IDENTIFICATION OF THE MECHANISM OF DIRECT THREE-PARTICLE INTERACTIONS

E. I. DUBOVOĬ and I. S. SHAPIRO

Submitted April 25, 1967

Zh. Eksp. Teor. Fiz. 53, 1395–1399 (October, 1967)

It is shown that the amplitudes for direct nuclear reactions leading to the formation of three particles and corresponding to triangular knock-out graphs with rearrangement of the nuclei at the vertices, can be expressed in terms of a universal function of two dimensionless variables for arbitrary masses of the virtual particles. The behavior of the differential cross section near the singularities is calculated and contour maps of the corresponding surfaces are drawn. Comparison of the data obtained with experiment permits one to establish the mechanism of the reaction.

1. INTRODUCTION

IN an earlier paper,[1] the amplitude for the reaction

$$A + x \to B + y + x \tag{1}$$

has been investigated as a function of the square of the momentum transfer $q^2 = |\mathbf{p}_x - \mathbf{p}'_x|^2$ ($\mathbf{p}_x$, $\mathbf{p}'_x$ are the momenta of the particle x before and after the reaction) and the energy ω of the particles B and y in the system of their center of mass. The behavior of the reaction amplitude was considered near the singularities of the triangular graph corresponding to the account of elastic and inelastic scattering of the particles in the final state. It was shown that the modulus of the amplitude and its real and imaginary parts have extremal geometric loci given by the projection of the curves of the complex singularities of the amplitude on the (q^2, ω) plane.

In the present paper we consider the amplitude for reactions of the more general type

$$A + x \to B + y + z. \tag{2}$$

The reaction (2) with $z \neq x$ cannot proceed without a "rearrangement" of the nuclei in the virtual processes corresponding to the vertices of the graphs. We must therefore consider the triangular graph (Fig. 1) with arbitrary masses of the virtual particles. The graph of Fig. 1 includes the process studied in[1] as a special case. The amplitude corresponding to this graph has singularities in the variables t and ω, where

$$\begin{aligned} t &= -(\mathbf{p}_x - \mathbf{p}_z)^2 + 2(m_x - m_z)(E_x - E_z), \\ \omega &= E_y + E_B - (\mathbf{p}_y + \mathbf{p}_B)^2 / 2(m_y + m_B) \end{aligned} \tag{3}$$

(m, p, and E are the mass, the momentum, and the kinetic energy of the corresponding particles).

The location of the regions of nonregular behavior of the reaction amplitude and the character of its behavior are determined by the masses of the virtual particles. Therefore the investigation of the behavior of the differential cross section near the singularities of the amplitude enables one, in principle, to identify the mechanism of the reaction.

2. REACTION AMPLITUDE

The amplitude M_Δ corresponding to the graph of Fig. 1 is a function of the two variables t and ω (for constant vertices of the graph). In analogy to[1], it is convenient for a general discussion to go over to dimensionless variables (everywhere in the following we take $\hbar = c = 1$) ξ and λ:

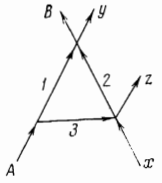

FIG. 1. Triangular graph corresponding to the reaction $x + A \to B + y + z$ which depends on t and ω

$$\begin{aligned} \xi &= \frac{m_2}{m_3}\frac{m_A}{m_B + m_y}\frac{\omega - Q}{\varepsilon}, \\ \lambda &= \frac{m_1}{m_3}\frac{2(m_x - m_z)(\omega - Q_0) - t}{2(m_B + m_y)\varepsilon}, \end{aligned} \tag{4}$$

where m_1, m_2, and m_3 are the masses of the virtual particles 1, 2, and 3,

$$\begin{aligned} \varepsilon &= m_1 + m_3 - m_A, \\ Q &= m_1 + m_2 - m_B - m_y, \quad Q_0 = m_A + m_x - m_B - m_y - m_z. \end{aligned} \tag{5}$$

The calculation of the Feynman integral corresponding to the graph of Fig. 1 (cf.[1,2]) yields the following expression for the amplitude:

$$M_\Delta = C f_\Delta(\xi, \lambda). \tag{6}$$

Here (as in[1])

$$\begin{aligned} C &= -i\frac{m_2 m_3}{2\pi\varkappa}\frac{m_1^2}{m_A(m_B + m_y)} M_A M_{xz} M_{2y}, \\ \varkappa^2 &= 2m_{13}\varepsilon, \quad m_{13} = m_1 m_3 / (m_1 + m_3) \end{aligned} \tag{7}$$

is a rectangular matrix in the spinor indices of the initial and final particles which is independent of the kinematic variables and is expressed through the amplitudes for the virtual processes

$$M_A(A \to 1 + 3), \quad M_{xz}(x + 3 \to 2 + z), \quad M_{2y}(1 + 2 \to B + y).$$

The amplitudes for the virtual processes in (6) are taken at values of the kinematic variables corresponding to a singular point of the graph (where the intermediate particles are on the mass shell). Formula (6) is correct only if near this point the quantities M_A, M_{xy}, and M_{2y} vary slowly compared to the function $f_\Delta(\xi, \lambda)$.

The amplitude $f_\Delta(\xi, \lambda)$ as a function of the complex variables ξ and λ has the form

$$f_\Delta(\xi, \lambda) = \frac{i}{\sqrt{-\lambda}} \ln \frac{1 + \sqrt{-\xi} + \sqrt{-\lambda}}{1 + \sqrt{-\xi} - \sqrt{-\lambda}}, \tag{8}$$

where $0 \leq \arctan X < \pi$. For $\lambda > 0$ expression (8) is identical with formulas (2.16) and (2.17) of [1].

In the general case considered by us, the variable λ can also take on negative values in the physical region (for $m_x < m_z$). For $\lambda < 0$ we obtain from (8) (for $\xi > 0$ we replace $\sqrt{-\xi}$ by $-i\sqrt{\xi}$, cf. [2]):

$$f_\Delta(\xi,\lambda) = \frac{-1}{\sqrt{-\lambda}}\left[\operatorname{arctg}\frac{2\sqrt{-\xi\lambda}}{\xi+1+\lambda} + \frac{i}{2}\ln\frac{\xi+(1-\sqrt{-\lambda})^2}{\xi+(1+\sqrt{-\lambda})^2}\right], \quad (9) \qquad (9)$$

$$0 \leqslant \operatorname{arctg} X < \pi, \quad \xi \geqslant 0, \quad \lambda < 0;$$

$$f_\Delta(\xi,\lambda) = \frac{i}{\sqrt{-\lambda}}\ln\frac{1+\sqrt{-\xi}+\sqrt{-\lambda}}{1+\sqrt{-\xi}-\sqrt{-\lambda}}, \quad \xi \leqslant 0, \quad \lambda < 0. \qquad (10)$$

It is seen from (9) and (10) that the reaction amplitude has, as a function of λ, a "moving" singularity λ_Δ, whose position is determined by the equation

$$\lambda_\Delta = \xi - 1 - 2\sqrt{-\xi}. \qquad (11)$$

For $\xi > 0$ this singularity is complex. Since in the general case under consideration λ may be negative in the physical region, one may, for $m_x < m_z$, get close to the singular point $\xi = 0$, $\lambda = -1$ (for small t and ω), in the neighborhood of which the amplitude varies rapidly.

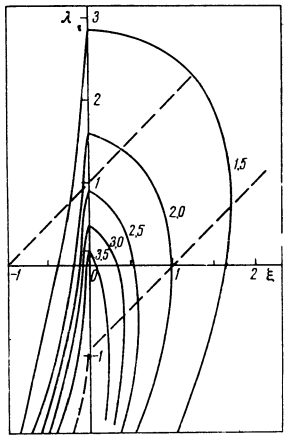

FIG. 2. Contour map of the surface $|f_\Delta(\xi,\lambda)|^2$ corresponding to the graph of Fig. 1.

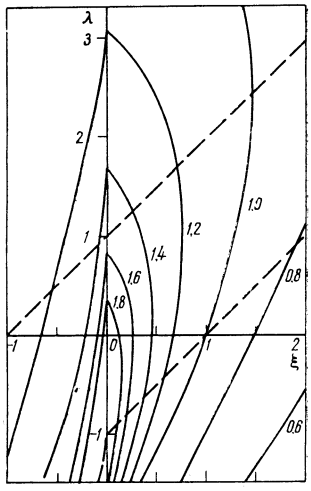

FIG. 3. Contour map of $\mathrm{Im}\, f_\Delta(\xi,\lambda)$

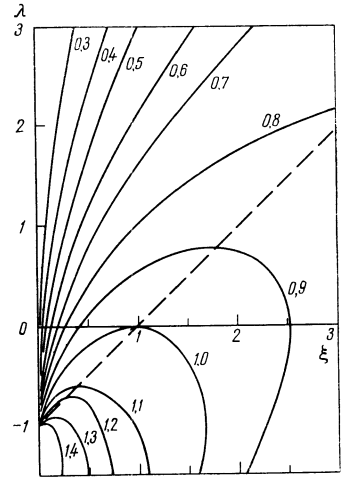

FIG. 4. Contour map of $\mathrm{Re}\, f_\Delta(\xi,\lambda)$

As a function of ξ, the reaction amplitude has a normal singularity along the straight line $\xi = 0$ and a "moving" singularity

$$\xi_\Delta = \lambda - 1 + 2\sqrt{-\lambda}. \qquad (12)$$

The differential cross section for the reaction, $\partial^2\sigma/\partial\xi\partial\lambda$ is determined [cf. [1], formulas (2.18) to (2.20), (2.23), and (2.24)] by the functions $|f_\Delta(\xi,\lambda)|^2$, $\mathrm{Re}\, f_\Delta$, and $\mathrm{Im}\, f_\Delta$. The contour maps (level lines) of the surfaces corresponding to these functions are shown in Figs. 2 to 4 for the region $\lambda \gtrless 0$ and $\xi \gtrless 0$. The dashed straight lines in these figures indicate the projections of the complex singularities (11) and (12). The dashed line starting at the point $\xi = 0$, $\lambda = -1$ is the curve of the real singularity (11).

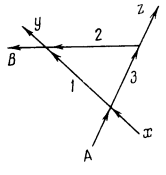

FIG. 5. Triangular graph corresponding to the reaction $x + A \to B + y + z$ which depends on ω and ω_0

3. CONCLUDING REMARKS

The triangular graph considered by us is the simplest of the graphs leading to singularities of the reaction amplitude with respect to the variables t and ω.

The main result of the present paper is that the amplitude corresponding to the graph of Fig. 1 is expressed through a universal function $f_\Delta(\xi, \lambda)$ of the dimensionless variables ξ and λ. The function $f_\Delta(\xi, \lambda)$ is independent of the masses of the virtual particles, so that the results obtained above apply to all possible virtual processes involved in the graph of Fig. 1. Thus the calculation and investigation of this class of graphs may be considered as accomplished exhaustively.

We note that this situation is not at all the same for all triangular graphs. For example, the triangular graph leading to singularities in ω and ω_0 (ω_0 is the energy of the colliding particles in the system of their center of mass) (cf. Fig. 5) does not permit such a reduction to a universal function of two variables. It reduces to a function of two kinematic variables and of one variable which depends on the masses of the virtual

particles, so that it is impossible to investigate it in detail and to give a graphic description for an arbitrary reaction.

Although the behavior of the amplitude for the reactions considered above is determined by a universal function of the two dimensionless variables ξ and λ, the behavior of the differential cross section as a function of t and ω depends essentially on the masses of the virtual particles [this follows from (4)]. Therefore a comparison of the results obtained above with the experimental data can be used for establishing the mechanism of the reaction.

The authors express their gratitude to I. Baranova for carrying out the numerical calculations.

[1] E. I. Dubovoĭ and I. S. Shapiro, Zh. Eksp. Teor. Fiz. 51, 1251 (1966) [Soviet Phys.-JETP 24, 839 (1967)].

[2] L. D. Blokhintsev, É. I. Dolinskiĭ, and V. S. Popov, Zh. Eksp. Teor. Fiz. 43, 2290 (1962) [Soviet Phys.-JETP 16, 1618 (1963)].

Translated by R. Lipperheide
161