CYCLOTRON RESONANCE IN A STATIONARY AND VARYING LIGHT-ELECTRIC FIELD

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It is shown that when an electromagnetic wave moves in a conductor in the absence or presence of a stationary magnetic field stationary electric fields arise. We call these fields light-electric fields: in some cases they depend in a resonant manner on the wave frequency, magnetic field strength, and angle between the magnetic field and wave vector. A double-frequency field is also produced besides the stationary field. The field strengths are estimated for various types of slow waves (with wave velocities $\ll c$) propagating in metals, semimetals, and semiconductors.

1. A transverse electromagnetic wave propagating in a conductor produces in the latter a constant electric field proportional to the Poynting vector of this wave. We shall call this field light-electric. The electromagnetic wave can be either "fast" (with phase velocity close to that of light) or "slow" (Alfven wave and helicons). In the case of a fast wave, this field was considered by Barlow[1] neglecting diffusion and recombination, and by us[2] without this neglect. In the present paper we consider the light-electric field produced by slow electromagnetic waves propagating in the presence of a strong external magnetic field (cyclotron frequency Ω larger than the collision frequency ν). We shall investigate not only a constant but also an alternating light-electric field of double frequency, which is also proportional to the Poynting vector.

If an external magnetic field is applied to the conductor, the light-electric field acquires an interesting singularity; it turns out that not only the alternating but even the constant light-electric field has in some cases a resonant character: for a constant field, the resonance takes place at a frequency determined by the cyclotron frequency and the angle between the direction of propagation of the wave and the magnetic field. In particular, when the wave propagates transversely to a magnetic field, in which the Larmor radius exceeds the wavelength, the constant light-electric field has resonances at frequencies that are multiples of the cyclotron frequency.

2. We shall write the kinetic equation for the distribution function in the relaxation-frequency approximation. This equation then takes the form

$$\frac{\partial f}{\partial t}+\nu f+\mathbf{v}\nabla f+[\mathbf{v}\Omega_0]\frac{\partial f_1}{\partial \mathbf{v}}+\frac{e}{m}(\mathbf{E}_0+\mathbf{E}_1+\mathbf{E}_2)\frac{\partial f}{\partial \mathbf{v}}=0, \quad (1)^*$$

$\mathbf{E}_1$ is the field of the wave, and $\mathbf{E}_0$ and $\mathbf{E}_2$ are respectively the constant part and the second harmonic of the light-electric field. The first and second approximations with respect to the wave-field amplitude are

$$\nu f_1+\frac{\partial f_1}{\partial t}+\mathbf{v}\nabla f_1+[\mathbf{v}\Omega_1]\frac{\partial f_0}{\partial \mathbf{v}}+\frac{e\mathbf{E}_1}{m}\frac{\partial f_0}{\partial \mathbf{v}}=0,$$

$$\frac{\partial f_2}{\partial t}+\nu f_2+\mathbf{v}\nabla f_2+[\mathbf{v}\Omega_0]\frac{\partial f}{\partial \mathbf{v}}+\frac{e(\mathbf{E}_0+\mathbf{E}_2)}{m}\frac{\partial f_0}{\partial \mathbf{v}}+[\mathbf{v}\Omega_1]\frac{\partial f_1}{\partial \mathbf{v}}+\frac{e\mathbf{E}_1}{m}\frac{\partial f_1}{\partial \mathbf{v}}=0. \quad (2)$$

*$[\mathbf{v}\Omega_0] \equiv [\mathbf{v}\times\Omega_0]$.

Multiplying these equations by the velocity $\mathbf{v}$, integrating, and taking into account the fact that $f_1 \sim \mathbf{E}_1 \sim \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, we obtain for the first-approximation current an expression in the form $\mathbf{j}_1(\omega, \mathbf{k}) = \hat{\sigma}(\omega, \mathbf{k})\mathbf{E}_1$, where $\hat{\sigma}$ is the conductivity tensor, and for the second-approximation current the equation

$$\frac{\partial \mathbf{j}_\pm'}{\partial t}+\nu_\pm\mathbf{j}_\pm'=[\mathbf{j}_\pm'\Omega_{0\pm}]-\frac{e^2n_{0\pm}}{m_\pm}(\mathbf{E}_2+\mathbf{E}_0)-\frac{e_\pm T}{m}\nabla n_\pm + \overline{[\mathbf{j}_\pm'\Omega_{1\pm}]}+\frac{e^2(n_\pm-n_{0\pm})}{m_\pm}\mathbf{E}_1. \quad (3)$$

Here $n_{0\pm}$, $m_\pm$, and $\mathbf{j}_\pm'$ are the equilibrium concentrations and masses of the carriers of both signs and the currents produced by them; the current $\mathbf{j}'$ consists of the second harmonic $\mathbf{j}_2$ and the dc component $\mathbf{j}_0$; the bar denotes time averaging.

3. For the time-independent part $\mathbf{j}'$ we obtain the equation

$$\nu_\pm\mathbf{j}_{0\pm}=[\mathbf{j}_{0\pm}\Omega_{0\pm}]+\frac{e^2n_{0\pm}}{m_\pm}\mathbf{E}_0-\frac{e_\pm T}{m_\pm}\nabla n_\pm+\overline{[\mathbf{j}_{1\pm}\Omega_{1\pm}]} \quad (4)$$

The term $eTm^{-1}\nabla n$ is the result of the fact that the electric field $\mathbf{E}_0$ displaces the carriers, producing a gradient of their concentration within the bounded crystal. In the case when carriers of only one sign are present we have $\mathbf{j}_0 = 0$ and the term with ∇n is negligibly small, as can be readily seen by comparing it with the term containing the electric field, and using the Poisson equation for the estimate. Multiplying Maxwell's equation

$$k^2\mathbf{E}_1-\mathbf{k}(\mathbf{k}\mathbf{E}_1)-\omega^2c^{-2}\mathbf{E}_1=4\pi i\omega c^{-2}\mathbf{j}_1.$$

vectorially by Ω_1, averaging the result over the period of the oscillations, and using (4) we obtain

$$\mathbf{E}_0=-\frac{\omega\,\mathrm{Im}\,N^2}{en_0c^2}\mathbf{\Pi}=-\frac{4\omega\,\mathrm{Im}\,N^2}{en_0Nc^2}\mathbf{\Pi}_0, \quad (5)$$

where $\mathbf{\Pi}$ is the Poynting vector in the medium and N is the refractive index of the medium. We are interested in the case when $\mathrm{Re}\,N \gg 1$ ("slow" waves), for which $\Pi \approx 4\Pi_0/\mathrm{Re}\,N$, where Π_0 is the flux density for a normally-incident wave, with allowance for the fact that $\mathrm{Re}\,N \gg \mathrm{Im}\,N$.

We proceed to the case when there are carriers of both signs (inequal amounts). The situation is different in semiconductors, where the carrier recombination is relatively slow, so that the recombination frequency $\nu_r \ll \nu$, and therefore we could neglect it in (2), and in metals (or semimetals), where $\nu_r \gg \nu$, so that the carrier density can be assumed to be in equilibrium and we can put $\nabla n = 0$ in (4).

In the former case we multiply Eqs. (4) by the masses $m_\pm$, we subtract one from the other, and take into account the fact that $j_{0+} = j_{0-} = j_0$. We substitute in the resultant equation the relation for the recombination kinetics, namely $e_\pm(n_0 - n_\pm)\nu_r = \operatorname{div} \mathbf{j}_{0\pm}$, we obtain, by projecting (4) on the wave vector (directed along the x axis):

$$\frac{\partial^2 j_0}{\partial x^2} + \frac{j_0}{l_D^2} = \frac{n_0 \nu_r}{2T} F_\pm; \tag{6}$$

here $l_D = \sqrt{D/\nu_r}$ is the diffusion length and $\mathbf{F}_\pm = \mathbf{j}_\pm \times \mathbf{H}_1/nc$.

In the case of volume recombination $j|_{x=0,\,l} = 0$ (l is the thickness of the sample), we have

$$j_0 = \frac{n_0(F_+ + F_-)}{\Sigma m\nu}\left[1 - \left(\operatorname{sh}\frac{x}{l_D} + \operatorname{sh}\frac{l-x}{l_D}\right)\Big/\operatorname{sh}\frac{l}{l_D}\right]. \tag{6'}$$

Adding Eqs. (4) we obtain

$$\mathbf{E}_0 = \frac{\mathbf{j}_0}{en_0}\left(\frac{1}{\mu_+} - \frac{1}{\mu_-}\right) - \frac{[\mathbf{j}\mathbf{H}_0]}{en_0 c} + \frac{\mathbf{F}_+ - \mathbf{F}_-}{2}, \quad \mu = \frac{e}{m\nu}, \tag{7}$$

$\mu_\pm$ are the static mobilities of the carriers. The potential difference in the propagation direction (x axis) is given by

$$\frac{V}{l} = \frac{\mu_+ F_+ - \mu_- F_-}{e(\mu_+ + \mu_-)} - \theta\frac{(\mu_+ - \mu_-)(F_+ + F_-)}{2e(\mu_+ + \mu_-)}. \tag{8}$$

In the perpendicular direction

$$\frac{V_\perp}{l_\perp} = \frac{|[\mathbf{j}_0\mathbf{H}_0]|}{en_0 c} = (1-\theta)(F_+ + F_-)\frac{H_{0\perp}}{c\Sigma m\nu}, \quad \theta = \frac{l_D}{l\operatorname{cth}(l/2l_D)}. \tag{8'}$$

When absorption is taken into account, the first term in (8) is multiplied by $(l_a/l)[1 - \exp(-l/l_a)]$, and θ is replaced by

$$\theta' = \theta\left(1 - \frac{l_D^2}{l_a^2}\right)^{-1}\left\{1 - \frac{1}{2}[1 - e^{-l/l_a}]'\left(1 + \frac{l_D}{l_a}\operatorname{cth}\frac{l}{2l_D}\right)\right\},$$

where l_a is the absorption length.

4. We now apply the results (5), (7), and (8) for the light-electric field in crystals having carriers of one or both signs to certain known types of slow waves.

In a helical wave (helicon) in a conductor with carriers of one sign[3] we have

$$\operatorname{Im} N^2 = \frac{\omega_0^2}{\omega(\omega - \Omega\cos\vartheta)}\left[\frac{\nu}{\Omega_0} + \frac{3\pi kR}{16}\sin^2\vartheta\right],$$

where ω_0 is the plasma frequency and R the Larmor radius; further,

$$\operatorname{Re} N = [\varepsilon + \omega_0^2/\omega(\omega - \Omega_0\cos\vartheta)]^{1/2},$$

ε is the static dielectric constant; in metals it can be neglected, and in semiconductors $\operatorname{Re} N = \sqrt{\varepsilon}$. In semiconductors when $\omega \ll \Omega_0 \cos\vartheta$ the field is

$$E_0 = \frac{16\pi}{\sqrt{\varepsilon}}\frac{\Pi_0}{\mu H_0^2}\left[\frac{\nu}{\Omega_0} + \frac{3\pi kR}{16}\sin^2\vartheta\right], \tag{9}$$

and near resonance ($\omega = \Omega_0 \cos\vartheta$)

$$E_0 = \frac{16\pi\mu}{\sqrt{\varepsilon}\,c^2}\Pi_0, \tag{10}$$

if the depth of penetration of the wave exceeds the dimensions of the crystal. Thus, the ratio of (10) to (9) is of the order of $(\Omega_0/\nu)^2 = (\mu H_0/c)^2$. At resonance when $\mu \sim 10^6$ absolute units and $\Pi_0 \sim 10^3$ W/cm^3 we have $E_0 \sim 10^{-4}$ absolute units = 0.03 V/cm.

In metals at $\omega \ll \Omega_0 \cos\vartheta$ we have

$$E_0 = \frac{16\pi\nu\sqrt{\omega}\,\Pi_0}{\sqrt{\Omega_0}\,\omega_0 c H_0}\left[\frac{\nu}{\Omega_0} + \frac{3\pi kR}{16}\sin^2\vartheta\right]; \tag{11}$$

near resonance

$$E_0 = \frac{4\sqrt{\omega\nu}}{\omega_0^2}\frac{\mu\Pi_0}{c^2}, \tag{12}$$

and the ratio of (12) to (11) is $\sim(\mu H_0/c)^{3/2}$. At resonance (in metals) $\omega_0 \sim 10^{16}$ sec^{-1}, $\omega \sim 10^{11}$, $\nu \sim 5\times10^9$, $\Pi_0 \sim 10^3$ W/cm^2, and $E_0 \sim 0.1$ mV/cm. At $\nu \sim 10^8$ sec^{-1} and $\omega_0 \sim 3\times10^{13}$ (Bi) we have $E_0 = 1$ V/cm (2.5 $\times 10^{-2}$ abs. un.).

5. The case of almost transverse propagation $\cos\vartheta \ll \Omega\omega/\omega_0 \ll 1$. We confine ourselves to metals. Here we can have, in turn, two cases.

a) When $kR \ll 1$ there propagate two waves of different polarization, for which $N^2 = \omega_0^2/\omega^2$, and the light-electric field is $E_0 = 4\pi\omega\nu\Pi_0 = \omega c^2 e n_0$.

In bismuth $n_0 \sim 10^{17}$, $\nu \sim 10^8$, and when $\Pi_0 \sim 10^3$W/cm^2 we have $E_0 = 10^{17}$ abs. un. = 30 μV/cm.

b) When $kR \gg 1$ there propagate, according to [3], two waves of different polarization, for which

$$(kR)^3 = \left(\frac{\omega_0 R}{c}\right)^3\left(1 - \frac{\omega}{n\Omega} - \frac{i\nu}{\omega}\right)^{-1},$$

n = 1, 2, 3, ... In this case resonances take place at frequencies that are multiples of the cyclotron frequency ($\omega = n\Omega$). Using (5), we find that when ω is not close to $n\Omega$ ($|\omega - n\Omega| \gg \nu$) the light electric field is

$$E_0 = \frac{4\nu\Pi_0}{en_0c^2}\left(\frac{c\omega_0^2}{\omega^3 R}\right)^{1/3}, \tag{13}$$

and near resonance

$$E_0 = \frac{4\Pi_0}{en_0c^2}\left(\frac{c\omega_0^2\omega}{\nu R}\right)^{1/3}. \tag{14}$$

Near resonance in metals, at $H_0 \sim 10^4$ Oe and $\Pi_0 \sim 10^3$ W/cm^2 we have $E_0 \sim 0.1$ mV/cm.

6. We now proceed to conductors containing carriers of opposite signs in equal concentrations. We consider Alfven and fast magnetosonic waves. For both metals and semiconductors we have in the first of these cases

$$N^2 = \varepsilon + \frac{\omega_0^2}{4\pi(\Omega\cos\vartheta)^2}\left(1 + \frac{i\nu}{\Omega}\right), \quad \Omega = \frac{eH}{(m_+ + m_-)c}, \quad \cos\vartheta > \frac{\omega}{\Omega}.$$

As before, ε can be neglected in metals, whereas in semiconductors, to the contrary, only ε need be retained. The expression for N^2 in magnetosonic waves differs only in the absence of $\cos\vartheta$. For the force we have

$$\mathbf{F}_\pm = \frac{1}{cn_0}\overline{[\mathbf{j}_{1\pm}\mathbf{H}_1]} = 16\pi m_\pm \nu_\pm \Pi_0/NH^2.$$

Using (8), we obtain in the case $l \ll l_D$ ($\theta = 1$) and a fast magnetosonic wave

$$E_\parallel = \frac{V_\parallel}{l} = \frac{16\pi}{N}\frac{m_+\nu_+ - m_-\nu_-}{eH_0^2}\Pi_0, \quad E_\perp = 0, \tag{15}$$

and in the case $l \gg l_D$ ($\theta = 0$)

$$E_{\|} = 0, \quad E_{\perp} = \frac{V_{\perp}}{l_{\perp}} = \frac{16\pi \sin \vartheta}{cH_0N} \Pi_0. \tag{16}$$

In the case of propagation x of Alfven waves we put in these formulas $H \to H \cos \vartheta$. In bismuth at $\Pi_0 \sim 10^3$ W/cm^2 and $H_{\perp} \sim H$ we have $E_0 \sim 0.3$ V/cm.

7. Kaner and Skobov have shown that in metals with carriers of both signs in equal concentrations ($n_+ = n_-$) helical waves can propagate, i.e., waves having a quadratic spectrum not only at the frequencies $\omega \gg \Omega_{0+}$, but also at frequencies $\omega \ll \Omega_{0\pm}$ when $c/\omega_0 R \ll kR \ll 1$. For these waves $N^2 = c^2/\alpha\Omega R^2(\omega + i\nu)$ if $\mathbf{H} \parallel \mathbf{k}$; $\alpha = \alpha_+ - \alpha_-$ ($\alpha_\pm$ are coefficients that depend on the form of the Fermi surface of the carriers); for a spherical surface $\alpha_+ = \alpha_- = 1/5$ (and in this case these waves are missing). The indicated waves are circularly polarized and act on the carriers with a force

$$F_{\pm} = \frac{16\pi e^2 v_{\pm} R\sqrt{\alpha\omega}}{c^2\Omega^{3/2}(m_+ + m_-)} \Pi_0. \tag{17}$$

Therefore, for example, when $l \gg l_D$ we get

$$E_0 = \frac{16\pi e^2 R\sqrt{\alpha\omega}}{c^2(m_+ + m_-)\Omega^{3/2}} \frac{(m_+ - m_-)v_+v_-}{m_+v_+ + m_-v_-} \Pi_0 \tag{18}$$

8. Let us consider the alternating light-electric field E_2 with frequency 2ω. From Eq. (3) follows an equation for the current j_2, with a solution in the form

$$\mathbf{j}_2 = \hat{\sigma}(2\omega)(\mathbf{E}_2 + \mathbf{F}/e)$$

($\hat{\sigma}$ is the conductivity tensor). Substituting this expression in Maxwell's equation, we get

$$(N^2(2\omega) - \varepsilon)\mathbf{E}_{2\perp} - \varepsilon\mathbf{E}_{2\|} = \frac{4\pi i\hat{\sigma}(2\omega)}{2\omega}(\mathbf{E}_2 + \mathbf{F}/e), \tag{19}$$

where $\mathbf{E}_{2\perp}$ and $\mathbf{E}_{2\|}$ are the components of E perpendicular and parallel to the force F, i.e., to the direction of the wave vector **k**. Solving this equation we obtain the following:

a) For metals in which $N^2 \gg \varepsilon$, so that the second term in (19) can be neglected, we have

$$E_2 = -F/e \tag{20}$$

in the case of carriers of one sign and

$$E_2 = \frac{\sigma_{xx}^+ F^+/e_+ + \sigma_{xx}^- F^-/e^-}{\sigma_{xx}^+ + \sigma_{xx}^-} \tag{21}$$

in the case of carriers of both signs; here $\mathbf{x} \parallel \mathbf{F}$.

b) For semiconductors, when $N^2 - \varepsilon \ll \varepsilon$, we obtain for the field components, by iterating with respect to the small parameters, the following equation ($\mathbf{k} \parallel \mathbf{x}$, H is in the xz plane):

$$E_x = -\frac{4\pi i\sigma_{xx}F}{\omega\varepsilon e}, \quad E_z = -\frac{\sigma_{zx}F}{\sigma_{zz}e},$$

$$E_y = \frac{4\pi i}{\omega(N^2 - \varepsilon)}\left(\sigma_{yz}E_z + \sigma_{yx}\frac{F}{e}\right)$$

$$= \frac{4\pi iF}{e\omega(N^2 - \varepsilon)}\left(\sigma_{yx} - \frac{\sigma_{yz}\sigma_{zx}}{\sigma_{zz}}\right). \tag{22}$$

These expressions hold for the cases of carriers of one sign; if carriers of both signs are present, then the equations are modified in analogy with the modification of (20) into (21).

In metals the field E_2 is essentially longitudinal, while in semiconductors (for a helical wave) it is essentially transverse. In order of magnitude, in both cases the field is equal to F/e.

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