DIFFUSION-DISLOCATION MECHANISM OF CRYSTAL FLOW WITH ALLOWANCE FOR ELASTIC INTERACTION BETWEEN DISLOCATION LOOPS

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The effect of elastic interaction between dislocation loops on the plastic flow velocity is taken into account. It is shown that in this case the plastic flow depends on the square of the load.

In an earlier paper[1] (henceforth cited as I) we considered a diffusion-dislocation mechanism of crystal flow, in which no account was taken of the elastic interaction between dislocation loops. If the plastic flow is sufficiently well developed, this interaction becomes significant under certain conditions.[1)] Then the dislocation loops no longer grow during the course of plastic flow to dimensions equal to those of the crystal, but to dimensions determined by this interaction of the dislocation loops with one another. Thus, in the kinetic equation derived in I it is necessary to take into account this interaction of the dislocation loops, which leads to the cessation of their diffusion growth. This signifies that segments of the dislocation loop will have neighbors in front of them and the interaction with the neighbors will stop the diffusion growth of the loop.

In an isotropic crystal, when the dislocation loops can grow in all planes, this will be the main mechanism limiting the growth of the loops.

In an anisotropic crystal, when the dislocation loops can grow only in a small number of selected planes, the principal role is assumed by the diffusion interaction of the loops situated in parallel planes. In this case the interaction of the loops not only stops their diffusion growth, but also leads to the appearance of neighbors with opposite Burgers vectors in the same plane. Indeed, if a certain loop has been stopped by interaction with loops in a parallel plane, then in the course of time the sources operating in this plane will be displaced and will emit loops situated in the same plane. The main qualitative result in all these cases will be the appearance of a certain average dimension, up to which the loops grow during the course of their evolution; this dimension is determined by the number of sources per unit volume or plane and by the distance at which the loops interact. The possibility of dislocation gliding under the influence of an external load will be neglected throughout.

Let us consider the case of an isotropic crystal and let us supplement the kinetic equation derived in I by a term that takes into account the possibility of stopping the loop as a result of its interaction with other loops. This term is the analog of the collision integral and is a functional of the distribution function of the moving loops $f(R, t, \varphi)$. We denoted by $J\{f\}$.

Thus, the kinetic equation describing the interstitial vacancies or vacancy loops can now be written in the form

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial R}=v\frac{\rho(R,\varphi)}{R}\theta(R-R_{cr}(\varphi))-J\{f\},\ \theta(x)=\begin{cases}1, & x>0\\ 0, & x<0\end{cases}. \quad (1)$$

The notation here is the same as in I: $f = f^i$ or f^v is the distribution function of the interstitial or vacancy loops, respectively, and $\rho(R, \varphi)$ is the volume density of the sources of dislocations with dimensions R and Burgers-vector orientation φ (for example, dislocation segments), $R_{cr}(\varphi)$—dimension beyond which a loop with orientation φ can grow (see I), v—diffusion rate of growth of the interstitial or vacancy loops,

$$R^v_{cr}(\varphi)=a\frac{a^3G}{4\pi(1-\nu)kT}\left\{\ln\frac{R}{r_0}+\beta\right\}\frac{1}{\Delta^*-\varkappa\cos^2\varphi},$$

$$R^i_{cr}(\varphi)=a\frac{a^3G}{4\pi(1-\nu)kT}\left\{\ln\frac{R}{r_0}+\beta\right\}\frac{1}{-\Delta^*+\varkappa\cos^2\varphi},$$

$$v^v=a^2D^vA(R)(\Delta^*-\varkappa\cos^2\varphi),\quad v^i=a^2D^iA(R)(-\Delta^*+\varkappa\cos^2\varphi),$$

$$D^*=C_0^vD^v+C_0^iD^i,\quad \varkappa=\frac{a^3\sigma}{kT},\quad \Delta^*=\frac{C_0^v\Delta^vD^v-C_0^i\Delta^iD^i}{D^*},$$

$$A(R)=2\pi/a^3\ln\frac{8R}{a},$$

where C_0^v, C_0^i, D^v, D^i, Δ^v, and Δ^m are respectively the equilibrium concentration, the diffusion coefficient, and the supersaturation of the vacancies and of the interstitial atoms, R is the radius of the dislocation loops, a is the lattice constant, $r_0 \cong a$, σ is the external load ($\sigma = \sigma_{zz}$), the crystal is assumed isotropic, G is the shear modulus, ν is the Poisson coefficient, $\kappa > 0$ corresponds to tension, $\kappa < 0$ to compression, and φ characterizes the angle between the direction of the Burgers vector of the dislocation loops and the load direction. The range of the angles φ for the vacancy and interstitial loops is limited to the definite regions that defined, as is well known, by the conditions $v^v > 0$ and by v^i (see I). This means that we take into account growing loops. Δ^* is a constant determined by the total balance of matter.

Let us calculate now the distances $h(\varphi)$ at which loops with a Burgers vector, making an angle φ with respect to the external load, interact strongly with the nearest loops.

1)This circumstance was pointed out to us by A. L. Roitburd.

Since the work needed to produce a point defect depends on the average stress field, which is determined by all the dislocations, this field can be included in the definition of the equilibrium concentrations of the point defects. This means that to calculate $h(\varphi)$ we need to take into account only the "close collisions," that is, the interaction of the loop with the nearest one. The quantity $h(\varphi)$ is determined from the condition that the rate of diffusion growth of the loop by interaction with the neighboring loop vanish. For the vacancy and interstitial loops we have respectively

$$\Delta^* - \varkappa\cos^2\varphi - \frac{b_l\bar{\sigma}_{lm}b_m}{kTb^2}a^3 = 0, \quad -\Delta^* + \varkappa\cos^2\varphi - \frac{b_l\bar{\sigma}_{lm}b_m}{kTb^2}a^3 = 0. \quad (2)$$

For the vacancy loops $\mathbf{b} = -b\mathbf{n}$ ($\mathbf{b}$—Burgers vector). For the interstitial loops $\mathbf{b} = b\mathbf{n}$ ($\mathbf{n}$—vector normal to the dislocation loop, defined in the circuiting direction). The bar denotes averaging over the positions of the neighboring loop, and $\bar{\sigma}_{lm}$ is the stress field produced by the nearest dislocation loop. Since the curvature of the dislocation loops is on the average sufficiently large for a developed plastic flow, $\bar{R} \gg h(\varphi)$, the stress field produced by it will differ little at a distance $h(\varphi)$ from the stress field of a linear dislocation.

The greatest contribution to the deceleration of a given loop is made by like loops moving head on towards another source, whose axes are almost parallel. As a result we obtain

$$h^v(\varphi) = \frac{h_0}{\Delta^* - \varkappa\cos^2\varphi}, \quad h^i(\varphi) = \frac{h_0}{-\Delta^* + \varkappa\cos^2\varphi},$$
$$h_0 = b\frac{a^3G}{2\pi(1-\nu)kT}p, \quad (3)$$

where p is a number of the order of unity.

We note also that the dislocation loops emitted by one source move with equal velocity. This means that the number of loops per unit volume, with which a given loop can interact, is equal to the number of sources of a given type:

$$n_\varphi = \int_{R_{cr}(\varphi)}^{R_0} \rho(z,\varphi)\,dz,$$

(R_0—dimension of the crystal). If $\rho(z, \varphi) = \rho(z)$ is a sharp function near a certain value $z = l$, which is a fairly natural assumption, we can assume approximately that $n_\varphi = n = \text{const}$.

We can now obtain the average loop dimension $\bar{R}(\varphi)$, beyond which a loop with a given orientation grows. To this end we note the volume allotted to each loop after it is out of the plastic-flow process is determined by the number of sources with which the given loop can interact. From this we get

$$V = \frac{1}{n} = \pi\bar{R}^2(\varphi)h(\varphi), \quad \bar{R}^2(\varphi) = \frac{1}{\pi n h(\varphi)}. \quad (4)$$

We shall see subsequently that segments that interact elastically most strongly with one another have almost opposite Burgers vectors and are dislocations of the same type. The elastic field of such a complex decreases like a dipole field. Thus, these complexes, which accumulate in time, will not interfere appreciably with the operation of the sources of the dislocation loops, all the more since each succeeding dislocation loop emitted by the source moves in a succeeding atomic plane. In addition, in our problem the distribution function is encountered only under the integral sign, therefore a detailed knowledge of this function is not so important, especially since it will differ little from the distribution obtained in I in the hydrodynamic approximation in the main interval of its variation $R/\bar{R} \approx 1$, where the "collisions" have still little effect.

Consequently, we can replace the "collision integral" (1) by an effective boundary condition. This reduces to a situation wherein the upper limit in all the integrations with respect to R is not R_0 but $\bar{R}(\varphi)$. It is clear that the interaction of the dislocation loops will be appreciable if $\bar{R}(\varphi) \ll R_0$ ($\bar{R}(\varphi)$—value of $\bar{R}(\varphi)$ averaged over the angle φ, and R_0 is the dimension of the crystal).

The function $F(\varphi) = 2\pi\int_0^{R_0} f(R, \varphi)RdR$ introduced in I will now take the form ($t \gg \bar{R}/v$)

$$F(\varphi) = \pi\bar{R}^2(\varphi)\int_{R_{cr}(\varphi)}^{\bar{R}(\varphi)} \frac{\rho(z)}{z}\,dz. \quad (5)$$

The equation of material balance, which determines the range of angles in which the dislocation loops can grow, will now have a somewhat different form as compared with formula (22) of I:

$$\int_{v^v(\varphi)>0} \bar{R}_v^2(\varphi)v^v(\varphi)F^v(\varphi)\sin\varphi\,d\varphi = \int_{v^i(\varphi)>0} \bar{R}_i^2(\varphi)v^i(\varphi)F^i(\varphi)\sin\varphi\,d\varphi. \quad (6)$$

If we introduce the notation used in I, namely $\cos\varphi = u$, $\Delta^* = u_0^2\kappa$, and $0 < u_0^2 < 1$, then (6) takes the form

$$\int_0^{u_0} (u_0^2 - u^2)^2\psi(u_0^2 - u^2)\,du = \int_{u_0}^1 (u^2 - u_0^2)^2\psi(u^2 - u_0^2)\,du,$$
$$\psi(u_0^2 - u^2) = \int_{R_{cr}(\varphi)}^{\bar{R}(\varphi)} \frac{\rho(z)}{z}\,dz. \quad (7)$$

Equation (7) differs from formula (24) of I in that the factor $u^2 - u_0^2$ under the integral sign goes over into $(u^2 - u_0^2)^2$.

Assuming that $a/l \gg \sigma/G$ (l—average dimension of the created loops), $\psi(u_0^2 - u^2)$ will depend little on its argument, for under this condition we have approximately

$$\int_{R_{cr}(\varphi)}^{\bar{R}(\varphi)} \frac{\rho(z)}{z}\,dz \cong \frac{1}{l}\int_0^\infty \rho(z)\,dz$$

($\rho(z)$ is a sharp function near $z = l$ and $\rho(0) = \rho(\infty) = 0$).

If $a/l \gg \sigma/G$, then $\dot{\varepsilon}_{ZZ} = 0$, for otherwise the sources cannot operate (see I). When $a/l \ll \sigma/G$, Eq. (7) simplifies to

$$\int_0^{u_0} (u_0^2 - u^2)^2\,du = \int_{u_0}^1 (u^2 - u_0^2)^2\,du.$$

Its approximate solution is $u_0^2 = 0.3$.

We now express the rate of plastic flow in the same approximation, by substituting in the exact formula (28) of I the altered values of $F^v(\varphi)$ and $F^i(\varphi)$:

$$\dot{\varepsilon}_{zz} = 2a\left\{-\int_{\varphi_0}^{\pi/2} v^v(\varphi)F^v(\varphi)\cos^2\varphi\sin\varphi\,d\varphi + \int_0^{\varphi_0} v^i(\varphi)F^i(\varphi)\cos^2\varphi\sin\varphi\,d\varphi\right\}$$

$$= 2\pi\left(\ln\frac{8\bar{R}}{a}\pi h_0\int_0^\infty \rho(z)\,dz\right)^{-1} D^*\varkappa\left\{-\int_0^\infty\frac{\rho(z)}{z}dz\int_0^{u_0}u^2(u^2-u_0^2)^2\,du\right.$$
$$\left.+\int_0^\infty\frac{\rho(z)}{z}dz\int_{u_0}^1 u^2(u^2-u_0^2)^2\,du\right\}=\frac{M}{\ln(8\bar{R}/a)}\frac{D^*}{n_0 l}\left(\frac{\sigma a^3}{kT}\right)^2, \quad (8)$$
$$\bar{R}=\left[\frac{\sigma a^3}{kT}\Big/\pi h_0\int_0^\infty\rho(z)\,dz\right]^{1/3}, \quad M\cong\frac{4\pi}{7}.$$

Thus, allowance for the interaction reduces to replacing R_0 by $\bar{R}$ with some numerical coefficient in formula (32) of I.

The applicability of (8) is determined by the obvious relation

$$\bar{R}=[\varkappa/\pi h_0 n]^{1/2}\ll R_0,$$

and can be rewritten as follows:

$$\sigma/G\ll R_0^2 na/2(1-\nu). \quad (9)$$

In the opposite case, the rate of plastic flow $\dot{\varepsilon}_{zz}$ is determined by formula (32) of I. We note that if the sources of the dislocation loops are the block boundaries, then the number of sources of the loops $n = 6k\theta/l^2a$ (θ—angle of block disorientation, l—linear dimension, the factor k is the relative number of dislocations on the block boundary, which can serve as sources of dislocation loops), and the inequality (9) takes the form $\sigma/G \ll \theta k R_0^2/l^2$. If $\sigma/G \gg \theta k R_0^2/l^2$ then, substituting this value of n in formula (32) of I, we get

$$\dot{\varepsilon}_{zz}=\frac{24\pi}{45}\frac{k}{\ln(8R_0/a)}D^*\frac{R_0^2}{l^3a}\theta\frac{\sigma a^3}{kT}. \quad (10)$$

We note that if the dislocation loops require, for some reason, a greater activation energy in order to grow through the block boundaries, then R_0 must be replaced by l. In this case the dependence of $\dot{\varepsilon}_{zz}$ on σ changes in the region $\sigma \approx G\theta$ (a transition takes place from a quadratic to a linear dependence).

Generally speaking, dislocation loops can grow in real crystals only in selected directions. This can lead to an appreciable quantitative change in the expressions derived above, but only in the case if the number of these selected directions is small. Allowance for these circumstances reduces to the substitution $2\pi\int\ldots\sin\varphi d\varphi\to\sum_i$, where $\sum_i$ denotes summation over all possible dislocation-loop growth directions determined by the same conditions as obtained above.

In conclusion we note that if we consider an anisotropic crystal we obtain for homogeneous external stresses a homogeneous elastic state regardless of the anisotropy of the elastic constants. The anisotropy of the crystal comes into play only when we try to determine the elastic energy connected with the dislocation loop (this determines $R_{cr}(\varphi)$), but since the loops are sufficiently large, automatic averaging takes place over the arrangement of the dislocation axis relative to the g crystallographic axes; we thus obtain a certain average elastic energy per unit length of dislocation. The diffusion fluxes per unit length of dislocation loop are also averaged for the same reason. This means that in such a crystal plastic-flow mechanism it is always possible to characterize an anisotropic crystal by means of certain average elastic constants of the isotropy crystal, and the loops can grow only in definite planes.

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[1] A. M. Kosevich, Z. K. Saralidze, and V. V. Slezov, Zh. Eksp. Teor. Fiz. **50**, 958 (1966) [Sov. Phys.-JETP **23**, 636 (1966)].

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