HELICOIDAL INSTABILITY IN METALS AND SEMICONDUCTORS

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The existence of helicoidal instability of sound oscillations in conducting solids is predicted. The instability is due to a Lorentz force in the presence of a stationary current and does not depend on the condition that the drift velocity be greater than the velocity of sound. The increment and amplitude of the stationary waves under nonlinear conditions are found.

1. IT is well known that weakly damped helical magnetic waves (helicons) exist in conducting solids placed in a strong magnetic field. In conductors having one group of carriers, amplification of these waves in an external electric field is impossible. Indeed, the drift motion of the electrons with constant velocity v_0 can be eliminated by going over to another reference frame, and therefore the electron drift leads only to a Doppler shift of the helicon $\omega' = \omega - kv_0$ (ω—frequency in the lab system, ω'—frequency in the moving reference frame, k—helicon wave number). The instability arises only when the carrier drift cannot be excluded by transforming the coordinate frame. In particular, amplification takes place in the presence of two groups of carriers with different mobility, or else when account is taken of the coupling of the electromagnetic wave with the lattice vibrations (sound). In the latter case there is a preferred reference frame connected with the lattice. The instability of the electromagnetic wave in a conductor with two types of carrier was investigated theoretically by Veselago, Glushkov, and Rukhadze.[1] It was observed experimentally in bismuth by Bartelink.[2]

The present paper is devoted to an investigation of the character of the instability of unstable coupled electromagnetic and sound waves in an isotropic metal or a semiconductor with one group of electrons, and to a determination of the amplitude of the stationary sound oscillations in the nonlinear mode.

2. Let us consider the propagation of transverse sound and electromagnetic waves along a magnetic field $\mathbf{H}_0 \parallel Oz$. Let the constant electric field $\mathbf{E}_0$ be parallel to the vector $\mathbf{H}_0$. In the region of low frequencies and a strong magnetic field

$$\omega, kv_0 \ll 1/\tau \ll \Omega \tag{1}$$

it is possible to neglect in the electron equations of motion the inertial force m dv/dt (v—velocity, m—effective mass, Ω—cyclotron frequency, and τ—mean free path time of the electrons). The complete system of equations takes the following form:

$$c\,\mathrm{rot}\,\mathbf{E} = -\dot{\mathbf{H}}, \qquad c\,\mathrm{rot}\,\mathbf{H} = 4\pi ne(\dot{\mathbf{u}} - \mathbf{v}_\perp), \tag{2}$$

$$\frac{e}{m}\left(\mathbf{E}_0 + \mathbf{E} + \frac{1}{c}[\mathbf{v}, \mathbf{H}_0 + \mathbf{H}]\right) + \frac{\mathbf{v} - \dot{\mathbf{u}}}{\tau} = 0, \tag{3*}$$

$$M(\ddot{\mathbf{u}} - s^2\Delta\mathbf{u}) = e\left(\mathbf{E} + \frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}_0]\right) - \frac{m}{\tau}(\dot{\mathbf{u}} - \mathbf{v}_\perp). \tag{4}$$

*$[\mathbf{v}, \mathbf{H}_0 + \mathbf{H}] \equiv \mathbf{v} \times (\mathbf{H}_0 + \mathbf{H})$.

Here E and H are the alternating electric and magnetic fields, u the transverse lattice-displacement vector, s the speed of transverse sound, M the mass of the ion with charge e, n the equilibrium energy density, and the dot denotes partial differentiation with respect to the time. The dissipative terms in the equations of motion (2) and (4) take into account the conservation of the momentum in collisions between the electrons and the lattice (ions).

In the approximation linear in the alternating field, it is easy to obtain a dispersion equation that determines the spectrum and the damping (growth) of the coupled waves:

$$\omega^2 - k^2s^2 = -\frac{m}{M}\omega\Omega\frac{\pm kv_0 - \Omega(kc/\omega_0)^2}{\omega - kv_0 + \Omega(kc/\omega_0)^2(\pm 1 + i\Gamma)} \tag{5}$$

where $\omega_0 = (4\pi ne^2/m)^{1/2}$ is the plasma frequency, $\Gamma = (\Omega\tau)^{-1}$ is the relative damping of the helicon, $v_0 = eE_0\tau/m$ the electron drift velocity in a constant electric field E_0, and $j_0 = -ne\tau E_0/m$ is the constant current. The upper sign pertains to a circularly polarized wave with $E_x = iE_y$ ("plus" polarization), and the lower sign to a wave with $E_x = -iE_y$ ("minus" polarization). It is obvious from (5) that waves with "minus" polarization are damped for all values of v_0.[1)] Instability sets in only for waves with "plus" polarization.

Let us investigate the character of the instability in the region of small wave numbers

$$ks \ll \omega, \qquad |kv_0 - \Omega(kc/\omega_0)^2| \ll \omega. \tag{6}$$

From the dispersion equation (5) we get

$$\omega = \pm i\gamma, \qquad \gamma = (m\Omega/M)^{1/2}[kv_0 - \Omega(kc/\omega_0)^2]^{1/2}. \tag{7}$$

Consequently, when $k < k_0 \equiv \omega_0^2 v_0/\Omega c^2$ one of the "waves" increases exponentially in time. The real part of the frequency ω is determined by the small dissipative terms of (5), which contain Γ. It is obvious that this instability is of the relaxation type ($\mathrm{Re}\,\omega \ll \gamma$) and has an absolute character.[3]

In order to understand the nature of this instability, let us consider the limiting case $k \ll k_0$. The system (2)–(4) is equivalent in this case to the system

1)If we put $M = \infty$ and neglect the interaction between the sound and the electromagnetic waves, then the dispersion equation for the helicons in the moving system, $\omega' = (kc/\omega_0)^2\Omega(\mp 1 - i\Gamma)$ has the same form as in the absence of drift.

$$c\,\mathrm{rot}\,\mathbf{E} = -\dot{\mathbf{H}}, \qquad \mathbf{u} = \mathbf{v}_\perp, \tag{2'}$$

$$c\mathbf{E} + [\mathbf{v}, \mathbf{H}_0 + \mathbf{H}] = 0, \qquad \mathbf{j}_0 = -ne\mathbf{v}_0, \tag{3'}$$

$$nM\ddot{\mathbf{u}} = c^{-1}[\mathbf{j}_0\mathbf{H}]. \tag{4'}$$

The growth of the amplitude of the oscillations is due to the Lorentz force $c^{-1}\mathbf{j}_0 \times \mathbf{H}$ in (4′), which results from the electron drift. The alternating magnetic field $\mathbf{H}$ produces, in the presence of the direct current $\mathbf{j}_0$, a transverse displacement of the ions, which leads in turn to an increase of the magnetic field. The amplitude of the circularly polarized oscillations increases, forming a peculiar helical structure. Therefore we shall call an instability of this type helicoidal, in analogy with the helicoidal plasma instability considered in [4]. This instability should obviously take place also in an ordinary electron-ion plasma ($s = 0$).

It must be noted that the elastic force $Ms^2\Delta\mathbf{u}$ and the Lorentz force $c^{-1}\mathbf{j} \times \mathbf{H}_0$ stabilize the helicoidal instability. If the inequalities (6) are reversed, there is no growth of the oscillations.

When $v_0 > s$ an oscillatory instability sets in, with $\mathrm{Re}\,\omega \neq 0$, connected with the possibility of resonant interaction between the sound and helical waves. According to [5], resonance occurs when

$$k = k_r \equiv k_0(1 - s/v_0), \qquad k_0 = \omega_0^2 v_0/\Omega c^2. \tag{8}$$

Near resonance we have

$$\omega = k_r s \pm i(m\Omega k_r s/M)^{1/2}. \tag{9}$$

It is easy to show that in this case the instability is convective.

The helicoidal instability of the relaxation (7) and oscillatory type (9) is not connected with dissipative effects, since it exists also if the quantity $i\Gamma$ is neglected in the dispersion equation (5). If account is taken in (5) of the electron collisions, then the oscillations whose wave numbers and frequencies satisfy the inequalities (see the figure)

$$\omega/\mathrm{Max}(s, v_0) < k \leqslant k_0. \tag{10}$$

will also grow. Far from resonance, the change in the spectrum and the growth increment of the sound wave are described by the formula

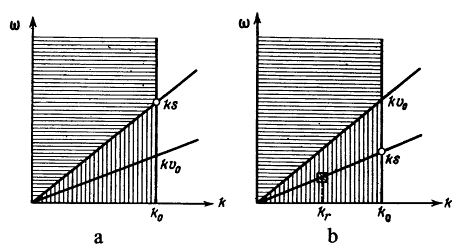
$$\delta\omega \equiv \omega - ks = -\frac{m\Omega}{2M}\frac{k_0 - k}{k_0(s/v_0 - 1) + k(1 + i\Gamma)}. \tag{11}$$

In this case the growth of the oscillations is due to collisions between the electrons and the lattice: $\mathrm{Im}\,\delta\omega \sim \Gamma$. We shall call this a dissipative helicoidal instability. The figure shows the boundaries of the instability regions.

It must be emphasized that the occurrence of the helicoidal instability is not connected with any definite relations between the drift velocity v_0 and the phase velocity of the wave. In the case considered by us the instability condition

$$k_0 > k \text{ for } v_0 > k\Omega c^2/\omega_0^2 \tag{12}$$

corresponds to the requirement that the Lorentz force $c^{-1}\mathbf{j}_0 \times \mathbf{H}$ be larger than the force $c^{-1}\mathbf{j} \times \mathbf{H}_0$. It can therefore be observed not only in semiconductors, but also in metals in which it is impossible to create a large drift velocity.[3]



Regions of helicoidal instability (horizontal shading – without allowance for dissipation, vertical – with allowance for dissipative instability). $k_0 = \omega_0^2 v_0/\Omega c^2$; a) $v_0 < s$; b) $v_0 > s$, $k_r = k_0(1 - s/v_0)$. The horizontally shaded region near $k_r s$ corresponds to resonance.

3. When the amplitude of the wave increases strongly, an important role is assumed by nonlinear effects that limit further growth of the oscillations. We are interested in the question of establishment of a stationary oscillatory mode, when a plane monochromatic wave of constant amplitude propagates in the conductor. The main nonlinear mechanism determining the stationary state is the nonlinear part of the Lorentz force $ec^{-1}\mathbf{v} \times \mathbf{H}$ in the equation of motion of the conduction electrons (3). Since the wave is transverse and circularly polarized, the nonlinear interaction does not lead to the appearance of higher harmonics. All that changes is the average drift velocity V. Indeed, from Eq. (3) for the z-projection of the electron velocity it follows that

$$V = v_0 - \Delta v, \qquad \Delta v = cH_0^{-2}[\mathbf{EH}]_z. \tag{13}$$

The quantity Δv represents the change of the drift velocity of the electron as a result of the alternating electric and magnetic fields. Using Maxwell's equations (2) we obtain

$$\Delta v = \mathrm{Re}\frac{k}{\omega}\frac{c^2}{H_0^2}(E_x^2 + E_y^2). \tag{14}$$

In the case of a circularly polarized stationary wave, $E_x = A\cos(kz - \omega t)$. $E_y = A(\sin kz - \omega t)$, the nonlinear "renormalization" of the drift velocity Δv depends only on the amplitude A of the oscillations. In the stationary state, the growth increment vanishes. Replacing in the dispersion equation (5) the drift velocity v_0 by the "renormalized" velocity V and equating the growth increment (11) to zero, we obtain an equation for the stationary amplitude:

$$V = \Omega k c^2/\omega_0^2. \tag{15}$$

From this condition and Eq. (5) it is obvious that the stationary wave represents transverse sound with a dispersion law $\omega = ks$ and with an amplitude

$$|u_0| = \frac{c}{\omega}\frac{A_0}{H_0} = \frac{1}{k}\left(\frac{v_0}{s} - \frac{\Omega c^2 k}{\omega_0^2 s}\right)^{1/2}. \tag{16}$$

Let us consider the evolution in time of an initial perturbation with a given wave number $k \ll k_0$. At first the amplitude increases rapidly with an increment $\gamma = (m\Omega k v_0/M)^{1/2}$, and the drift velocity V decreases (see (14) and (15)). Accordingly, a decrease takes place in the nonlinear increment

$$\gamma_A = \left(\frac{m}{M}\Omega\right)^{1/2}\left[kV - \left(\frac{kc}{\omega_0}\right)^2\Omega\right]^{1/2}.$$

The growth will continue until γ_A vanishes. When $\gamma_A = 0$ there propagates in the conductor an undamped sound wave with amplitude u_0. Since the nonlinear increment γ_A is smaller than γ_0, the establishment of the stationary state is "soft".[6] Consequently, this stationary state is stable.

It is possible to determine in the same manner the stationary amplitude of electromagnetic wave in conductors with two types of carrier.[1, 2]

We present a numerical estimate for a typical method with electron density $n \sim 10^{22}$ cm^{-3} and collision frequency $\nu \sim 10^9$ sec^{-1}. The inequality (12) for the current density j_0 can be represented in the form

$$j_0 > H_0 c\omega / 4\pi s. \tag{17}$$

For $H_0 \approx 10^3$ Oe ($\Omega = 10^{10}$ sec^{-1}) and $\omega \approx 2\pi \times 10^6$ the critical current density (17) is of the order of 10^5 A/cm^2, and the power dissipated per unit volume is of the order of 0.1 W/cm^3. The drift velocity v_0 is then of the order of several cm/sec.

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