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PROPAGATION OF ELECTROMAGNETIC WAVES IN SEMICONDUCTORS WHEN THE ELECTRON TEMPERATURE IS NOT A UNIQUE FUNCTION OF THE INCIDENT-FIELD AMPLITUDE

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The propagation of strong electromagnetic waves in a semiconductor is investigated in the case when the effective electron temperature is not a unique function of the incident-field amplitude. The problem is solved for weak damping of the waves. A qualitative investigation, using a model is performed for arbitrary damping. It is shown that under certain conditions the reflection coefficient becomes an oscillating function of the incident-field amplitude. The results can easily be applied to a plasma.

We have previously investigated nonlinear effects produced in the propagation of electromagnetic waves in a semiconductor when the electron gas is heated by an alternating magnetic field propagating in the sample.[1] It was assumed there that the electron temperature is a single-valued function of the field. Yet, in many cases this may not be so, and qualitatively new effects result. The present communication is devoted to the investigation of these phenomena.

The complete system of equations describing the propagation of strong electromagnetic waves in a semiconductor consists of the wave equation and the equation for the determination of the temperature ([1]). For simplicity we assume that the wave propagation and the magnetic field are directed along the z axis. We confine ourselves to the normal skin effect, since it can be shown that the phenomena considered in the present communication do not appear in the limiting case of a strong anomalous skin effect.

According to formulas (1.19), (1.21), (1.22), and (3.1) of [1], the complete system of equations of the problem has the following form:

$$A(v)(v-1) = B(v)u^2, \tag{1}$$

$$d^2E / dz^2 + k^2\varepsilon(v)E = 0. \tag{2}$$

Here $v = \Theta/T$, Θ is the electron temperature, T the lattice temperature, u the modulus of the electric field E, $k = \omega/c$, and ω is the frequency of the incident field. By E is meant $E = E_x \pm iE_y$—the field of the normal wave. For concreteness we shall consider a resonant normal wave $E = E_x + iE_y$. The nonresonant wave is investigated in similar fashion. A(v) describes the transfer of energy from the electron gas to the lattice and takes the form[1]

$$A(v) = A_0 v^{r-1}, \tag{3}$$

where A_0 and r are determined by the mechanism of this transfer. If there are l such mechanisms, then

$$A(v) = \sum_{i=1}^{l} A_{0i} v^{r_i - 1}. \tag{4}$$

The nomenclature for r_i is given in [2], and the coefficient A_{0i} is given in [1].

From formulas (1.11) and (1.22) of [1] we obtain for B(v) and $\varepsilon(v)$ the expressions

$$B(v) = \frac{4\omega_0^2}{3\pi^{1/2}v^{5/2}} \int_0^\infty \frac{\nu(x)x^{3/2}e^{-x/v}}{(\omega_H-\omega)^2+\nu^2(x)}dx, \tag{5}$$

$$\varepsilon(v) = \varepsilon_0 + \frac{4\omega_0^2}{3\pi^{1/2}v^{5/2}} \int_0^\infty \frac{x^{3/2}e^{-x/v}}{\omega[\omega_H-\omega-i\nu(x)]}dx, \tag{6}$$

where ω_0 is the nonlinear frequency, $\nu(x)$ is the frequency of the collisions causing the momentum transfer, $x = \varepsilon/T$, ε is the electron energy, $\omega_H = |e|H/mc$, e is the electron charge, m is the electron mass, H is the external magnetic field, and ε_0 is the dielectric constant of the lattice. If there are p mechanisms for momentum transfer, then

$$\nu(x) = \sum_{i=1}^{p} \nu_i(x). \tag{7}$$

Formulas (1)–(6) were obtained under the assumption that the electrons have a Maxwellian distribution, that is, the electron concentration is so large that the distribution over the energies is controlled by the interelectron collisions.

The equation for the temperature (1) for a specified value of u can, in general, have several solutions. This was first noted by A. Gurevich,[3] who presented a physical explanation of this phenomenon.

We rewrite (1) in the form

$$D(v) = u^2, \tag{8}$$

where $D(v) = A(v)(v-1)/B(v)$. If Eq. (8) has several roots, then there must be among them such roots v(u) which decrease with increasing field u (see Fig. 1). As seen from (8), this takes place if the function A(v) increases slowly or B(v) increases sufficiently rapidly with increasing v. This is tantamount to stating that the electrons do not have time to transfer to the lattice the energy acquired by them from the field. It is obvious that such a state is nonstationary. The only stable branches of the curve v(u) are those in which v increases with u, that is, dv/du > 0. It is seen from (8) that the stability condition can also be written in the form

$$dD(v) / dv > 0. \tag{9}$$

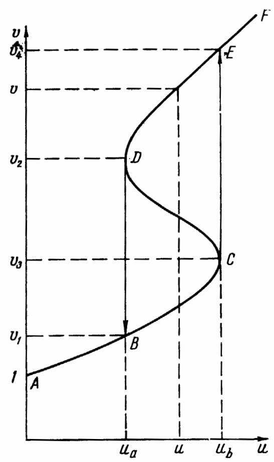

FIG. 1.

The transition from the stable branches to the unstable ones occurs at a temperature determined from the equation $dv/du = \infty$ or the equivalent equation $du/dv = 0$, or

$$dD(v) / dv = 0. \tag{10}$$

The field u corresponding to the transition of the temperature from one branch to the other, is determined from (8) by substituting in it the roots of Eq. (10).

Let us consider now the incidence of a plane electromagnetic wave from vacuum on the half-space $z > 0$ filled with a semiconductor. For concreteness let us assume that in the case under consideration Eq. (8) has three roots (see Fig. 1). As seen from the figure, the curve describing the dependence of the temperature and the field has two stable sections, AC and DF, and one unstable section CD.

Let us stop to discuss the change of the temperature of the electron gas as a function of the electric field $u_0 = u(+0)$, which in turn is determined by the amplitude of the field E_0 incident from the vacuum. When the field increases adiabatically from zero, the electron temperature, as a function of the field deep in the sample, is described by the lower branch AC so long as $u_0 < u_b$. At $u_0 = u_b$ the temperature of the electron gas on the boundary v_0 changes abruptly from v_3 to v_4, bypassing the unstable part of CD the curve. With further increase of u_0, v_0 moves to the right along the curve DF. The electric field attenuates with increasing distance z from the boundary of the semiconductor, owing to dissipation, to zero as $z \to \infty$. The electron temperature decreases together with the field and tends to the value T. At a certain point $z = a$, the function $v(z)$ changes abruptly from v_2 to v_1, leading to a discontinuity in the dielectric constant $\varepsilon(v)$ at this point. Since an electromagnetic wave is reflected from a discontinuity of the dielectric constant, the semiconductor behaves in the investigated case like a plate of thickness a.

It is well known that the coefficient P of reflection from the plate into vacuum is an oscillating function of the plate thickness. The plate thickness a should be determined from the equation $u(a, E_0) = u_a$, which determines the field at the discontinuity point. Thus, a is a function of E_0 and u_a, and consequently P oscillates with variation of E_0. If we now decrease u_0, then at $u_0 = u_a$ the value of v_0 again changes abruptly from v^2 to v_1. The dependence of v on u is then described by the section of the curve BA, and the dependence of P on E_0 is monotonic (see [1]). It will be shown that T depends on E_0 via v_0. Consequently, discontinuities in v_0 lead to discontinuities in T.

Let us proceed to the calculations. We confine ourselves first to weakly damped waves. In order for the waves to be weakly damped, the real part of $\varepsilon(v)$ should be much larger than the imaginary part, that is, we assume that the dielectric constant can be written in the form

$$\varepsilon(v) = n^2 + i\alpha\Phi(\alpha, v), \tag{11}$$

where the smallness of the parameter $\alpha \ll 1$ corresponds to weak damping. It is also assumed that $n^2 \gg \alpha$. The function $\Phi(\alpha, v)$ is of the order of unity. We shall assume that $\Phi(\alpha, v)$ can be represented in the form of a series

$$\Phi(\alpha, v) = \sum_{l=1}^{\infty} \alpha^{l-1}\Phi_l(v). \tag{12}$$

The coefficients $\Phi_l(v)$ are also of the order of unity.

It is seen from formula (6) that the dielectric constant can be transformed to the form (11) in one of the following three cases:

I. $\nu / |\omega_H - \omega| \ll 1$. Expanding (6) in powers of ν, we get

$$n^2 = \varepsilon_0 - \frac{\omega_0^2}{\omega(\omega - \omega_H)} \qquad \alpha = \frac{I}{\omega_H - \omega}, \qquad I = \int_0^\infty \frac{\nu(x)\, x^{3/2}}{e^x} dx,$$

$$\Phi_l = \frac{4}{3\pi^{1/2}} \frac{\omega_0^2}{\omega(\omega_H - \omega)} \frac{i^{l-1}}{v^{5/2} I^l} \int_0^\infty \frac{x^{3/2}}{e^{x/v}} \nu^l dx. \tag{13}$$

II. Cyclotron resonance, $\omega = \omega_H$. We have

$$n^2 = \varepsilon_0, \quad \alpha = \frac{4\omega_0^2}{3\pi^{1/2}\omega_H} I, \quad I = \int_0^\infty \frac{x^{3/2}}{e^x \nu(x)} dx,$$

$$\Phi_1 = \frac{1}{v^{5/2} I} \int_0^\infty \frac{x^{3/2}}{e^{x/v}\nu(x)} dx, \quad \Phi_l = 0 \text{ for } l > 1. \tag{14}$$

In order for the second term in (11) to be much smaller than the first, the inequality $\omega_0^2 / \omega_H \nu \ll 1$ should be satisfied.

III. $\omega_H = 0$, $\omega/\nu \ll 1$. Under this assumption, the expansion takes the form

$$n^2 = \varepsilon_0, \quad \alpha = \omega I, \quad I = \int_0^\infty \frac{x^{3/2}}{e^x \nu(x)} dx,$$

$$\Phi_l = \frac{4}{3\pi^{1/2}} \frac{\omega_0^2}{\omega^2} \frac{i^{l-1}}{v^{5/2} I^l} \int_0^\infty \frac{x^{3/2}}{\nu^l e^{x/v}} dx. \tag{15}$$

In order for the expansion to be valid, it is necessary to satisfy the inequality $\omega_0^2 / \omega\nu_0 \ll 1$.

We shall need in what follows the dependence of the dielectric constant on the modulus of the field u. To this end, it is necessary to substitute v as a function of u in formula (11). We introduce the following notation: Let $\varphi(u)$ denote the function $\Phi(v(u))$ if $v > v_2$, and let $\psi(u)$ denote the same function if $v < v_3$. In the new notation, Eq. (2) takes the form

$$d^2E / dz^2 + k^2[n^2 + i\alpha\varphi(u)]E = 0 \quad \text{for } v > v_2, \tag{16}$$

$$d^2E / dz^2 + k^2[n^2 + i\alpha\psi(u)]E = 0 \quad \text{for } v < v_3. \tag{17}$$

We can add to these equations the ordinary boundary conditions of electrodynamics at the point $z = 0$

$$\frac{\partial E(-0)}{\partial z} = \frac{\partial E(+0)}{\partial z}, \quad E(-0) = E(+0), \tag{18}$$

and at the point a:

$$\frac{\partial E(a-0)}{\partial z} = \frac{\partial E(a+0)}{\partial z}, \quad E(a-0) = E(a+0). \tag{19}$$

If $v_0 < v_3$, then $\varepsilon(v(u))$ is a single-valued function of u. This case was considered in detail in [1]. We shall therefore assume directly that $v_0 > v_2$.

To solve Eqs. (16) and (17) we shall use a method similar to the method of varying the constants. We seek a solution of (16) and (17) in the form

$$E = E_0(e^{ikz} + Pe^{-ikz}), \qquad z < 0, \tag{20}$$

$$E = w_1(z)e^{iknz} + w_2(z)e^{-iknz}, \quad 0 < z < a, \tag{21}$$

$$E = w_3(z)e^{iknz}, \qquad z > a. \tag{22}$$

The structure of the field in (21) takes account of the fact that there are both an incident and reflected wave in the region of space $0 < z < a$.

Let us obtain equations for $w_1(z)$, $w_2(z)$, and $w_3(z)$. Substituting (21) in (16) we get

$$\left[\frac{d^2w_1}{dz^2} + 2ikn\frac{dw_1}{dz} + ik^2\alpha\varphi(u)w_1\right]e^{iknz} + \left[\frac{d^2w_2}{dz^2} - 2ikn\frac{dw_2}{dz} + ik^2\alpha\varphi(u)w_2\right]e^{-iknz} = 0. \tag{23}$$

We expressed the field E in terms of two unknown functions of w_1 and w_2. Inasmuch as now one unknown function is expressed in terms of two, we can impose additional conditions on them. We stipulate that the expression in the first square bracket in (23) vanish. It follows from this that the expression in the second square bracket vanishes. Ultimately we get for w_1 and w_2 the system of equations

$$\frac{d^2w_1}{dz^2} + 2ikn\frac{dw_1}{dz} + ik^2\alpha\varphi(u)w_1 = 0, \tag{24}$$

$$\frac{d^2w_2}{dz^2} - 2ikn\frac{dw_2}{dz} + ik^2\alpha\varphi(u)w_2 = 0. \tag{25}$$

Substituting (22) in (17) we get for w_3 the expression

$$\frac{d^2w_3}{dz^2} + 2ikn\frac{dw_3}{dz} + ik^2\alpha\psi(u)w_3 = 0. \tag{26}$$

Equations (24)–(26) are best solved by successive approximations in α. We must bear in mind the following circumstance. When $\alpha = 0$, the quantities w_l ($l = 1, 2, 3$) are constants. When α differs from zero, the w_l vary like functions of z, depending on the smallness of α, that is, it can be assumed that $w_l(z) = w_l(\alpha kz)$. It follows therefore that differentiation of w_l with respect to z increases the order of smallness of w_l, and integration lowers the order of smallness of the quantity, that is, we have the following estimates

$$\frac{dw_l}{dz} \sim \alpha k w_l, \quad \int w_l dz \sim \frac{w_l}{\alpha k}. \tag{27}$$

We confine ourselves in the calculation to terms of order α. We write w_l in the following fashion:

$$w_l = u_l(z)e^{ikS_l(z)} \tag{28}$$

and we seek u_l and S_l in the form

$$u_l = u_l^{(0)} + \alpha u_l^{(1)}, \quad S_l = S_l^{(0)} + \alpha S_l^{(1)}.$$

We note that the expansion of $u_2(z)$ begins with a term that is linear in α, since the jump causing the appearance of the reflected wave vanishes when $\alpha = 0$. Calculations show that in order to find quantities of the order α in the expansion of $\varepsilon(v)$ it is necessary to retain terms of order α^2. Substituting (28) in (24)–(26) and taking (27) into account, and also the boundary conditions (18) and (19), we obtain with the aid of an iteration method the following first- and second-approximation values:

$$z = -\frac{2n}{k\alpha}\int_{u_1^{(0)}(0)}^{u_1^{(0)}}\frac{du}{\varphi_1(u)u}, \quad S_1^{(0)} = S_3^{(0)} = u_2^{(0)} = 0,$$

$$z - a = -\frac{2n}{k\alpha}\int_{u_1^{(0)}(a)}^{u_3^{(0)}}\frac{du}{\psi_1(u)u}, \quad S_2^{(0)} = -\frac{\pi}{2k} - 2na,$$

$$S_1^{(1)}(z) = S_1^{(1)}(0) - \frac{1}{k}\int_{u_1^{(0)}(0)}^{u_1^{(0)}}\left[\frac{1}{4n^2}\frac{1}{u}\frac{d(\varphi_1(u)u)}{du} + \frac{\varphi_2(u)}{\varphi_1(u)u}\right]du,$$

$$S_3^{(1)}(z) = S_3^{(1)}(a) - \frac{1}{k}\int_{u_1^{(0)}(a)}^{u_3^{(0)}}\left[\frac{1}{4n^2}\frac{1}{u}\frac{d(\psi_1(u)u)}{du} + \frac{\psi_2(u)}{\psi_1(u)u}\right]du, \tag{29}$$

$$u_1^{(1)}(z) = u_1^{(1)}(0)\varphi_1(u_1^{(0)})u_1^{(0)}(z)/\varphi_1(u_1^{(0)}(0))u_1^{(0)}(0),$$

$$u_3^{(1)}(z) = u_3^{(1)}(a)\psi_1(u_3^{(0)})u_3^{(0)}(z)/\psi_1(u_3^{(0)}(a))u_3^{(0)}(a),$$

$$u_2^{(1)}(z) = u_2^{(1)}(0)u_1^{(0)}(0)/u_1^{(0)}(z).$$

The procedure described above for finding the solution is not applicable in the direct vicinity of the point v_2. Indeed, it follows from (29) that

$$\frac{du_1^{(0)}}{dz} = -\frac{k\alpha}{2n}\Phi_1(v)u_1^{(0)}.$$

Differentiating this expression, we can easily show that

$$\frac{d^2u_1^{(0)}}{dz^2} = \frac{k^2\alpha^2}{4n^2}\Phi_1^2(v)u_1^{(0)}\left[1 + \frac{u_1^{(0)}}{\Phi_1(v)}\frac{d\Phi_1(v)}{dv}\frac{dv}{du}\right]. \tag{30}$$

However, near the point v_2 the derivative dv/du increases without limit, in connection with which the expansion (28) becomes meaningless.[2] However, near the point $v = v_2$ Eq. (16) can be solved by an iteration method, using the smallness of $(v - v_2)/v_2$. To this end we determined first the dependence of v on the coordinate z near the point $v = v_2$.

Expanding $D(v)$ in powers of $v - v_2$ (see formula (8)) and recognizing that $dD(v_2)/dv = 0$, we get

$$v - v_2 = 2[u_a / D''(v_2)]^{1/2}(u - u_a)^{1/2}; \tag{31}$$

The prime denotes differentiation with respect to v. The modulus of the field near the point a has no singularities whatever, and therefore $u(z)$ can be expanded in powers of $a - z$. We ultimately have

$$v - v_2 = \gamma(a - z)^{1/2}, \quad \gamma = 2\left[\frac{u_a}{D''(v_2)}\left|\frac{du(a)}{dz}\right|\right]^{1/2}. \tag{32}$$

Expanding $\Phi_1(v)$ in powers of $v - v_2$ and substituting

2) Allowance for the thermal conductivity in the balance equation (1) leads to a finite but large value of dv/du. This circumstance is inessential in what follows.

this expansion into the equation for determining w_1 and w_2, we get

$$\frac{d^2 w_{1,2}}{dz^2} \pm 2ikn\frac{dw_{1,2}}{dz} + i\alpha k^2\left[\Phi_1(v_2) + \gamma\frac{d\Phi_1(v_2)}{dv}(a-z)^{1/2}\right]w_{1,2} = 0. \quad (33)$$

Using the smallness of the second term in the square bracket in (33), we solve it by successive approximations with respect to this term. The result is

$$w_{1,2} = A_{1,2}\exp\left\{\mp\frac{\alpha k}{2n}\Phi_1(v_2)(z-a)\right\} \mp \frac{\alpha k}{n}\Phi_1'(v_2) \times \left[\frac{u_a}{D''(v_2)}\left|A_{1,2}\frac{\alpha k}{2n}\Phi_1(v_2)\right|\right]^{1/2} A_{1,2} \times\left[\int_a^z (a-z)^{1/2}dz - e^{\mp 2iknz}\int_a^z e^{\pm 2iknz}(a-z)^{1/2}dz\right]. \quad (34)$$

The solution of (33) can be made continuous at the point z = a with the solution of (26) with the aid of the boundary conditions (19), and it is possible to go over to the solution of (24) and (25) when z is to the left of a. With this aid of this procedure (which will not be presented here, owing to its complexity), we can determine all the arbitrary constants that enter into the solutions. In particular, we can find the reflection coefficient P. Here, naturally, we must use also the boundary conditions (18). The expressions for the arbitrary constants which enter into the solution are

$$u_1^{(0)}(0) = 2E_0/(1+n), \quad u_3^{(0)}(a) = u_1^{(0)}(a),$$

$$u_1^{(1)}(0) = \frac{n-1}{4n^2(n+1)}\frac{u_1^{(0)2}(a)}{u_1^{(0)}(0)}[\Phi_1(v_1) - \Phi_1(v_2)]\sin 2kna,$$

$$u_3^{(1)}(a) = \frac{n-1}{4n^2(n+1)}\frac{\Phi_1(v_2)}{\Phi_1(v_0)}\frac{u_1^{(0)3}(a)}{u_1^{(0)2}(0)}[\Phi_1(v_1) - \Phi_1(v_2)]\sin 2kna,$$

$$u_2^{(1)}(0) = \frac{1}{4n^2}\frac{u_1^{(0)2}(a)}{u_1^{(0)}(0)}[\Phi_1(v_2) - \Phi_1(v_1)],$$

$$A_1 = u_1\ (a) + \alpha[u_1\ (a) + ikS_1\ (a)u_1\ (a)]$$

$$A_2 = \alpha\frac{u_1^{(0)}(0)}{u_1^{(0)}(a)}u_2^{(1)}(0)\exp\{-ikS_2^{(0)}(0)\}, \quad (35)$$

$$S_1^{(1)}(0) = \frac{1}{2n(n-1)k}\left\{\frac{n-1}{2n}\frac{u_1^{(0)2}(a)}{u_1^{(0)2}(0)}[\Phi_1(v_2) - \Phi_1(v_1)]\cos 2kna - \Phi_1(v_0)\right\},$$

$$S_3^{(1)}(a) = \frac{1}{4n^2k}[\Phi_1(v_2) - \Phi_1(v_1)]\left[1 + \frac{n-1}{n+1}\frac{u_1^{(0)2}(a)}{u_1^{0(2)}(0)}\cos 2kna\right] - \frac{1}{2n(n+1)k}\Phi_1(v_0) - \frac{1}{k}\int_{u_1^{(0)}(0)}^{u_1^{(0)}(a)}\left[\frac{1}{4n^2}\frac{1}{u}\frac{d(\varphi_1(u)u)}{du} + \frac{\varphi_2(u)}{\varphi_1(u)u}\right]du.$$

The reflection coefficient is described by the following formula

$$P = \frac{1-n}{1+n} - \frac{i\alpha}{n}\left\{\frac{\Phi_1(v_0)}{(1+n)^2} + \frac{u_a^2}{4E_0^2}[\Phi_1(v_1) - \Phi_1(v_2)]e^{2ikna}\right\}. \quad (36)$$

It can be shown that if dv/du is finite at the discontinuity point, then formula (36) still remains in force. It is seen from (36) that the reflection coefficient is a periodic function of a.

We now proceed to determine the dependence of a on the amplitude of the incident field. As already indicated above, the thickness a is determined from the equation that the amplitude of the field at the point a be equal to the field u_a. The equation for finding a, as follows from (21) and (34), is

$$|A_1 + A_2\exp\{-2ikna\}| = u_a, \quad (37)$$

with A_1 and A_2 determined by formula (35).

We shall seek a solution of (37) in the form of a power expansion, starting with α^{-1}:

$$a = a_{-1}\alpha^{-1} + a_0 + a_1\alpha + \dots \quad (38)$$

We shall retain in this expansion only the first two terms, since the third term adds to formula (36) an inessential correction of the order of α^2. Substituting (38) in (37), we get a_{-1}

$$a_{-1} = \frac{n}{k}\int_{v_2}^{v_0}\Phi_1^{-1}(v)\frac{d\ln D(v)}{dv}dv. \quad (39)$$

In finding a_{-1}, we have gone over in all the expressions from integration with respect to u to integration with respect to v, using formula (8). The next approximation yields an equation for the determination of a_0:

$$d = q\sin(d+b), \quad (40)$$

where

$$d = 2kna_0, \quad b = \frac{2kna_{-1}}{\alpha}, \quad q = \frac{1-n^2}{4}\frac{u_a^2}{E_0^2}\frac{\Phi(v_2) - \Phi(v_1)}{\Phi(v_0)}.$$

It is seen from (40) that d (meaning also a_0) is a periodic function of $a_{-1}\alpha^{-1}$, and consequently the reflection coefficient P contains all the harmonics in the Fourier expansion in $a_{-1}\alpha^{-1}$. An exception is the case n = 1, when, as follows from (40), q = 0 and $a = a_{-1}\alpha^{-1}$.

Let us investigate Eq. (40). A simple analysis shows that the equation has one root at $|q| \le 1$, and a finite number of roots when $|q| > 1$. If $|q| > 1$, then the only root of this equation with a physical meaning is the smallest one, since a wave reaching the point a determined by this root transforms the semiconductor in the region z > a from a regime corresponding to the upper branch into a regime corresponding to the lower branch. It follows therefore that none of the transitions caused by the other roots can be realized. In the general case, Eq. (40) must be solved graphically, but when $|q| < 2/e$ it can be solved by expansion in a Lagrange series.[4] When $|q| < 2/e$, the reflection coefficient is reduced to the form

$$P = \frac{1-n}{1+n} - \frac{i\alpha}{n}\left[\frac{\Phi_1(v_0)}{(1+n)^2} + \frac{u_a^2}{4E_0^2}\Phi_1(v_1) - \Phi_1(v_2)\sum_{l=-\infty}^{\infty}B_l \times\exp\left\{\frac{2iknla_{-1}}{\alpha}\right\}\right]. \quad (41)$$

Here

$$B_l = \begin{cases}\sum_{k=l-1}^{\infty}{}' Q_{lk}, & l \geqslant 2\\ \sum_{k=|l|+1}^{\infty} Q_{lk}, & l \leqslant 0,\\ 1 + \sum_{k=0}^{\infty}{}' Q_{1k}, & l = 1\end{cases}$$

$$Q_{lk} = (-1)^{(k-l+1)/2}\, l^{k-1} q^k \Big/ 2^k\left(\frac{k-l+1}{2}\right)!\left(\frac{k+l-1}{2}\right)!;$$

The prime at the summation sign denotes that the sum includes only terms containing k with parity $l \pm 1$.

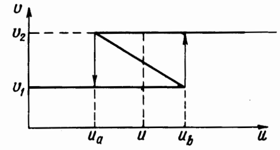

FIG. 2.

Let us now find the period of the oscillations of the coefficient of reflection with respect to the amplitude of the incident electric field E_0. We denote this period by δE_0. It is obvious that it should be determined from the relation

$$\frac{2kn}{\alpha}[a_{-1}(E_0+\delta E_0)-a_{-1}(E_0)]=2\pi. \qquad (42)$$

Assuming that $\delta E_0 \ll E_0$ and expanding the numerator of formula (42) in a series, we get

$$\delta E_0=\pi\alpha \Big/ kn\frac{da_{-1}}{dE_0}. \qquad (43)$$

The quantity da_{-1}/dE_0 can be obtained from formula (39). Ultimately we get for the period δE_0 the expression

$$\delta E_0=\pi\alpha\Phi_1(v_0)\Big/ n^2\frac{d\ln D(v_0)}{dE_0}. \qquad (44)$$

The dependences of v_0 on E_0 and on other parameters, for various cases, and also of D on v_0 are given in [1]. We note that the period of the oscillations of the reflection coefficient does not depend on the field u_a at the discontinuity point.

It is difficult to solve a similar problem for the case when the nonlinearity in the wave equation is not small, but a qualitative investigation can be carried out by using the following model. Let the dependence of v on u have the form shown in Fig. 2. The transition from the upper branch ($v = v_2$) to the lower branch ($v = v_1$) is similar to that described in the case of weak damping. Then we have $\varepsilon(v) = \varepsilon(v_2) \equiv \varepsilon_2$ and $\varepsilon(v) = \varepsilon(v_1) \equiv \varepsilon_1$ when $v = v_1$. In this case Maxwell's equation becomes linear both in the region $z < a$ and in the region $z > a$, and its solution is written in the form

$$E = u_1[\exp\{ik\sqrt{\varepsilon_2}z\}+P_1\exp\{-ik\sqrt{\varepsilon_2}z\}], \quad 0<z<a,$$
$$E = u_2\exp\{ik\sqrt{\varepsilon_1}(z-a)\}, \quad z>a. \qquad (45)$$

Assuming that the field incident from the vacuum is of the form $E = E_0(e^{ikz} + Pe^{-ikz})$, and making the fields continuous at the point $z = 0$ and $z = a$ with the aid of boundary conditions (18) and (19), we obtain for the reflection coefficient P and for the other constants the following expressions:

$$P=\frac{(1-\sqrt{\varepsilon_2})(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})+(1+\sqrt{\varepsilon_2})(\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1})\exp\{2ika\sqrt{\varepsilon_2}\}}{(1+\sqrt{\varepsilon_2})(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})+(1-\sqrt{\varepsilon_2})(\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1})\exp\{2ika\sqrt{\varepsilon_2}\}},$$

$$P_1=\frac{\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1}}\exp\{2ika\sqrt{\varepsilon_2}\},$$

$$u_1=\frac{2E_0(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})}{(1+\sqrt{\varepsilon_2})(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})+(\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1})(1-\sqrt{\varepsilon_2})\exp\{2ika\sqrt{\varepsilon_2}\}}$$

$$u_2=\frac{4E_0\sqrt{\varepsilon_2}\exp\{ika\sqrt{\varepsilon_2}\}}{(1+\sqrt{\varepsilon_2})(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})+(\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1})(1-\sqrt{\varepsilon_2})\exp\{2ika\sqrt{\varepsilon_2}\}}. \qquad (46)$$

The value of a must be determined from the condition $|E(a)| = |u_2| = u_a$. Alternately, using (46), we get

$$\left|\frac{4E_0\sqrt{\varepsilon_2}\exp\{ika\sqrt{\varepsilon_2}\}}{(1+\sqrt{\varepsilon_2})(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})+(\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1})(1-\sqrt{\varepsilon_2})\exp\{2ika\sqrt{\varepsilon_2}\}}\right|=u_a. \qquad (47)$$

It will be convenient in what follows to introduce the notation $\sqrt{\varepsilon_1} = n_1 + i\kappa_1$ and $\sqrt{\varepsilon_2} = n_2 + i\kappa_2$. If $n_1 = n_2 = n$ and $\kappa_{1,2} \sim \alpha n$, that is, the attenuation is weak, then formula (46) follows from (36). On the other hand, if the attenuation κ is of the same order as n, and $ka\kappa_2 > 1$, then we can neglect in (47) the term with the exponential in the denominator, and we get for the determination of a

$$e^{-k\varkappa_2 a}=\frac{u_a}{4E_0}F(\varkappa_1,\varkappa_2,n_1,n_2), \qquad (48)$$

where

$$F(\varkappa_1,\varkappa_2,n_1,n_2)=\left[\frac{[(n_1+n_2)^2+(\varkappa_1+\varkappa_2)^2][(1+n^2)^2+\varkappa_2^2]}{n_2^2+\varkappa_2^2}\right]^{1/2}$$

Expanding (46) in terms of $\exp(-2ik\kappa_2 a)$, we get for the reflection coefficient P

$$P=\frac{1-\sqrt{\varepsilon_2}}{1+\sqrt{\varepsilon_2}}+\frac{4(\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1})\sqrt{\varepsilon_2}}{(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})(1+\sqrt{\varepsilon_2})^2}e^{-2k\varkappa_2 a}e^{2ikn_2 a}. \qquad (49)$$

Substituting a from (48) in (49), we get finally

$$P=\frac{1-\sqrt{\varepsilon_2}}{1+\sqrt{\varepsilon_2}}+\frac{u_a^2}{4E_0^2}\frac{\sqrt{\varepsilon_2}(\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1})}{(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})(1+\sqrt{\varepsilon_2})^2}F^2(\varkappa_1,\varkappa_2,n_1,n_2)$$
$$\times\exp\left\{-\frac{2in_2}{\varkappa_2}\ln\frac{u_a}{4E_0}F(\varkappa_1,\varkappa_2,n_1,n_2)\right\}. \qquad (50)$$

The period of the oscillations δE_0 is given by the formula

$$\delta E_0=E_0(e^{\pi\varkappa_2/n_2}-1). \qquad (51)$$

when $\kappa_2 \sim n_2$ the value of δE_0 is of the same order as or larger than E_0, and is likewise independent of u_a. In this case the amplitude of the oscillating term decreases rapidly when the field increases by an amount on the order of the period of the oscillations, unlike in (36), where the amplitude, by virtue of main assumptions, remains unchanged. Under different conditions imposed on n_1, n_2, κ_1, and κ_2, Eq. (46) can be investigated analogously.

It is obvious that in order for the effect of the oscillation of the reflection coefficient P as a function of E_0 to take place it is necessary that the wave be damped, and also that $u_0 > u_a$. Indeed, if the amplitude of the incident wave E_0 is such that $u_0 < u_a$, then the amplitude of the field at any point in the half-space corresponds to the upper branch of the temperature and there is no discontinuity of v anywhere. On the other hand, if $u_0 > u_a$, then there is no damping and the field amplitude deep within the sample is $u \equiv u_0$ and never reaches the value u_a, that is, there will be no transition to the lower branch. This is seen also from the equation. Putting $\kappa_1 = \kappa_2 = 0$, we get from (47) the following equation for the determination of a:

$$\cos 2kn_2a=\frac{16E_0^2n_2^2u_a^{-2}-(1+n_2)^2(n_2+n_1)^2-(n_2-n_1)^2(1-n_2)^2}{2(1+n_2)(n_2+n_1)(n_2-n_1)(1-n_2)}. \qquad (52)$$

This equation has a solution if the right side of the equation is smaller than unity, that is, if the inequality $E_0/u_a < (1 + n_1)/2$ is satisfied. It can be usually shown, however, that to have $u_0 > u_a$ it is necessary to satisfy the inequality $E_0/u_a > (1 + n_1)/2$. Inasmuch as the last two inequalities are incompatible, it is impossible to determine a from (52), since the quantity a itself is meaningless in this case, that is, there is no effect of the oscillations. We note that it is possible to get rid of some of the limitations imposed above.

In the beginning of the paper we assumed that the electromagnetic waves propagate along the magnetic field. The character of the calculation shows that this limitation is not essential. The results will be the same for an arbitrary mutual orientation of the magnetic field and the electromagnetic wave propagation direction.

Besides the reflection coefficient, it is of interest to investigate also the thermal emf and other thermomagnetic characteristics connected with the ambiguity of the electron temperature. It is obvious that by virtue of the fact that the temperature at the point $z = a$ is discontinuous, the thermomagnetic characteristics are likewise discontinuous. This circumstance makes it possible to determine a by direct experiment. We note that, generally speaking, owing to the finite fluctuation, the discontinuity from the upper branch to the lower one can occur at any point in the interval $u_a \le u \le u_b$. The entire analysis presented above remains in force here if we take u_a to mean the field amplitude field at which the temperature of the electron gas goes over from the upper branch to the lower one. The discontinuity point itself should be determined by a theory that takes into account the transient of the electron temperature in the sample. However, in view of the fact that the period of the oscillations of the temperature with the field (see formulas (44) and (51)) does not depend on the field at the point where the temperature goes from one branch to the other, the main features of the effect remains unchanged regardless of where the discontinuity takes place. The quantity u_a can be determined by measuring the amplitude of the oscillations of the reflection coefficient.

All the obtained results can be transferred without modification to the case of a plasma.

Let us estimate now the semiconductor and the plasma parameters for which the foregoing assumptions are valid. Some of the results of the present article were obtained under the assumption that the nonlinearity in the wave equation is small. Let us indicate when this takes place.

We stop first to discuss formulas (14) and (15). In order for the nonlinearity in case II (formula (14)) to be small, it is necessary to satisfy the inequality $\omega_0^2/\omega_H \nu \ll 1$, from which we get the following upper bound for the concentration: $N \ll \nu H/4\pi ec$. On the other hand, the concentration is bounded from below by the condition for the establishment of the electron temperature, which takes the form[5]

$$N > \frac{1}{4\pi} \frac{T^{1/2} v^{3/2} m^{3/2} s^2 \nu_a}{10 e^4} \tag{53}$$

(here s is the speed of sound and ν_a the frequency of collisions with the acoustic phonons). Putting $m = 10^{-28}$ g, $s = 10^5$ cm/sec, $\nu_a = 10^{11}$ sec^{-1}, $T = 10^{-15}$ erg, $H = 10^2$ Oe, and $v = 10$, we find that in order to satisfy both inequalities N must lie in the range $10^8 \ll N \ll 10^{11}$. In addition, in order for a stationary temperature to be established, it is necessary to satisfy the inequality $\omega_H = \omega \ll \nu_{ee}$, where ν_{ee} is the frequency of the interelectron collisions, given by the formula[5]

$$\nu_{ee} \sim 40\pi e^4 N / \sqrt{2} m^{1/2} T^{3/2} v^{3/2}. \tag{54}$$

At the assumed values of the parameters $\nu_{ee} = 10^{10}$ sec^{-1} and $\omega_H = 10^{10}$ sec^{-1}, so that this inequality is also satisfied.

In the case III (formula (15)) we should have the limitation $\omega_0^2/\omega\nu \ll 1$, hence $\omega \gg \omega_0^2/\nu$ and $\omega \ll \nu_{ee}$. Putting $N = 10^{13}$ cm^{-3}, $m = 10^{-27}$ g, $T = 10^{-15}$ erg, $v = 10$, and $\nu = 10^{12}$ sec^{-1}, we find that ω should lie in the interval $10^{10} \ll \omega \ll 10^{12}$.

For a plasma, let us consider the case when $\omega_H = 0$ and the plasma is fully ionized, that is, $\nu = \nu_{ei} \sim \nu_{ee}$ (ν_{ei} denotes the frequency of electron-ion collisions). For a fully ionized plasma, a temperature can always be introduced.[6] Putting $\nu_{ee} = 10^6$ sec^{-1}, $\omega = 10^5$ sec^{-1}, and $N = 10^9$ cm^{-3}, we get $\omega_0^2/\nu_{ee}\omega \approx 3 \times 10^7$. This means that the nonlinearity in the plasma is large and we must use formulas (46)–(51).

Let us obtain now estimates for the relative period of the oscillations $\delta E/E_0$. We start from the formula (51) and consider two cases.

1) $\pi\kappa_2/n_2 \ll 1$. This case corresponds to small nonlinearity and is realized in semiconductors. In this case it follows from (51) that $\delta E/E_0 = \pi\kappa_2/n_2$. Determining n and κ from formula (11) for $\delta E/E_0$, we obtain ultimately

$$\frac{\delta E}{E_0} = \frac{\pi\alpha\Phi_1}{2n^2} \tag{55}$$

Let us use formula (13) and assume that ε_0 is small compared with $\omega_0^2/\omega(\omega - \omega_H)$. For case I we have the formula

$$\frac{\delta E}{E_0} \sim \frac{\nu(v)}{\omega_H - \omega} \ll 1. \tag{56}$$

In case II (see formulas (14))

$$\frac{\delta E}{E_0} = \frac{2\pi^{1/2}}{3} \frac{\omega_0^2}{\omega_H \nu(v) \varepsilon_0}. \tag{57}$$

When $N \sim 10^{10}$ cm^{-3} and $H \sim 10^2$ Oe, we get $\delta E/E_0 \sim 10^{-2}$. Similarly, for the case III (formulas (15))

$$\frac{\delta E}{E_0} = \frac{4\pi^{1/2}}{3} \frac{\omega_0^2}{\omega \nu(v) \varepsilon_0}. \tag{58}$$

Taking $\omega = 10^{11}$ sec^{-1}, $\nu = 10^{12}$ sec^{-1}, and $\omega_0^2 = 10^{22}$ sec^{-1}, we get from (58) $\delta E/E_0 \sim 10^{-2}$.

2) $\kappa_2 \sim n_2$. This corresponds to plasma and cyclotron resonance at large carrier densities and to a clearly pronounced skin effect in the solid. The real part of ε is in this case much smaller than the imaginary part. Under these assumptions, we get from (51) the relation

$$\delta E / E_0 \sim e^{\pi} - 1 \sim 22. \tag{59}$$

In conclusion we indicate that the present analysis, generally speaking, does not exhaust all the possible stationary regimes and yields no information on the stability of the obtained solutions.

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