(\epsilon)$), and nuclear spins ($P_{sn}(\epsilon)$), respectively:

$$P(\varepsilon)=P_{sph}(\varepsilon)+P_{sd}(\varepsilon)+P_{sn}(\varepsilon), \tag{13}$$

$$P_{sph}(\varepsilon)=\sum_{\nu'\sigma',\nu\sigma,q}\hbar\Omega_qW_{sph}^q(\nu'\sigma'|\nu\sigma)\delta(\varepsilon_{\nu\sigma}-\varepsilon)$$

$$\approx\left[T\frac{\partial f(\varepsilon)}{\partial\varepsilon}+f(\varepsilon)\right]\frac{2\pi}{\hbar}\sum_{\nu'\sigma',\nu\sigma,q}\hbar\Omega_q|U_{sph}^q(\nu'\sigma'|\nu\sigma)|^2\cdot\delta(\varepsilon_{\nu'\sigma'}-\varepsilon-\hbar\Omega_q+\hbar q_xV)\delta(\varepsilon_{\nu\sigma}-\varepsilon); \tag{14}$$

$$P_{sd}(\varepsilon)=\sum_{\nu'\sigma',\nu\sigma,q}\Delta_d(\sigma'-\sigma)W_{sd}^q(\nu'\sigma'|\nu\sigma)\delta(\varepsilon_{\nu\sigma}-\varepsilon)$$

$$\approx\left[T\frac{\partial f(\varepsilon)}{\partial\varepsilon}+f(\varepsilon)\right]\frac{2\pi\Delta_d^2}{\hbar T}\sum_{\nu'\sigma',\nu\sigma,q}(\sigma-\sigma')^2|U_{sd}^q(\nu'\sigma'|\nu\sigma)|^2\cdot n_d\delta(\varepsilon_{\nu'\sigma'}-\varepsilon-(\sigma'-\sigma)\Delta_d+\hbar q_xV)\delta(\varepsilon_{\nu\sigma}-\varepsilon); \tag{15}$$

$$P_{sn}(\varepsilon)=\sum_{\nu'\sigma',\nu\sigma,q}\Delta_n(\sigma'-\sigma)W_{sn}^q(\nu'\sigma'|\nu\sigma)\delta(\varepsilon_{\nu\sigma}-\varepsilon)$$

$$\approx\left[T\frac{\partial f(\varepsilon)}{\partial\varepsilon}+f(\varepsilon)\right]\frac{2\pi\Delta_n^2}{\hbar T}\sum_{\nu'\sigma',\nu\sigma,q}(\sigma-\sigma')^2|U_{sn}^q(\nu'\sigma'|\nu\sigma)|^2\times N_n\delta(\varepsilon_{\nu'\sigma'}-\varepsilon-(\sigma'-\sigma)\Delta_n+\hbar q_xV)\delta(\varepsilon_{\nu\sigma}-\varepsilon). \tag{16}$$

In the expressions for the dissipative currents and the power loss of the hot electrons, we have neglected the contribution of the function $S(\epsilon)$, assuming that $S(\epsilon)\Delta_s\ll\bar{\epsilon}$. Thus, the function $f(\epsilon)$ is determined directly from the differential equation (8). To find the function $S(\epsilon)$ we transform (7) in similar fashion; in the lowest order in the inelasticity of the scattering this yields

$$\sum_{\nu'\sigma',\nu\sigma,q}(\sigma-\sigma')\sum_iW_i^q(\nu'\sigma'|\nu\sigma)\delta(\varepsilon_{\nu\sigma}-\varepsilon)=0. \tag{17}$$

Finally, from (3) we obtain the following expression for the relative increase I^* of the nuclear magnetization, due to the interaction between the nuclei and the hot electrons:

$$I^*=\sum_\sigma\sigma N_\sigma\Big/\sum_\sigma\sigma N_{0\sigma},$$

$$I^*=\operatorname{cth}\left(\frac{\Delta_n}{T}\right)\frac{\Sigma_1^-}{\Sigma_2^+}\frac{\omega_{ns}(E)}{\omega_{ns}(E)+\omega_{nl}}+\frac{\omega_{nl}}{\omega_{ns}(E)+\omega_{nl}} \tag{18}$$

where

$$\omega_{ns}(E)=2\pi\hbar^{-1}\Sigma_3^+ \tag{19}$$

is the frequency of the relaxation of the nuclear magnetization on the hot electrons; we have introduced above the notation

$$\Sigma_\alpha^\pm=\sum_{\nu'\sigma',\nu\sigma,q}\rho_\alpha|U_{sn}^q(\nu'\sigma'|\nu\sigma)|^2[f_{\nu'\sigma'}\pm f_{\nu\sigma}]\delta(\varepsilon_{\nu'\sigma'}-\varepsilon_{\nu\sigma}-(\sigma'-\sigma)\Delta_n+\hbar q_xV),\quad\alpha=1,2,3,$$

$$\rho_1=\sigma-\sigma',\quad\rho_2=\sigma(\sigma-\sigma'),\quad\rho_3=(\sigma-\sigma')^2.$$

3. The system of equations (8), (17), and (18) is valid both in quantizing magnetic field $\hbar\omega_0\gtrsim\bar{\epsilon}$ and in the classical regions $\hbar\omega_0\ll\bar{\epsilon}$. We confine ourselves here to the case $\hbar\omega_0<T$ and $\omega_0>\tilde{\omega}$, when the quantization of the orbital motion of the conduction electrons can be neglected. We then have in the foregoing formulas $(\nu,\nu')=(\mathbf{p},\mathbf{p}')$, and $\epsilon_\nu=p^2/2m$. We furthermore take into account the fact that the scattering by the nuclear spins has little influence on the kinetics of the conduction electrons. Then (8) takes the form

$$\left[\frac{\partial f(\varepsilon)}{\partial\varepsilon}+\frac{1}{T}f(\varepsilon)\right]\left[1+\frac{\tilde{\omega}_{sd}(\varepsilon)}{\tilde{\omega}_{sph}(\varepsilon)}\frac{2\Delta_d^2}{ms^2\varepsilon}\right]=-\frac{\partial f(\varepsilon)}{\partial\varepsilon}\frac{\beta^2}{3}\left[1+\frac{\tilde{\omega}_{si}(\varepsilon)+\tilde{\omega}_{sd}(\varepsilon)}{\tilde{\omega}_{sph}(\varepsilon)}\right] \tag{20}$$

where $\tilde{\omega}\ldots(\epsilon)$ are the momentum-relaxation frequencies of electrons with energy ϵ, and $\beta = cE/sH$ is the Mach number of the electron beam. A similar equation was obtained by Kazarinov and Skobov [3]. Writing down the solution of this equation in the form

$$f(\varepsilon) = C\exp\left\{-\int_0^\varepsilon \frac{d\varepsilon'}{T(\varepsilon')}\right\},$$

$$C = n_0\left[\int_0^\infty d\varepsilon g(\varepsilon)\exp\left\{-\int_0^\varepsilon \frac{d\varepsilon'}{T(\varepsilon')}\right\}\right]^{-1} \tag{21}$$

(n_0 is the concentration of the conduction electrons), we get

$$T(\varepsilon) = T\left[1 + \frac{\beta^2}{3}\frac{\tilde{\omega}_{sph}(\varepsilon) + \tilde{\omega}_{si}(\varepsilon) + \tilde{\omega}_{sd}(\varepsilon)}{\tilde{\omega}_{sph}(\varepsilon) + \dfrac{2\Delta_d^2}{ms^2\varepsilon}\tilde{\omega}_{sd}(\varepsilon)}\right]. \tag{22}$$

To calculate the dependence of the frequencies $\tilde{\omega}\ldots(\epsilon)$ on the energies we take into account in formulas (9)--(12) only processes of scattering with conservation of the spin orientation. Then

$$|U^{q}_{\ldots}(\mathbf{p} + \hbar\mathbf{q}, +|\mathbf{p}, +)|^2 \sim q^{t'}, \tag{23}$$

where $t' = 1$ and $\tilde{\omega}_{sph}(\epsilon) = \tilde{\omega}_{sph}(\epsilon/T)^{1/2}$ for acoustic phonons, $t' = 0$ and $\tilde{\omega}_{si,sd}(\epsilon) = \tilde{\omega}_{si,sd}(\epsilon/T)^{1/2}$, for neutral and magnetic impurities and $t' = -4$ and $\tilde{\omega}_{si}(\epsilon) = \tilde{\omega}_{si}(\epsilon/T)^{-3/2}$ for ionized impurities. Here $\tilde{\omega}\ldots$ is the momentum relaxation of frequency of electrons with energy $\epsilon = T$.

From (17) we can now readily calculate the functions $S(\epsilon)$. In the case of scattering of conduction electrons by phonons and nonmagnetic impurities, the matrix elements of the interaction with spin flip, which enter in this equation, are the result of the spin-orbit coupling. The dependence of these matrix elements on the momentum is determined by the crystal symmetry. We shall assume that the extremum of the carrier energy is situated at the center or on the border of the Brillouin zone; then [4]

$$|U_{\ldots}^{q}(\mathbf{p} + \hbar\mathbf{q}, +|\mathbf{p}, -)|^2 \sim |2\mathbf{p} + \hbar\mathbf{q} + 2m\mathbf{V}|^2 q^t, \tag{24}$$

where $t = 3$ for acoustic phonons in crystals with inversion center and $t = 1$ if there is no inversion center; $t = 2$ for nonmagnetic impurities.

For scattering by magnetic impurities and nuclear spins, we shall assume, as usual, that $|U^{q}_{sd,sn}(\mathbf{p} + \hbar\mathbf{q}, +|\mathbf{p}, -)|^2 = \text{const}$ (contact interaction). We then obtain from (17)

$$S(\varepsilon) = \sum_i S_i(\varepsilon)T_{1i}^{-1}(\varepsilon)\Big/\sum_i T_{1i}^{-1}(\varepsilon), \tag{25}$$

where $S_i(\epsilon)$ and $T_{1i}(\epsilon)$ are the values of the function $S(\epsilon)$ and the time of longitudinal spin-lattice relaxation of the conduction electrons with energy ϵ for the i-th scattering mechanism. We present below expressions obtained from (17) for $S_i(\epsilon)$ and $T_{1i}(\epsilon)$,

a) acoustic phonons:

$$S_{ph}(\varepsilon) = \gamma^2\beta^2\frac{t+3}{3} - \gamma^3(t+1)(t+3)\left[\frac{T(\varepsilon)}{T} - 1\right]$$
$$\times\left[1 + \frac{2\beta^2}{3} - \frac{4\beta^2\varepsilon}{3(t+2)T(\varepsilon)}\right],$$
$$T_{1ph}^{-1}(\varepsilon) = T_{1ph}^{-1}(\varepsilon/T)^{(t+2)/2}; \tag{26}$$

b) nonmagnetic impurities:

$$S_i(\varepsilon) = \gamma^2\beta^2(t+4)/3, \quad T_{1i}^{-1}(\varepsilon) = T_{1i}^{-1}(\varepsilon/T)^{(t+3)/2}; \tag{27}$$

c) magnetic impurities:

$$S_d(\varepsilon) = -\frac{\Delta_d}{\Delta_s}\left[\frac{T(\varepsilon)}{T} - 1\right], \quad T_{1d}^{-1}(\varepsilon) = T_{1d}^{-1}\left(\frac{\varepsilon}{T}\right)^{1/2}. \tag{28}$$

Here $T_1\ldots$ are the values of the longitudinal spin-lattice relaxation times $T_1\ldots(\epsilon)$ for $\epsilon = T$; $\gamma = (ms^2/2\epsilon)^{1/2}$.

4. By calculating the integrals encountered in formula (18), with the exception of the integrals with respect to the energy, we obtain an explicit expression for the nuclear polarization I^* in terms of the obtained functions $f(\epsilon)$ and $S(\epsilon)$:

$$I^* = \frac{\omega_{ns}(E)}{\omega_{ns}(E) + \omega_{nl}}\left(\int_0^\infty d\varepsilon f(\varepsilon)\varepsilon\right)^{-1}\int_0^\infty d\varepsilon f(\varepsilon)\varepsilon\cdot$$
$$\times\left\{1 - \frac{\Delta_s}{\Delta_n}\left[S(\varepsilon) - \frac{2}{3}\gamma^2\beta^2\right]\right\}\frac{T}{T(\varepsilon)} + \frac{\omega_{nl}}{\omega_{ns}(E) + \omega_{nl}}; \tag{29}$$

$$\omega_{ns}(E) = \omega_{ns}(0)\left(\frac{\pi}{4T}\right)^{1/2}\int_0^\infty f(\varepsilon)\varepsilon d\varepsilon\Big/\int_0^\infty f(\varepsilon)\varepsilon^{1/2}d\varepsilon. \tag{30}$$

Formulas (29) and (30) determine the change of the nuclear magnetization and of the frequency of the hyperfine relaxation of the nuclear spins in the presence of a strong current of hot electrons.

Let us consider the case when the energy of the hot electrons is scattered by acoustic phonons and the momentum is scattered by phonons and neutral impurities. Then

$$\frac{T(\varepsilon)}{T} \equiv T^* = 1 + \frac{\beta^2}{3}\frac{\tilde{\omega}_{sph} + \tilde{\omega}_{si}}{\omega_{sph}}. \tag{31}$$

A. The magnetization of the electron current relaxes on nonmagnetic impurities. Then

$$I^* = \frac{1}{T^*}\left[1 - \frac{\Delta_s}{\Delta_n}\frac{\bar{\gamma}^2\beta^2}{T^*}\frac{t+2}{3}\right]\frac{\omega_{ns}(0)T^{*1/2}}{\omega_{ns}(0)T^{*1/2} + \omega_{nl}}$$
$$+ \frac{\omega_{nl}}{\omega_{ns}(0)T^{*1/2} + \omega_{nl}}, \quad \bar{\gamma} = (ms^2/2T)^{1/2}. \tag{32}$$

when the magnetic moments of the nuclei and of the electrons are positive, the relative nuclear polarization first decreases like β^2, after which it reverses sign and reaches a maximum absolute value

$$I^*_{max} \sim \frac{\Delta_s}{\Delta_n}\bar{\gamma}^2\frac{\tilde{\omega}_{sph}}{\tilde{\omega}_{si} + \tilde{\omega}_{sph}}\frac{\omega_{ns}(0)}{\omega_{ns}(0) + \omega_{nl}} \tag{33}$$

at electron-current Mach numbers of the order of

$$\beta^2_{max} \sim \frac{\tilde{\omega}_{sph}}{\tilde{\omega}_{si} + \tilde{\omega}_{sph}} \tag{34}$$

and then begins to decrease in absolute magnitude like β^{-2} at $\omega_{ns} > \omega_{nl}$ and like β_{-1} at $\omega_{ns} < \omega_{nl}$ (see the figure, case a).

B. The magnetization of the electron current relaxes on acoustic phonons. Then

$$I^* = \frac{1}{T^*}\left\{1 - \frac{\Delta_s}{\Delta_n}\left[\frac{\bar{\gamma}^2\beta^2}{T^*}\cdot\frac{t-1}{3} - \bar{\gamma}^3(t+1)(t+3)\pi^{1/2}\frac{T^* - 1}{T^{*3/2}}\right.\right.$$
$$\left.\left.\times\left(1 + \frac{2\beta^2}{3}\frac{t+1}{t+2}\right)\right]\right\}\frac{\omega_{ns}(0)T^{*1/2}}{\omega_{ns}(0)T^{*1/2} + \omega_{nl}} + \frac{\omega_{nl}}{\omega_{ns}(0)T^{*1/2} + \omega_{nl}}. \tag{35}$$

If $\gamma(\tilde{\omega}_{si} + \tilde{\omega}_{sph})/\tilde{\omega}_{sph} < 1$, the main contribution to the polarization is made by the first term in the square bracket of (35), and then I* behaves, at Mach numbers that are not too large, just as in the case of impurity spin scattering. At low temperatures, the electron momentum is scattered principally by impurities, so that the inverse inequality $\overline{\gamma}(\tilde{\omega}_{si} + \tilde{\omega}_{sph})/\tilde{\omega}_{sph} > 1$ may be satisfied. Then the main contribution to the polarization of the nuclei is made by the second term in the square bracket of (35). I* first increases like β^2, and then reaches a maximum at β_{max} (34), with

$$I^*_{max} \sim \frac{\Delta_s}{\Delta_n}\overline{\gamma}^3 \frac{\omega_{ns}(0)}{\omega_{ns}(0)+\omega_{nl}}. \tag{36}$$

With further increase of the Mach number, I* decreases like β_{-1} if $\omega_{ns} > \omega_{nl}$, and tends to a constant limit

$$\sim \frac{\Delta_s}{\Delta_n}\overline{\gamma}^3 \frac{\tilde{\omega}_{sph}}{\tilde{\omega}_{si}} \frac{\omega_{ns}(0)}{\omega_{nl}},$$

if $\omega_{ns} < \omega_{nl}$ (case b in the figure).

C. The magnetization of the electron beam relaxes on magnetic impurities. In this case

$$I^* = \frac{1}{T^*}\left[1+\frac{\Delta_d}{\Delta_n}(T^*-1)\right]\frac{\omega_{ns}(0)T^{*1/2}}{\omega_{ns}(0)T^{*1/2}+\omega_{nl}} + \frac{\omega_{nl}}{\omega_{ns}(0)T^{*1/2}+\omega_{nl}}. \tag{37}$$

The relative polarization of the nuclei increase for small β like β^2; when $\beta > \beta_{max}$ as given by formula (34), I* tends to a constant value $\sim\Delta_d/\Delta_n$ if $\omega_{ns} > \omega_{nl}$, and increases in proportion to β if $\omega_{ns} < \omega_{nl}$ (case c in the figure).

Finally, let us consider the case when the energy of the hot electrons relaxes on acoustic phonons, and the momentum relaxes on ionized impurities.

$$T(\varepsilon) = T\left[1+\frac{\beta^2}{3}\frac{\tilde{\omega}_{si}}{\tilde{\omega}_{sph}}\left(\frac{\varepsilon}{T}\right)^{-2}\right]. \tag{38}$$

When $\frac{1}{3}\beta^2\tilde{\omega}_{si}/\tilde{\omega}_{sph} < 1$, the relative polarization I* of the nuclei varies like β^2 for all the electron-magnetization relaxation mechanisms. When $\frac{1}{3}\beta^2\tilde{\omega}_{si}/\tilde{\omega}_{sph} > 1$ we have

$$\omega_{ns}(E) = \omega_{ns}(0)\frac{\Gamma(2/3)}{2}\left(\frac{\tilde{\omega}_{si}}{\tilde{\omega}_{sph}}\right)^{1/6}\beta^{1/3}, \tag{39}$$

and the absolute value of I* is determined by the following expressions (we have written out the formulas for the case $\omega_{ns} > \omega_{nl}$ only, when hyperfine interaction is the main mechanism of nuclear-spin relaxation).

A′. The electron magnetization relaxes on nonmagnetic impurities. Then

$$I^* \approx -\frac{\Delta_s}{\Delta_n}\overline{\gamma}^2\left(\frac{\tilde{\omega}_{sph}}{\tilde{\omega}_{si}}\right)^{2/3}\beta^{2/3}. \tag{40}$$

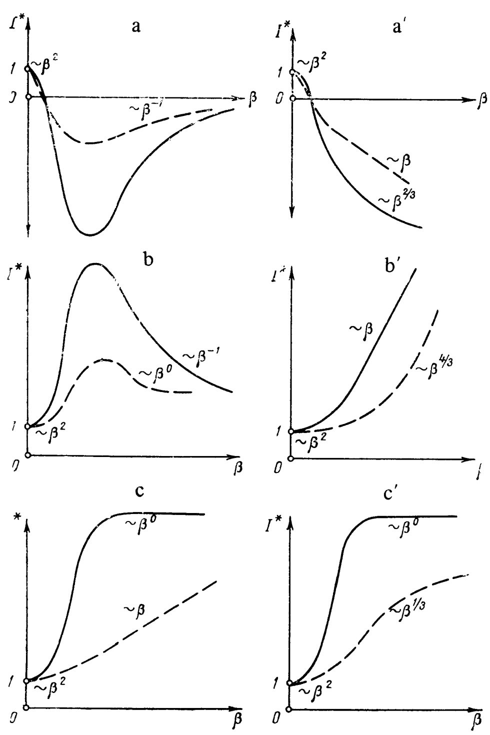
B′. The electron magnetization relaxes on acoustic phonons and $\gamma\omega_{si}/\tilde{\omega}_{sph} > 1$. Then

$$I^* \approx \frac{\Delta_s}{\Delta_n}\overline{\gamma}^3\left(\frac{\tilde{\omega}_{sph}}{\tilde{\omega}_{si}}\right)^{1/2}\beta. \tag{41}$$

C′. The electron magnetization relaxes on magnetic impurities. Then

$$I^* \approx \Delta_d/\Delta_n. \tag{42}$$

When $\omega_{ns}(E) < \omega_{nl}$, the expressions (40)–(42) must be multiplied by $\omega_{ns}(E)/\omega_{nl}$, where $\omega_{ns}(E)$ is given by (39). The dependence of the relative polarization of the nuclei on the Mach number for the case under consideration is shown in the figure, cases a) b), and c).



Relative increase of nuclear polarization as a function of the Mach number of the current of hot electrons. a, a′ – case of relaxation of magnetization of the hot electrons on nonmagnetic impurities. b, b′ – on acoustic phonons, c, c′ – on magnetic impurities; a, b, c – the electron energy relaxes on acoustic phonons and the momentum on phonons and neutral impurities; a′, b′, c′ – the same for acoustic phonons and ionized impurities. The solid curves correspond to predominance of the hyperfine interaction in the relaxation of the nuclear spins, and the dashed lines correspond to predominance of the spin-lattice interaction.

Let us estimate the order of magnitude of the nuclear polarization for the case of n-InSb at helium temperatures. Under these conditions, the momentum of the electron current relaxes on the impurities; assuming that the relaxation of the energy is due to scattering by acoustic phonons, we have $\omega_{si}/\tilde{\omega}_{sph} \sim 10-10^2$. The parameter γ equals under these conditions $\sim 10^{-1}$; for the nuclei In^{115}, Sb^{121}, and Sb^{123} we have $\Delta_s/\Delta_n \sim 10^5$. Thus, when $\omega_{ns} > \omega_{nl}$ the maximum polarization of the nuclei is $I^*_{max} \sim 10-10^2$ in scattering of the electron magnetization by nonmagnetic impurities, and $I^*_{max} \sim 10^2$ in the scattering of the spin by acoustic phonons. On the other hand, if the fluctuation of the spin of the electron current relaxes on magnetic impurities, then $I^*_{max} \sim 10^4$ (it is assumed that $g_d \sim 2$). According to (34), these values of the nuclear polarization are obtained at relatively small values of the Mach number of the current of hot electrons, $\beta \sim 10^{-1}$. In magnetic fields $H \sim 10^5$ Oe, this corresponds to an electric field intensity $E \sim 10$ V/cm.

The greatest nuclear polarization corresponds for the mechanisms under consideration to the case of relaxation of electron spins on magnetic impurities. In our calculation, however, we did not take into account the finite relaxation time of the impurity fluctuations due to impurity interaction with the lattice. Allowance for this fact leads to a decrease in the nuclear polarization; the stronger the deviation of the impurity magnetization from the thermodynamic-equilibrium value, the greater this decrease[2]. However, if the conduction-electron concentration is much smaller than the concentration of the magnetic impurities, the latter remain practically in equilibrium. In such a case our results are valid.

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