MOTION OF CHARGED PARTICLES IN AN AXIALLY-ASYMMETRIC MIRROR TRAP

A. N. DUBININA and Yu. N. YUDIN

Submitted February 21, 1967

Zh. Eksp. Teor. Fiz. 53, 1206–1212 (October, 1967)

The confinement time of electrons in an axially-symmetric magnetic mirror trap is investigated experimentally. The value of the critical non-adiabaticity parameter ρ_L/R is determined and found to be close to that of the nonadiabaticity parameter for an electron moving in an axially-symmetric trap. The result obtained apparently permits one to affirm that the degree of nonadiabaticity in a trap is characterized by the parameter ρ_L/R and not by the deviation from symmetry of the magnetic field.

THE question of the motion of individual charged particles in a static spatially-inhomogeneous magnetic field was considered in a number of papers [1-6].

Bogolyubov and Mitropol'skiĭ [1] used the so-called asymptotic method. If, for example, we speak of the motion of the charged particle in a magnetic trap, then the asymptotic theory states that when the parameter ρ_L/R is sufficiently small (ρ_L—Larmor radius, R—radius of curvature of the magnetic-field force line) the particle can execute an arbitrary number of oscillations between magnetic mirrors, but the theory does not give the quantitative connection between the value of the small parameter ρ_L/R and the number of oscillations, merely stipulating that $\rho_L/R \to 0$.

Rodionov [2] has shown that: a) in certain magnetic-field configurations, slow cummulative changes of the magnetic moment (10^4—10^5 oscillations) actually occur and can lead to the escape of the electron from the trap within a relatively short time that depends on the configuration of the magnetic field; b) in other magnetic-field configurations which are easy to realize in practice, the lifetime of the electrons is very large, $\sim 10^7$ oscillations. However, the physical picture of the motion of the charged particle in a magnetic trap during very large time intervals remains very unclear.

Among the first papers in which it was considered were those of Chirikov [3,4]. The picture of the motion, in its most general outlines, is as follows: There exists a certain critical value of the parameter ρ_L/R such that at larger values of ρ_L/R the motion becomes stochastic, i.e., so to speak, random. At smaller values of ρ_L/R the motion is stable, i.e., it differs little from motion with a constant magnetic moment.

Unfortunately, however, all the available mathematical papers suffer from that essential shortcoming that they do not offer even methods for estimating the critical value of ρ_L/R, in other words, they give only the sufficient instability criteria, but in no case the necessary criteria.

As to the stable region (small ρ_L/R), it is likewise completely unclear here how long the stability is maintained during the course of motion. The hope for a rigorous answer to this question was raised by the papers of Kolmogorov [5] and Arnol'd [6], in which conditions were obtained for absolute stability, i.e., stability when the small parameter (ρ_L/R in our case) is finite. A shortcoming of the theory is the absence of an accurate estimate of the critical parameter. The motion of the particle in an axially asymmetrical trap does not lend itself to investigation either by the methods of Arnol'd [6] or by any other method known to us. We therefore consider it of interest to investigate experimentally one such system—a charged particle in a magnetic trap.

During the course of the experiments, we investigated the dependence of the lifetime of the electrons captured in the space between the mirrors on the magnetic field. The purpose of the investigation was to find the numerical value of the critical parameter $(\rho_L/R)_1$, i.e., a value of ρ_L/R such that when $\rho_L/R < (\rho_L/R)_1$ the lifetime of the electron is determined essentially by the scattering from the residual gas which, according to Fermi (7), is equal to

$$\tau = \frac{\overline{\theta}^2 W^{3/2}}{4p \ln(2,7\,W)} \cdot 10^{-8}\ (\text{sec}), \tag{1}$$

where W is the energy of the charged particles in kV, p the pressure of the residual gas in mm Hg, and $\overline{\theta}^2$ is given in the case of a trap with magnetic mirrors, according to [10], by

$$\overline{\theta}^2 = 2\left(\arccos\frac{1}{\sqrt{\gamma}} + \frac{\pi}{2} - \alpha_0\right)^2,$$

where γ is the mirror ratio and α_0 the half-width of the loss cone.

When $\rho_L/R > (\rho_L/R)_1$, the electron lifetime is essentially determined, according to [3-6], by scattering not from the residual gas but by the inhomogeneities of the magnetic field, for now the change occurring in the magnetic field when the electron covers a path equal to the Larmor radius is so large that $\mu = w_\perp/H$ ceases to be an invariant of the motion. As shown earlier [8], at large values of ρ_L/R the electron lifetime τ varies with decreasing magnetic field like $\tau = A\exp(BR/\rho_L)$, where $A \approx 10^{-8}$ sec and B = 0.8—0.9. In [8] they considered the case of electron motion in an axially-asymmetrical magnetic trap. As indicated above, the results of Chirikov [3] and Arnol'd [6] (which pertain to axially asymmetrical traps) make it possible to trace qualitatively the motion of the particles over long periods of time. On the other hand, the motion of particles in an axially-asymmetrical trap does not lend itself to investigation by methods known to us.

The experiments described in [8] have made it possible to calculate the critical adiabaticity parameter in the case of an axially-symmetrical trap. It was found that the value of the parameter is $(\rho_L/R)_1 \approx (4.0 \pm 0.5) \times 10^{-2}$.

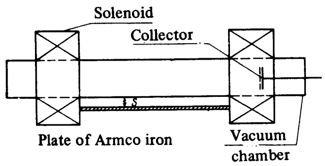

FIG. 1.

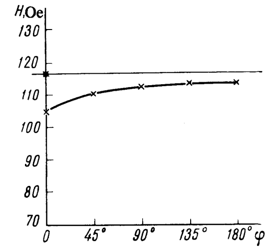

FIG. 2.

The purpose of the present investigation was to find the critical adiabaticity parameter in the case of electron motion in an axially asymmetrical magnetic trap, and to compare its value with that corresponding to the case of an axially-symmetrical trap. The apparatus used for the experiments constituted a magnetic trap of mirror configuration. The maximum field at the center of each mirror reached 1500 Oe and a mirror ratio ranging from 2.66 to 4.44. A detailed description of the setup is given in[9]. The injector was an electron gun located outside the working volume behind one of the mirrors, and a special electrode in the form of a hollow cylinder was placed in the center of the gun. The electrons were injected by rapidly changing the electric field applied to the electrode. The electrons captured in the space between the mirrors and then leaving the working volume as a result of entering the loss cone were observed by measuring the current in the collector located behind the magnetic mirror (see[9]).

Two insulated grids, to which different voltages were applied to suppress the secondary electron emission from the collector and to separate the required component of the particle current from the trap, were placed ahead of the collector. When suitable potentials were applied to the grid, we observed the current of the electrons that were injected by the gun, captured in the space between the mirrors, entered the loss cone, and then left the working volume through the mirror and reached the collector. The lifetime of the electrons was defined not as the time necessary for the collector current pulse to decrease by a factor e, but as a value three times larger.

The axial asymmetry was produced by placing a plate of "Armco" iron at a distance S from the vacuum chamber (Fig. 1).

Figure 2 shows the experimentally observed dependence of the magnetic field on the angle φ in the median plane.

The presence of a magnetic-field gradient in the direction of φ makes it possible to employ the drift theory of electron motion in a magnetic trap with simultaneous satisfaction of the following three conditions that impose limitations on the spatial variation of the magnetic field

$$\left(\frac{\rho_L}{R}\right)_{\parallel}=\frac{\rho_L}{H}\frac{V_{\parallel}}{V_{\perp}}\frac{\partial H}{\partial z}\ll 1, \tag{2a}$$

$$\left(\frac{\rho_L}{R}\right)_{\perp}=\frac{\rho_L}{H}\frac{\partial H}{\partial r}\ll 1, \tag{2b}$$

$$\left(\frac{\rho_L}{R}\right)_{\varphi}=\frac{\rho_L}{H}\frac{1}{r_0}\frac{\partial H}{\partial \varphi}\ll 1, \tag{2c}$$

where ρ_L is the Larmor radius of the particle, $\partial H/\partial z$, $\partial H/\partial r$, and $\partial H/\partial\varphi$ are the gradients of the magnetic field in the directions of the corresponding axes, z is the symmetry axis of the system, r is perpendicular to z, φ is the angle in the r plane at $z = 0$, and r_0 is the distance of the motion center of the particle from the z axis.

When the electron moves in the trap, the quantities ρ_L, R and ρ_L/R vary with the coordinate z (the z axis coincides with the symmetry axis of the system). The decisive influence on the motion of the electron is exerted by the largest of the three quantities $(\rho_L/R)_{\parallel}$, $(\rho_L/R)_{\perp}$, and $(\rho_L/R)_{\varphi}$. Recognizing that

$$\rho_L=\frac{3.4\sqrt{W(\text{eV})}\sin\theta_0}{H(\text{Oe})}\ (\text{cm}),$$

we rewrite expression (2c) in the form

$$\left(\frac{\rho_L}{R}\right)_{\varphi}=\frac{3.4\sqrt{W}\sin\theta_0}{H^2}\frac{1}{r_0}\frac{\partial H}{\partial\varphi}. \tag{3}$$

From symmetry considerations and in accordance with Fig. 2 we see that the Larmor radius and the field gradient $\partial H/\partial\varphi$ reach their maximum values in the median plane ($z = 0$) as $\varphi \to 0$. Inasmuch as the decisive role in the motion of the electron is the largest value of the parameter ρ_L/R, we have calculated the values of $(\rho_L/R)_{\varphi}$ as $z \to 0$ and $\varphi \to 0$. The maximum value of $(\rho_L/R)_{\varphi}$ is denoted $(\rho_L/R)_{\varphi_{max}}$:

$$\left(\frac{\rho_L}{R}\right)_{\varphi_{max}}=3.4\sqrt{W(\text{eV})}\,H_{01}^{-2}\sin\theta_0\frac{1}{r_0}\left.\frac{\partial H}{\partial\varphi}\right|_{\varphi\to 0;\ z\to 0}, \tag{4}$$

where θ_0 is the angle between the electron velocity vector and the magnetic field direction at $z = 0$, r_0 is the distance from the electron to the z axis, and H_{01} is the magnetic field at $z = 0$ and $r = r_0$.

To determine the critical adiabaticity parameter $(\rho_L/R)_{\varphi_{max}}$, let us find the dependence of the lifetime τ of the captured electrons on the magnetic field H at the center of the mirror (Fig. 3). The experimental points shown in Fig. 3, marked $\times$, ∇, and $\bigcirc$, were ob-

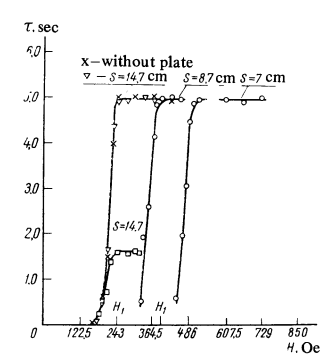

FIG. 3.

Exper. No.	W, kV	S, cm	H_{01}, Oe	$10^2\left(\frac{\rho_L}{R}\right)_{\varphi max}$	ρ, cm	τ, sec	$10^8\,p$, mm Hg
1	7.05	5.5	263	3.5	0.74	5	1
2	7.05	7.0	166	3.84	1.2	5	1
3	7.05	8.7	122	3.96	1.65	5	1
4	7.05	11.7	114	1.73	1.85	4.5	1.1
5	7.05	14.7	92	0.5	2.25	1.6	3
6	14.2	7.0	234	4.1	1.1	3.0	1.7
7	14.2	8.7	180	3.9	1.45	3.0	1.7
8	14.2	10.0	163	3.8	1.6	3.3	1.5
9	14.2	11.3	142	2	2.14	1.4	3.5
10	14.2	14.7	132	0.5	2.3	3.0	3

tained for electrons of energy W = 7.05 kV at a mirror ratio (in the absence of the plate) γ = 2.66 and at a residual gas pressure P = 10^{-8} mm Hg. At first we obtained the τ(H) plot for the case when there was no iron plate (the experimental points are designated by crosses). We then placed the iron plate at a distance S (see Fig. 1) from the chamber, plotted τ(H), increased the distance S from the plate to the chamber, and again plotted τ(H). Figure 3 shows these curves for distances S = 8.7 cm and S = 7 cm. Continuing to move the plate farther from the chamber (the plate remained at all times parallel to the chamber axis), we found the distance S at which the plots of τ(H) with the plate (∇) and without the plate ($\times$) coincided. In our case this distance turned out to be S = 14.7 cm. The maximum asymmetry amounted in this case to ~ 1.0% (in the case of an axially-symmetrical trap this value was approximately 0.5%). Figure 3 shows also the τ(H) plot for the same distance S = 14.7 cm., but at P = 3 × 10^{-8} mm Hg (the experimental points are denoted by squares). As seen from Fig. 3, when the iron plate comes closer to the chamber the τ(H) plots shift towards the region of stronger magnetic fields. The plots of Fig. 3 were used to obtain that value of the magnetic field at which the lifetime of the captured electrons begins to depend on the magnetic field (H_1 in Fig. 3), and calculated the value of $(\rho_L/R)_{\varphi_{max}}$ in accordance with the expression (4). The table lists the numerical values of the adiabaticity parameter $(\rho_L/R)_{\varphi_{max}}$ calculated in this manner. The same table lists the measurement data obtained for two values of the electron energy (W = 7.05 kV and W = 14.2 kV) and a mirror ratio γ = 2.66 (in the absence of a axial asymmetry).

In Fig. 3, the abscissas are the values of the magnetic field at the center of the magnetic mirror. Since in order to obtain τ(H) we varied the magnetic field by increasing or decreasing the current in the magnets, we marked on the abscissa axis of the τ(H) plots the values of the magnetic field at the center of the solenoid, since the ratio of the magnetic field at the center of coil to that at any point in space between the mirrors is a constant.

It should be noted that the oscillograms of the collector current are perfectly identical (of course, under identical initial conditions) in that part of the τ(H) plots where the transition to the horizontal section takes place, regardless of whether τ(H) plots were obtained with or without the iron. This indicates that when $\rho_L/R < (\rho_L/R)_1$ the electrons move in an identical manner (from the adiabatic point of view).

Formula (4) for the calculation of $(\rho_L/R)_{\varphi_{max}}$ contains the field in the median plane at the point where the electron is located. The table lists these values of a magnetic field H_{01} at that value of the field at the center of the solenoid, for which the lifetime of the captured electron becomes a function of the magnetic field (this is denoted H_1 in Fig. 3). The table lists also the values of the Larmor radius for the same value of the magnetic field (H_{01}).

It is seen from the table that when the plate is brought closer to the chamber (i.e., when the azimuthal inhomogeneity increases) the Larmor radius at which the lifetime of the electron becomes a function of the magnetic field decreases. But then the numerical value of the adiabaticity parameter $(\rho_L/R)_{\varphi_{max}}$ remains practically unchanged. On the other hand, when the aximuthal asymmetry becomes of the order of 1–2 percent, $(\rho_L/R)_{\varphi_{max}}$ becomes small (Nos. 4, 5, 9, and 10 in the table), and the electron motion is now determined by the value of $(\rho_L/R)_{\parallel max}$, the numerical value of which is $\approx (4.0 \pm 0.5) \times 10^{-2}$.

Thus, the data in the table indicate that when the electron moves in an axially-asymmetrical trap of mirror configuration, the critical adiabaticity parameter $(\rho_L/R)_{\varphi_{max}}$ is close to the critical adiabaticity parameter $(\varphi_L/R)_{\parallel max}$ when the electron moves in an axially-symmetrical trap.

Although the experimental results of Fig. 3 show that the adiabaticity parameter $(\rho_L/R)_{\varphi_{max}}$ does not change when the axial asymmetry is increased, the magnetic field at which the orbital magnetic moment of the particle $\mu = w_\perp/H$ ceases to be an adiabatic invariant increases with increasing axial asymmetry.

It must be noted that the presence of an iron plate decreases the magnetic field in the electron-capture region. The increase of the magnetic field with change of the angle occurs in the direction indicated in Fig. 4 by the arrows. Owing to the finite value of $\partial H/\partial\varphi$, the electrons will drift in an axially-asymmetrical field in a radial direction. This drift does not cause the electrons to go off to the chamber walls. This can be understood from the following considerations.

Owing to the presence of the radial gradient, the particle drifts in azimuth around the symmetry axis, en-

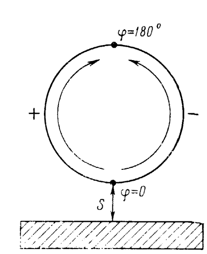

FIG. 4.

tering either in the (+) or in the (−) region (Fig. 4). Whereas in the (−) region the electron drift due to $\partial H/\partial \varphi$ and H is directed towards the chamber wall and the particle goes over to a magnetic surface situated closer to the wall, in the (+) region the same drift leads to a motion of the electron towards the axis of the system. Thus, in our case the presence of $\partial H/\partial \varphi$ does not produce a new mechanism for the electrons to go off to the chamber walls.

Control measurements aimed at determining the current to a cyclindrical probe located 3 mm away from the chamber wall and comparison of the collector-current oscillograms in axially-symmetrical and axially-asymmetrical traps, have shown that the foregoing reasoning is actually valid in our case.

In conclusion we note that our experiments may possibly lead to the conclusion that the important factor in the motion of an electron in a magnetic trap is not the presence or absence of axial symmetry, but only satisfaction of conditions (2a), (2b), and (2c), and the adiabaticity parameter ρ_L/R, which is defined in terms of the gradient of the magnetic field along the field or in terms of the axial asymmetry, should not be larger than $(\rho_L/R)_1 \cong (4.0 \pm 0.5) \times 10^{-2}$.

Thus, the parameter which characterizes the degree of non-adiabaticity in the trap is not the deviation of the magnetic field from symmetry, but the parameter ρ_L/R. It follows therefore that the parameter ρ_L/R can be used for different estimates also in the case of asymmetrical traps. This conclusion may be of definite interest in connection with the fact that there is still no developed general theory of motion of charged particles in axially-asymmetrical traps.

The authors are deeply grateful to B. V. Chirikov for suggesting the topic and for continuous interest and valuable advice, which greatly contributed to the performance of the work.

[1] M. G. Bogolyubov and Yu. A. Mitropol'skiĭ, Asimptoticheskie metody v teorii nelineĭnykh kolebaniĭ (Asymptotic Methods in Theory of Nonlinear Oscillations), Fizmatgiz, 1958 [Gordon and Breach, 1962].

[2] S. N. Rodionov, Atomnaya énergiya 6, 623 (1959).

[3] B. V. Chirikov, ibid. 6, 630 (1956).

[4] B. V. Chirikov, Dokl. Akad. Nauk SSSR **125**, 1015 (1959) [Soviet Phys.-Dokl. 4, 390 (1959)].

[5] A. N. Kolmogorov, ibid. 98, 527 (1954).

[6] V. I. Arnol'd, Usp. matem. nauk (Adv. in Math Sciences) 18, No. 6 (114), 91 (1963).

[7] E. Fermi, Nuclear Physics, U. of Chicago, 1950 (Russian Translation, IIL, 1951).

[8] A. N. Dubinina and L. S. Krasitskaya, ZhETF Pis. Red. 5, 230 (1967) [JETP Lett. 5, 184 (1967)].

[9] A. N. Dubinina and L. Ya. Traĭnin, and B. V. Chirikov, Zh. Eksp. Teor. Fiz. 49, 373 (1965) [Soviet Phys.-JETP **22**, 260 (1966)].

[10] V. M. Balebanov and N. N. Semashko, IAÉ Preprint 1073, Moscow, 1966.

Translated by J. G. Adashko
136